



Characterization of micro-/nano- structures

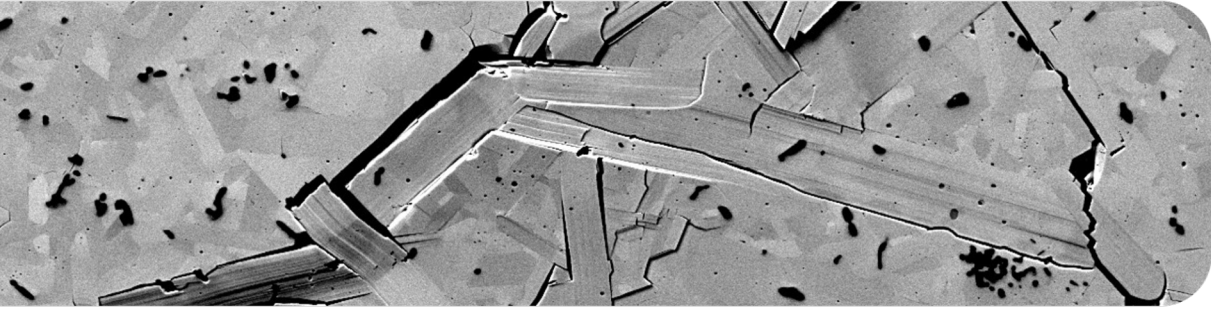
www.antoine-guitton.fr

Prof Antoine GUITTON

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antoine.guitton@univ-Lorraine.fr

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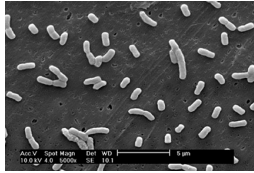
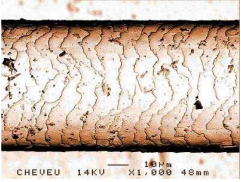
- ❖ Chapter #1: Some micro-/nano-structural features
- ❖ Chapter #2: Characterization by X-rays diffraction
- ❖ Chapter #3: Electron microscopy
- ❖ Chapter #4: Near-field scanning microscopy
- ❖ Chapter #5: Atom Probe Tomography (APT)



Some micro-/nano-structural features

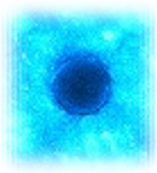
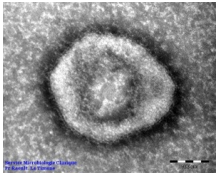
Some orders of magnitude

❖ Micrometer ($1 \mu\text{m} = 0.000001 \text{ m} = 10^{-6} \text{ m}$)

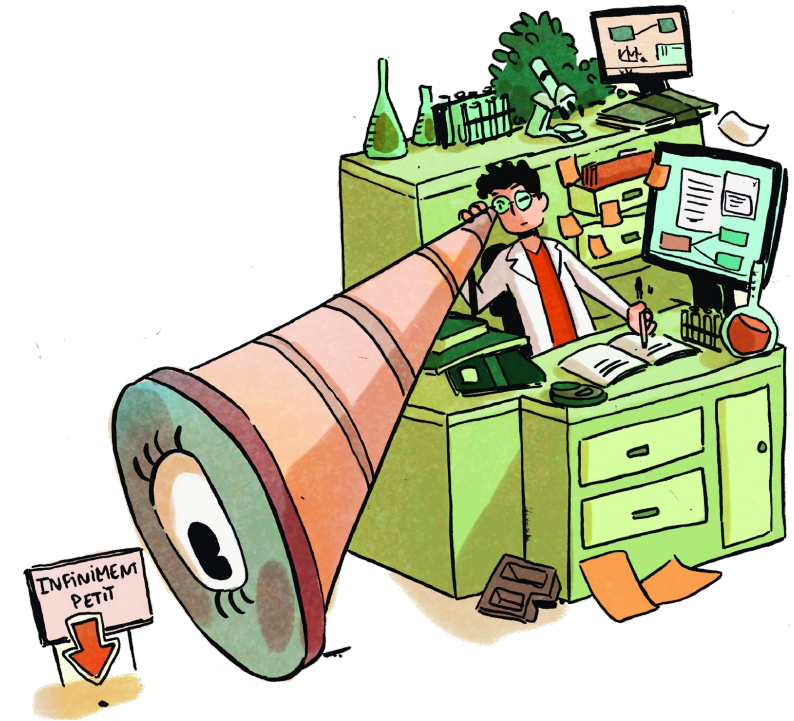


- Hair diameter = 50-100 μm
- Size of a bacteria = 0.1-10 μm

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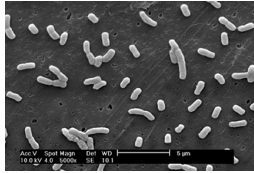
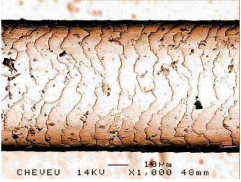


- Size of a virus = 20-450 nm



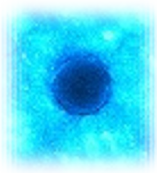
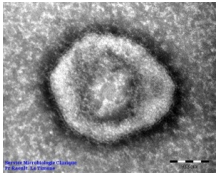
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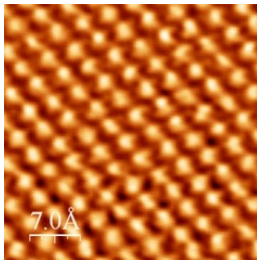
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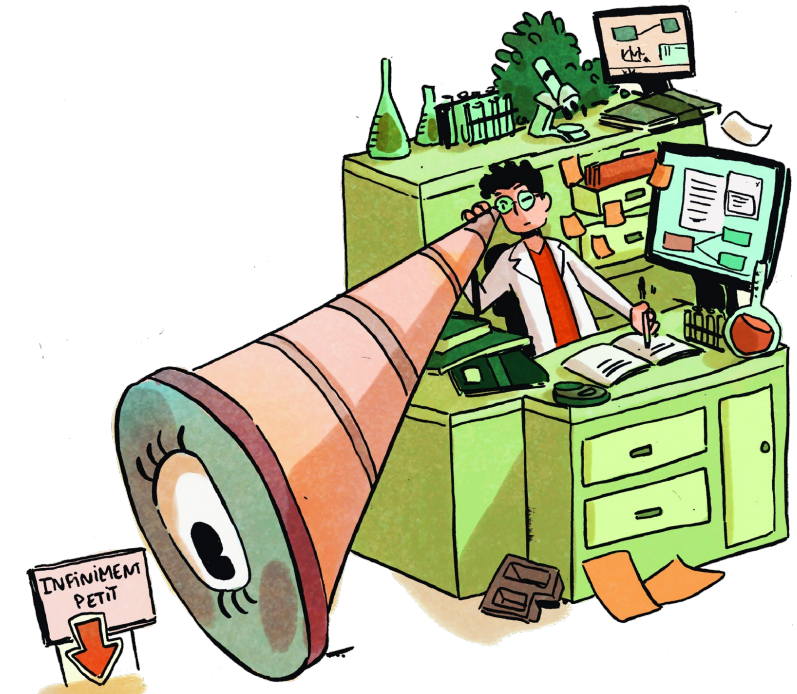


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❖ Atomic scale ($1 \text{ \AA} = 0.0000000001 \text{ m} = 10^{-10} \text{ m}$)

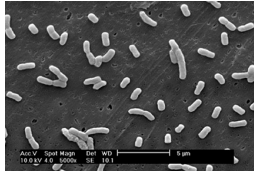
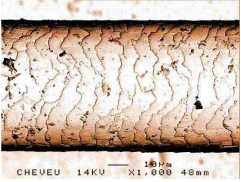


- Distance between atoms = 0.2 nm = 2 \AA
- Atom radius = 0.1 nm = 1 \AA



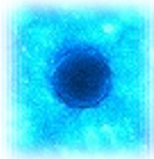
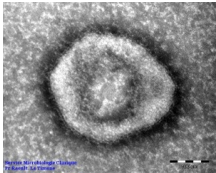
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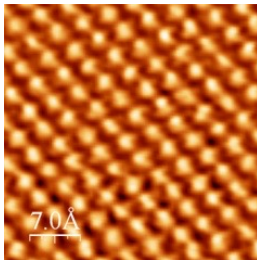
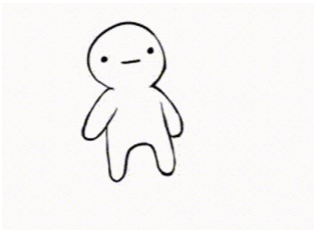
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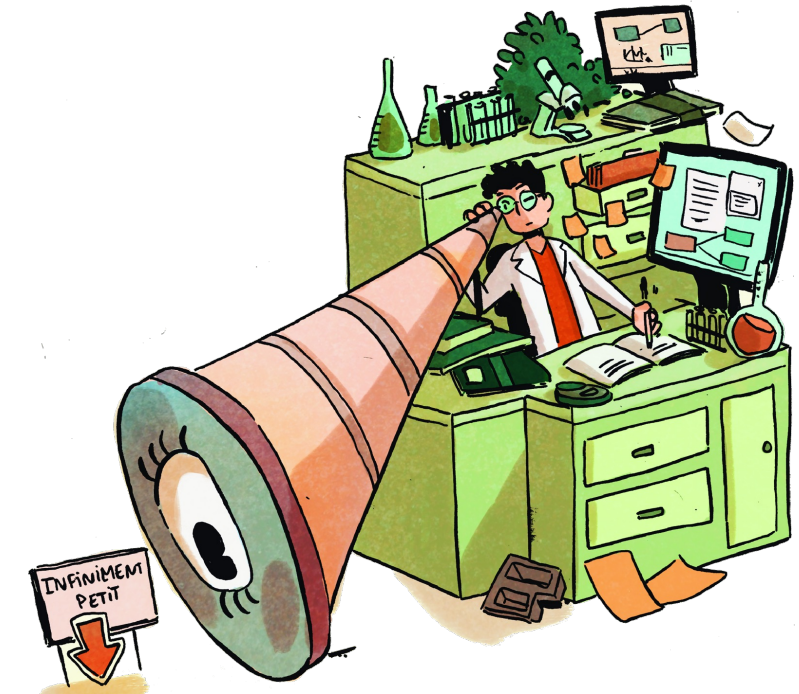


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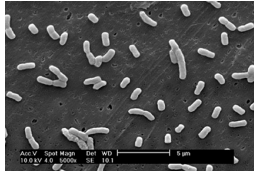
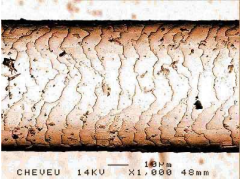
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This is neither copper nor gold!!!

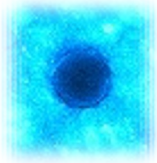
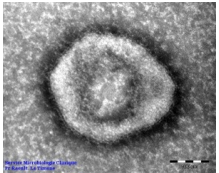
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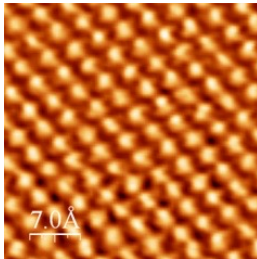
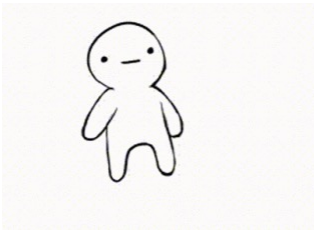
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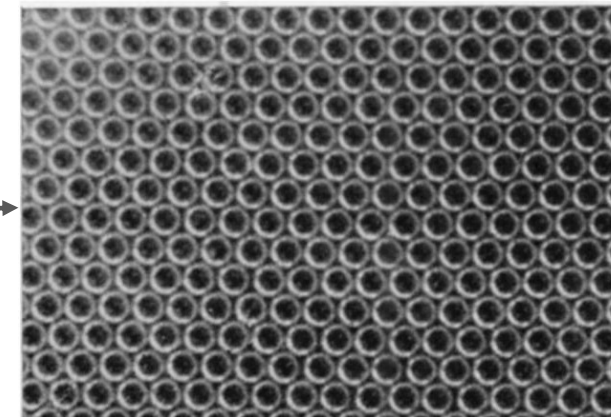
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Stacking of spheres

L. Bragg, J.F. Nye, *Proc. R. Soc. Lond.*, 1947

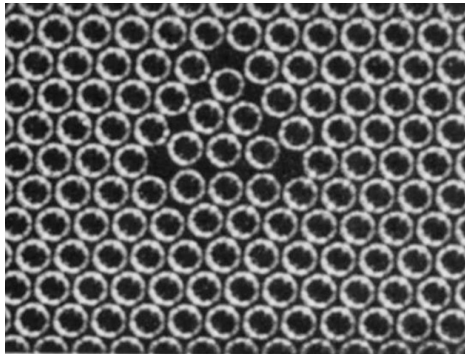


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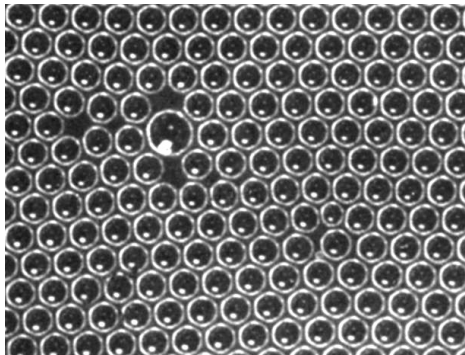
Defects in crystalline materials

0 dimension

Vacancy



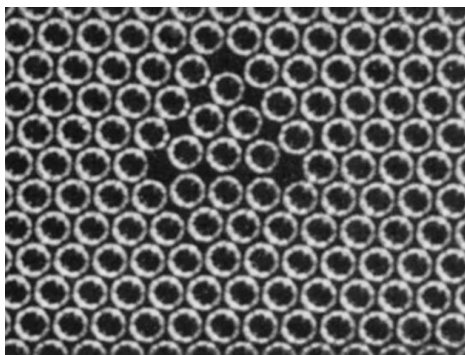
Interstitial



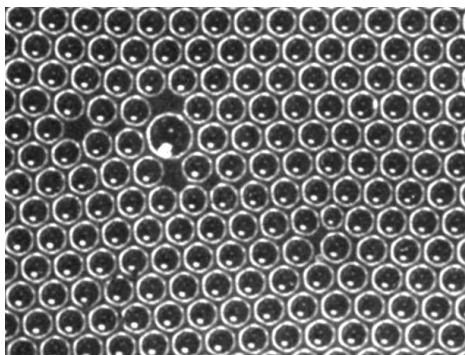
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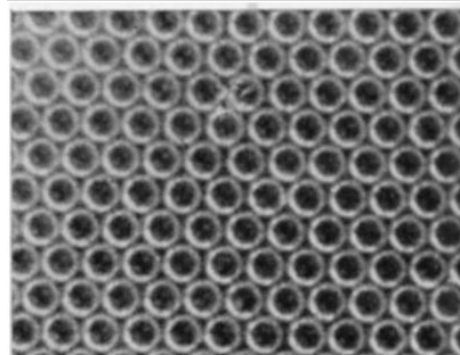


Interstitial



1 dimension

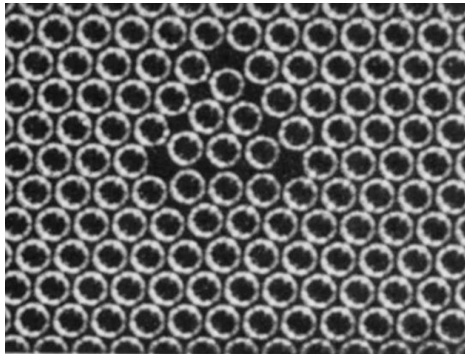
Dislocation



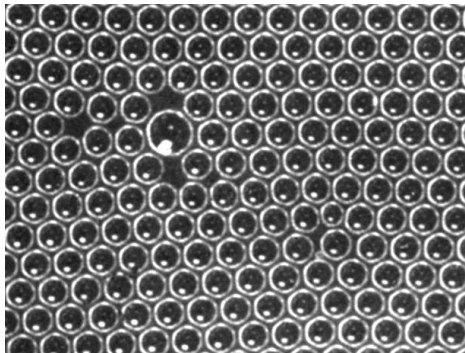
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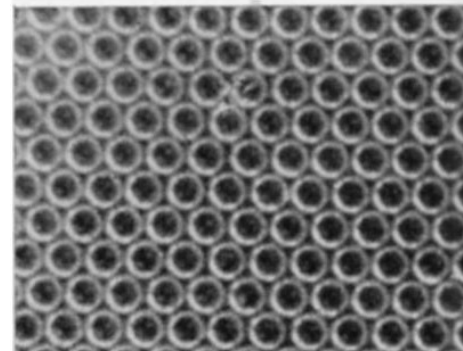


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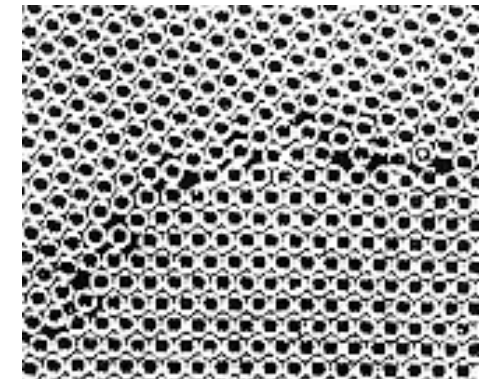


2 dimensions

Stacking fault



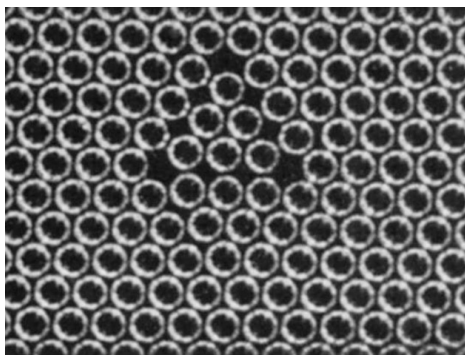
Grain boundaries



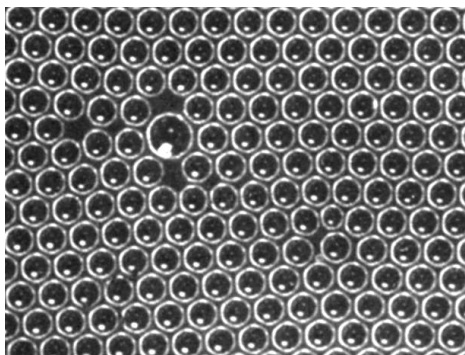
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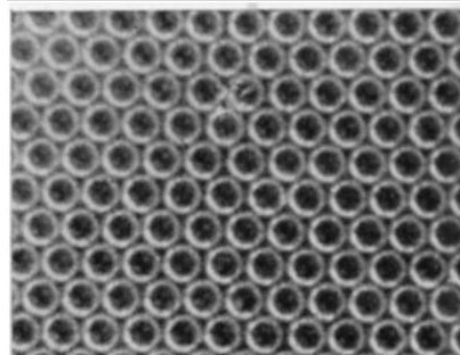


Interstitial



1 dimension

Dislocation



3 dimensions

Inclusions, precipitates...

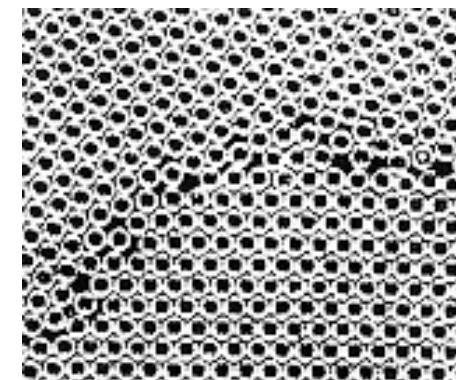
"Crystals are like people; it is the defects in them which tend to make them interesting!"
(Colin J. Humphreys)

2 dimensions

Stacking fault

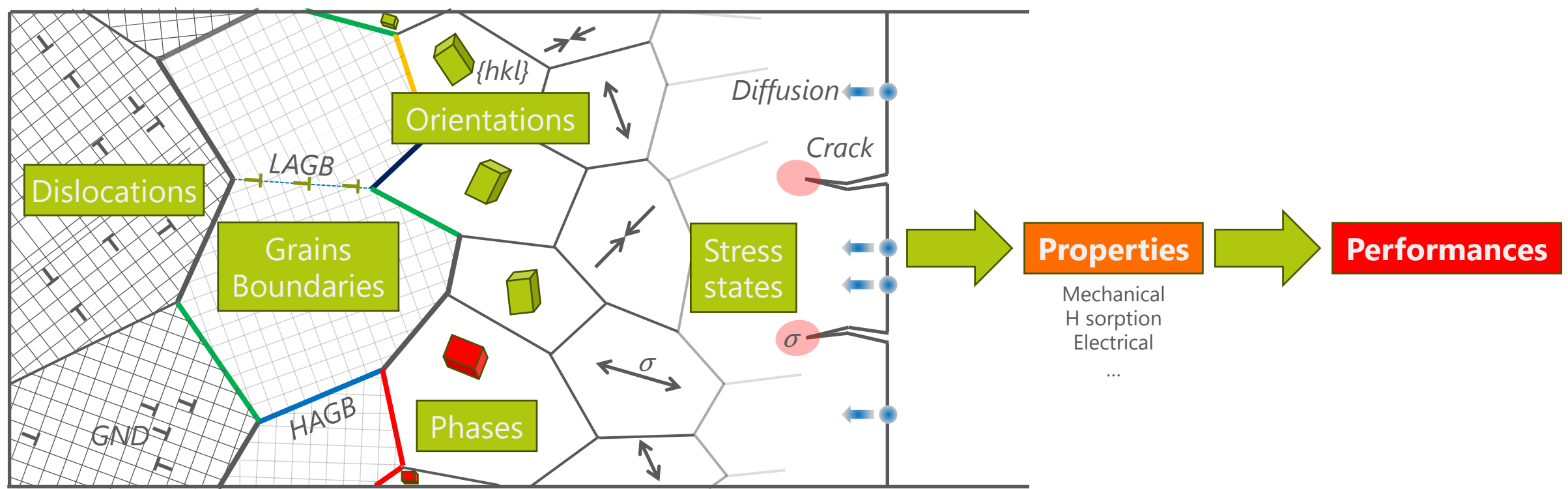


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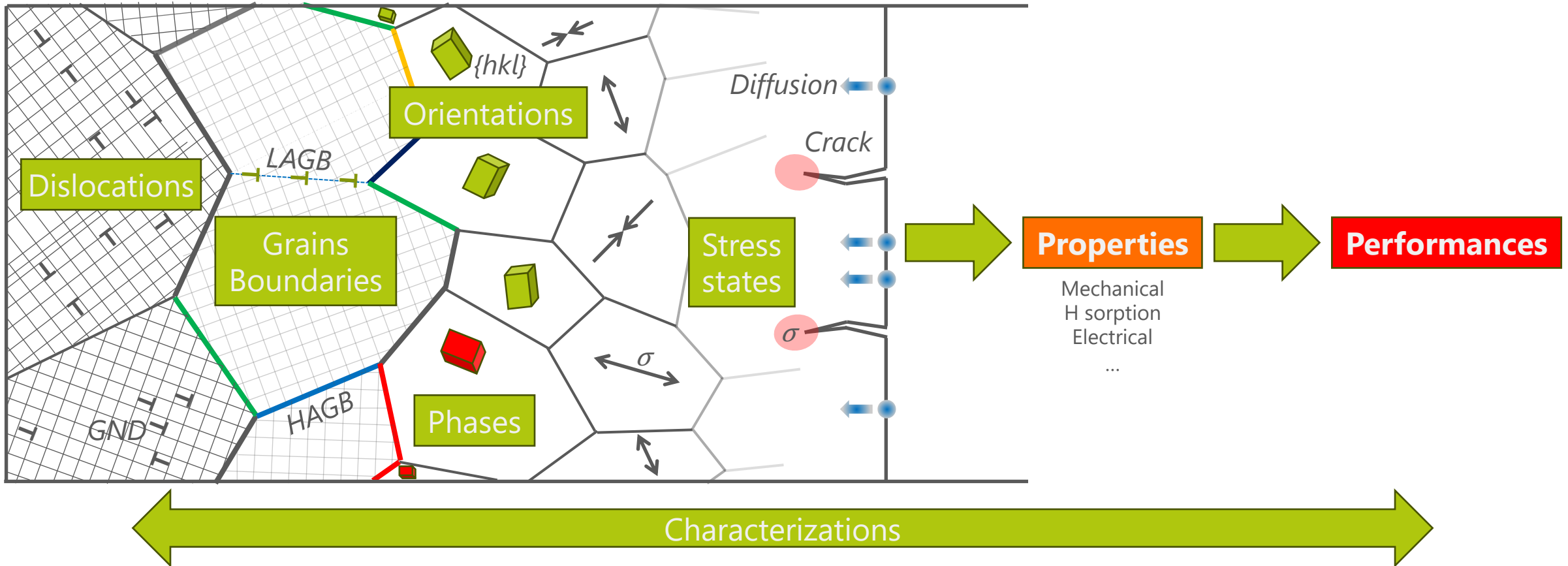
Micro-/nano-structures

Microstructure (~1 μm → 1 mm) and nanostructure (~1 nm → 100 nm) refer to the internal features of a material, at the micrometer and nanometer scales respectively, influencing its properties

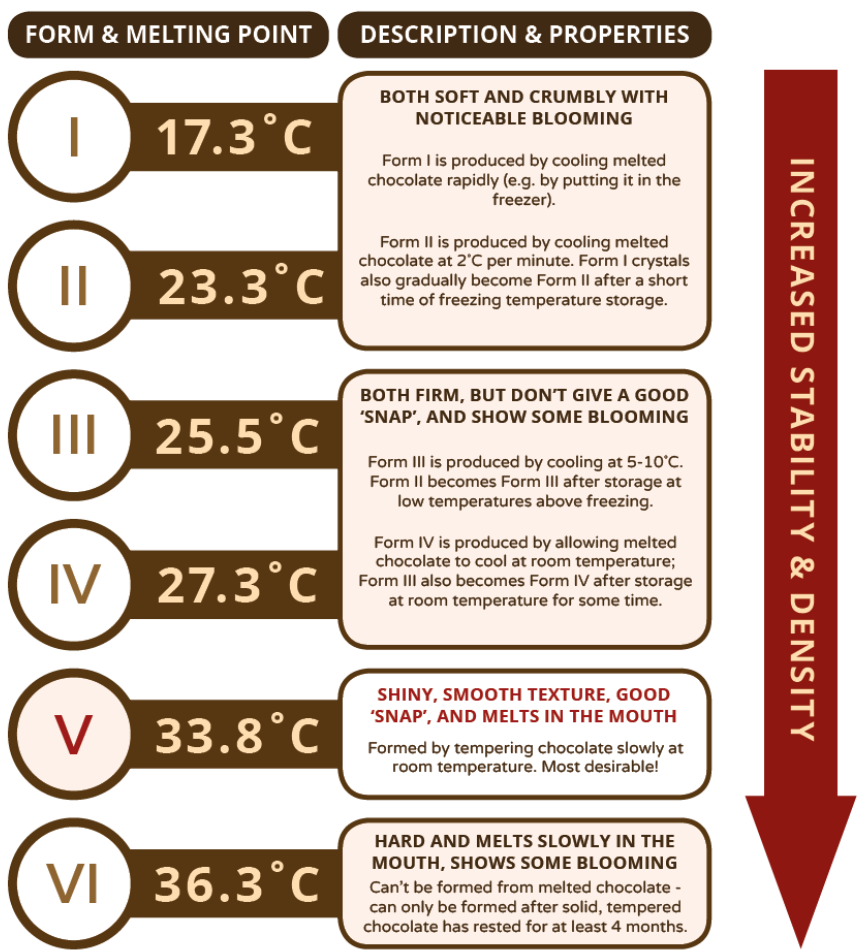


Micro-/nano-structures

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Phases



❖ Definition:

- Distinct physical states of a material with uniform composition and structure.
- Example:
 - ✓ cocoa butter polymorphism: exists in six polymorphic forms (I to VI), each with different melting points and stability.
 - ✓ Most desirable phase: Form V (β') – gives chocolate its smooth texture, glossy finish, and ideal snap.

❖ Phase transitions:

- Heating and cooling control polymorphic transformation

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❖ Phase diagram:

- Graphically represents phase stability as a function of temperature and composition.

❖ Tie line (isothermal line):

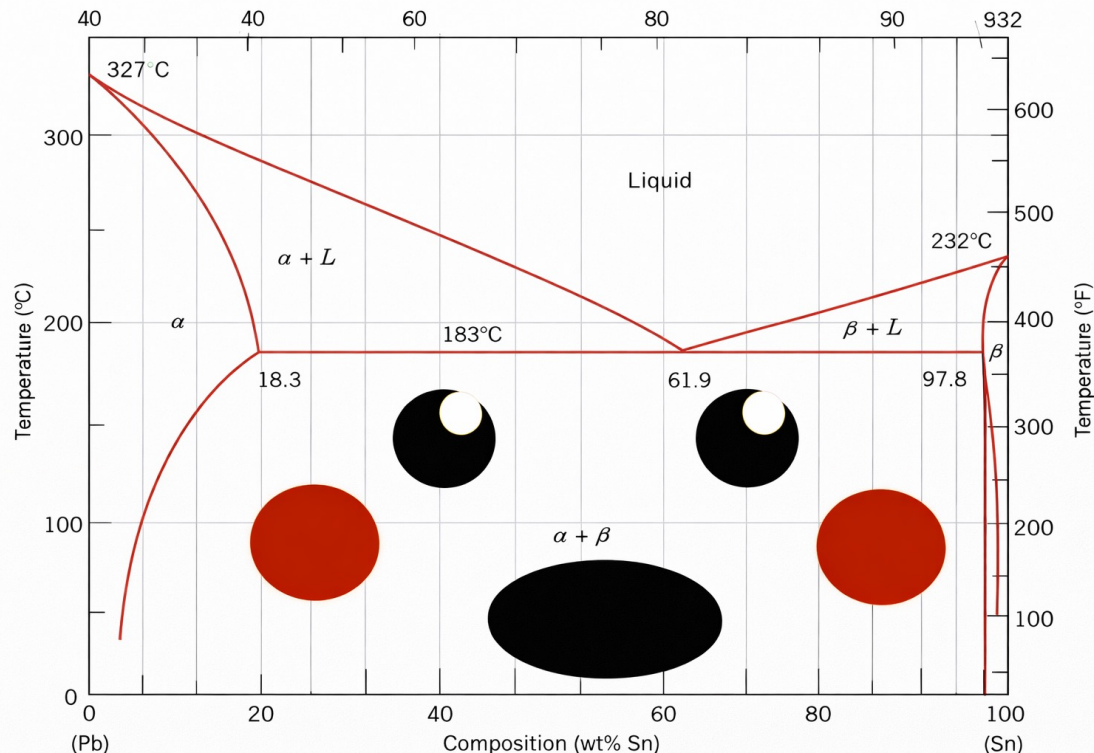
- A horizontal line drawn in a two-phase region to determine the composition of coexisting phases.

❖ Lever rule:

- Used in two-phase regions to determine phase fractions; applies to both metals and chocolate crystallization.

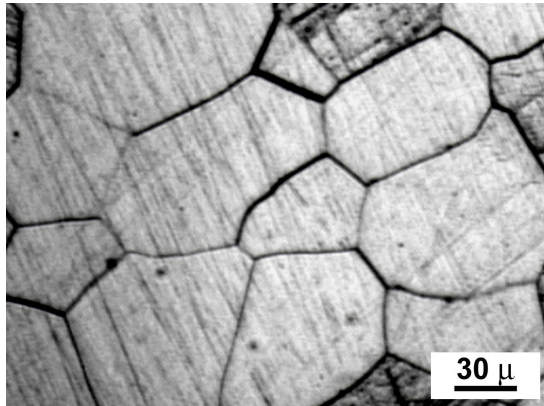
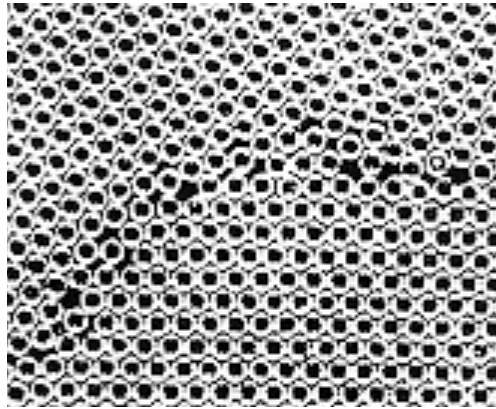
❖ Importance in processing:

- Helps control microstructure, texture, and mechanical properties in both food and engineering materials.



Grains

L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)



❖ Definition:

- Grains are individual crystallites in a polycrystalline material, each with a distinct crystallographic orientation.
- Can range from nanometers (nanocrystalline materials) to millimeters (coarse-grained materials).

❖ Description:

- Shape
- Size

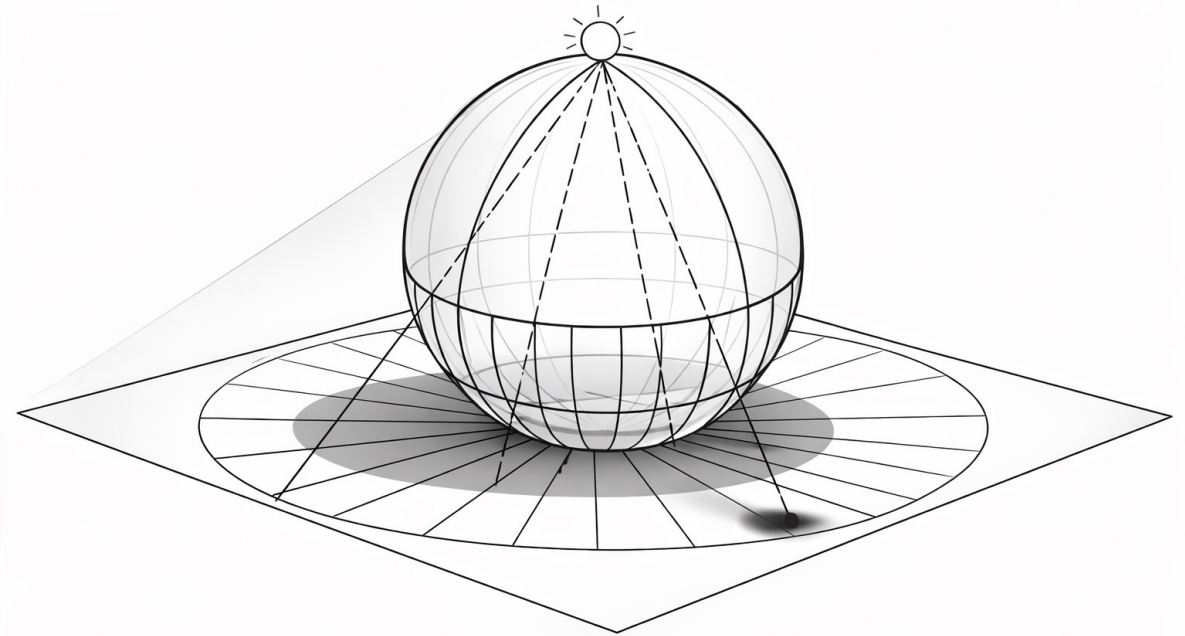
❖ Texture:

- Texture refers to a preferred orientation of grains in a polycrystalline material.

❖ Light and shadow analogy:

- A sphere is illuminated by a point light source
- The light source is placed at the north pole of the sphere
- Each point on the sphere is connected to the light by a straight ray
- The ray intersects a flat plane and creates a shadow point

↪ All shadow points form the stereographic projection



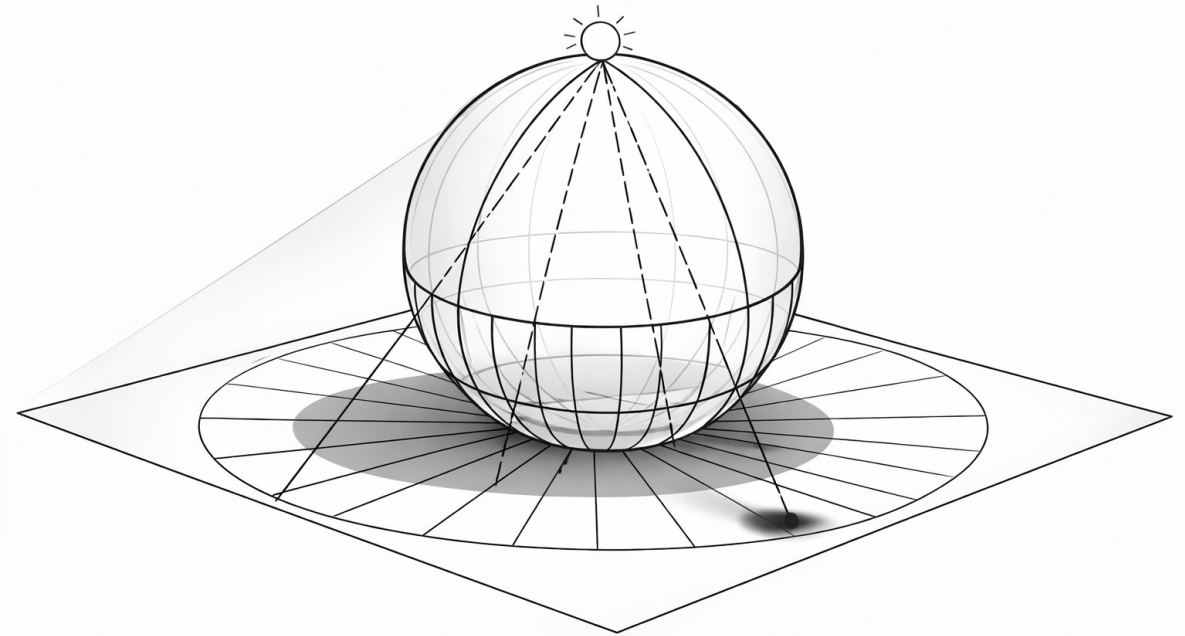
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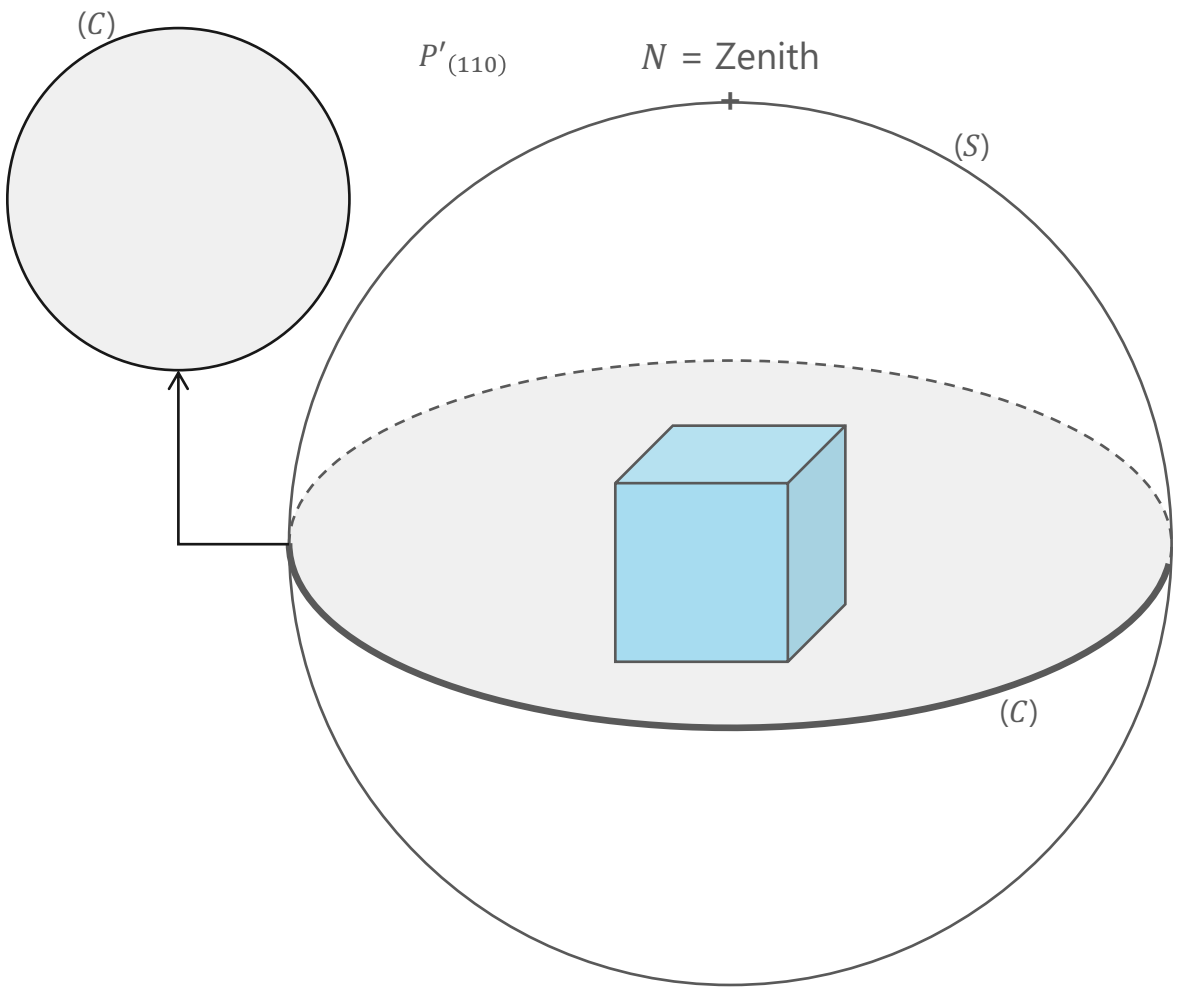
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❖ Key geometric property:

- Angles between directions are preserved
- Lengths and areas are not preserved

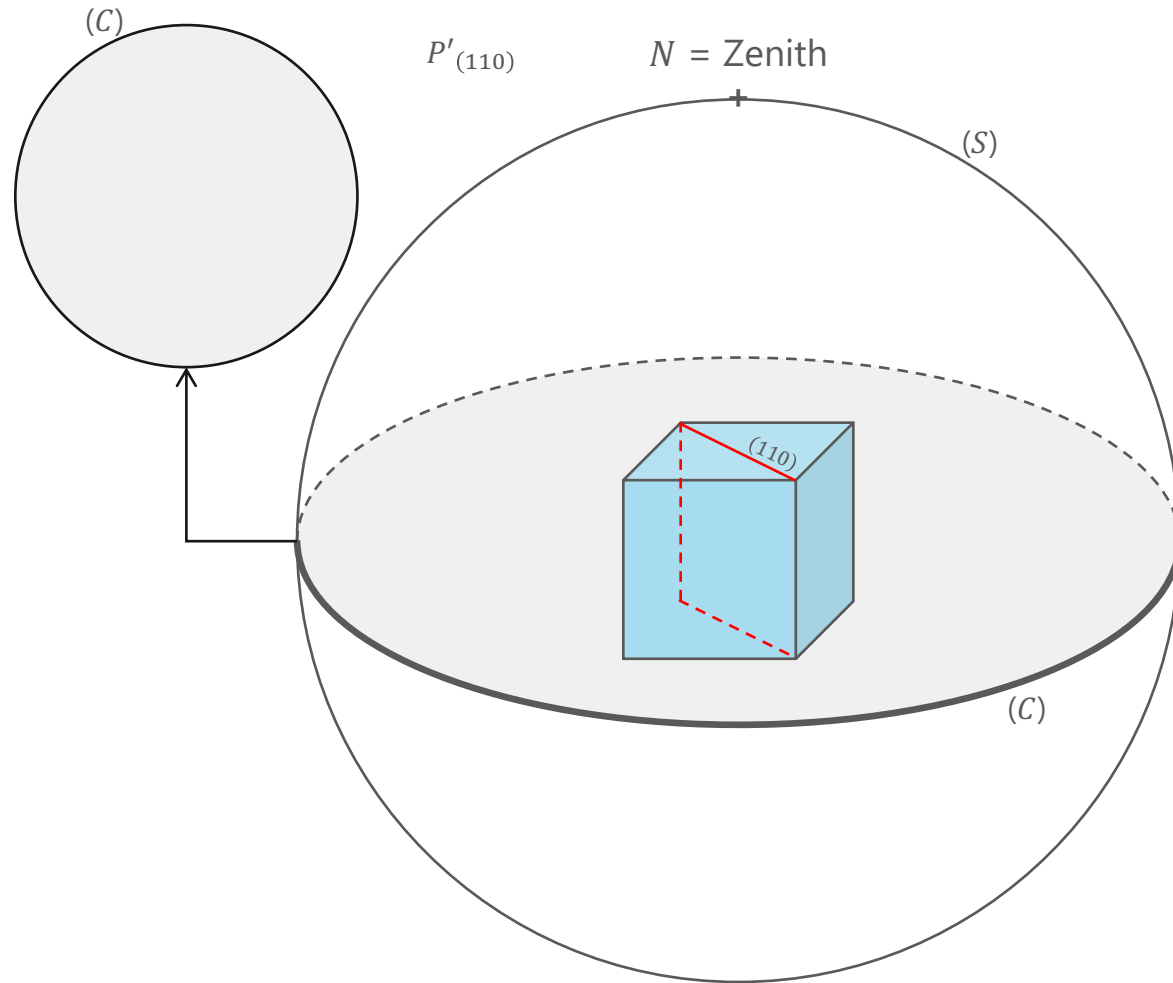


Stereographic projection of a plane via its normal



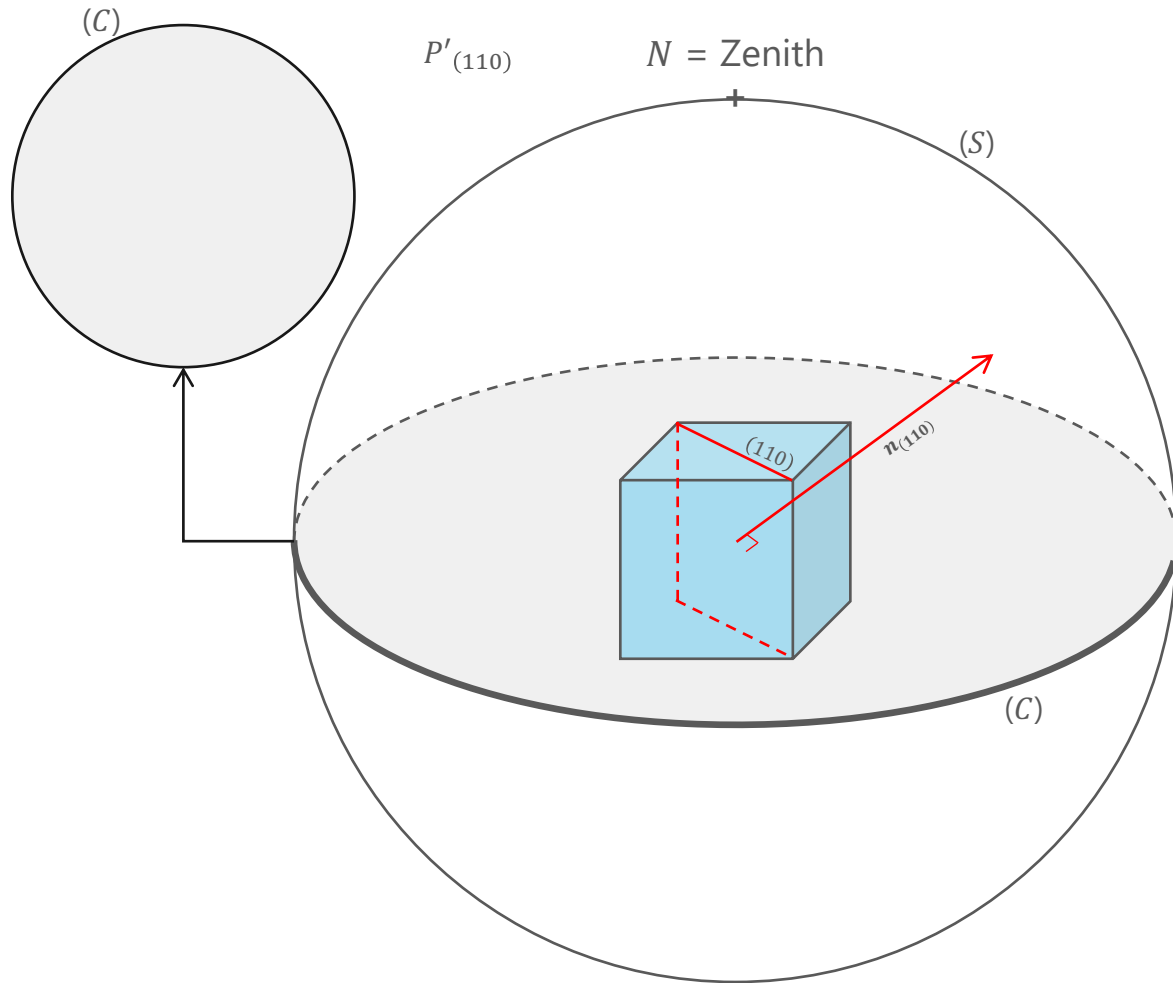
❖ Consider the crystal placed at the center of a reference sphere (S) .

Stereographic projection of a plane via its normal



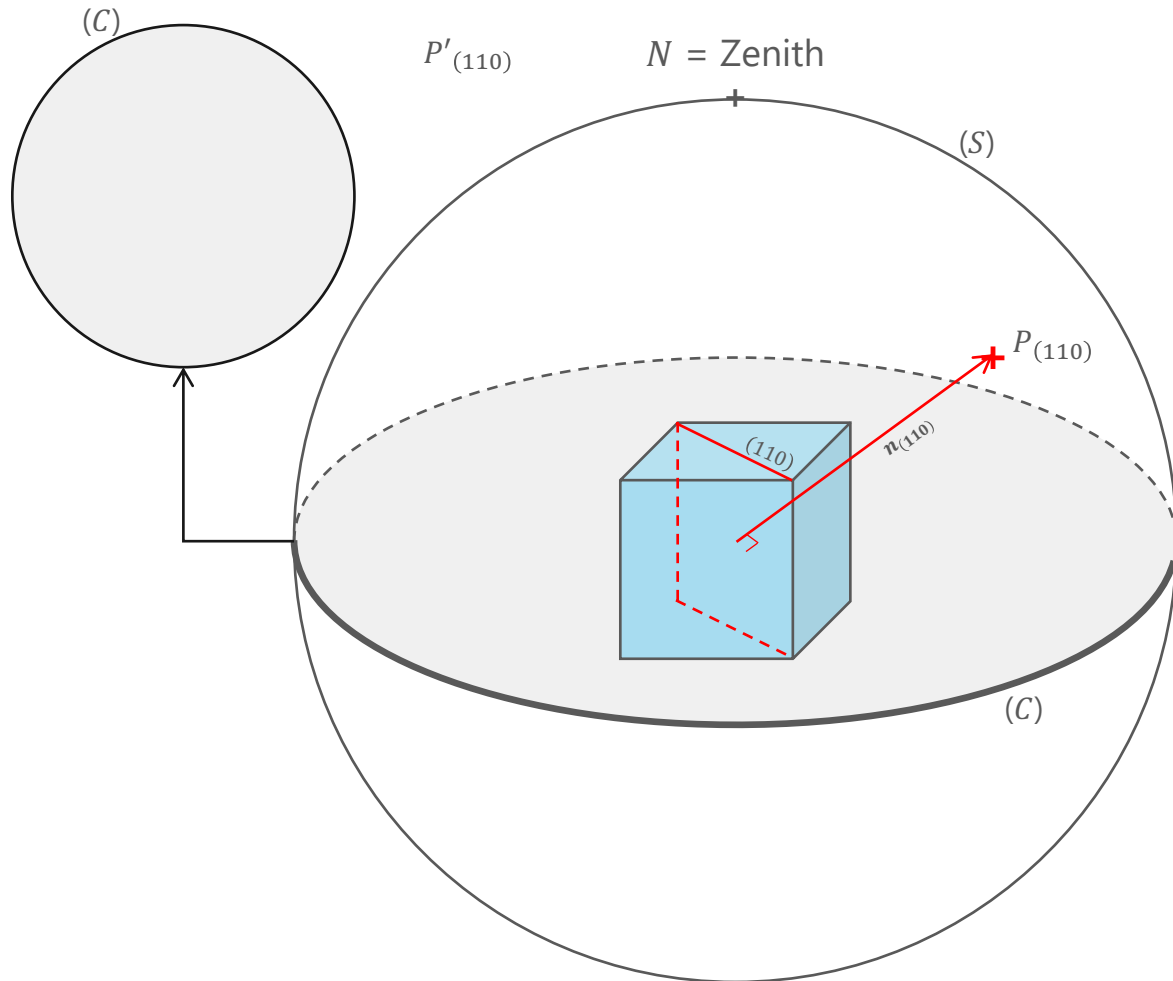
- ❖ Consider the crystal placed at the center of a reference sphere (S).
- ❖ Identify the crystallographic plane of interest, e.g., the (110) plane.

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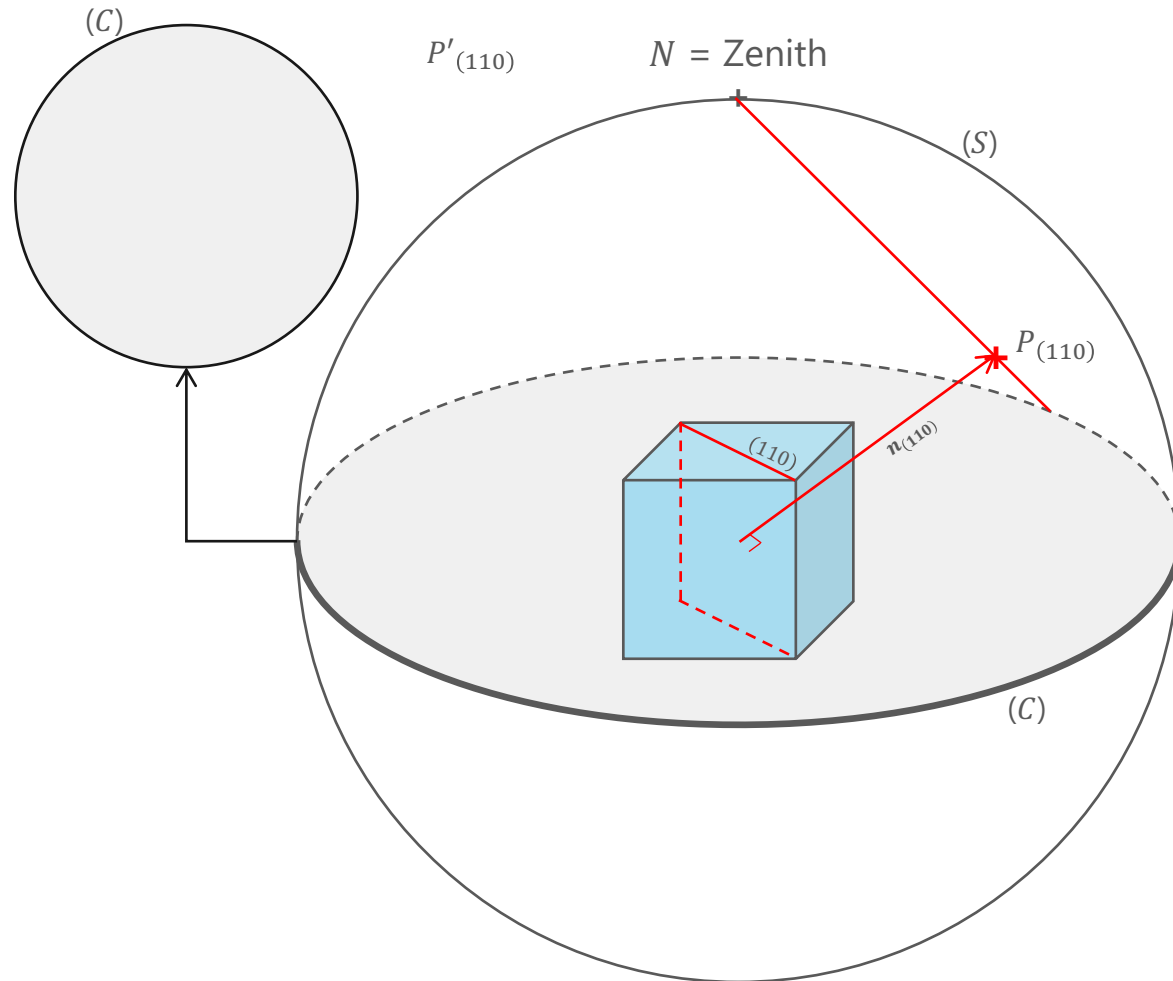
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- ❖ This normal intersects the sphere at a point $P_{(110)}$ on its surface.

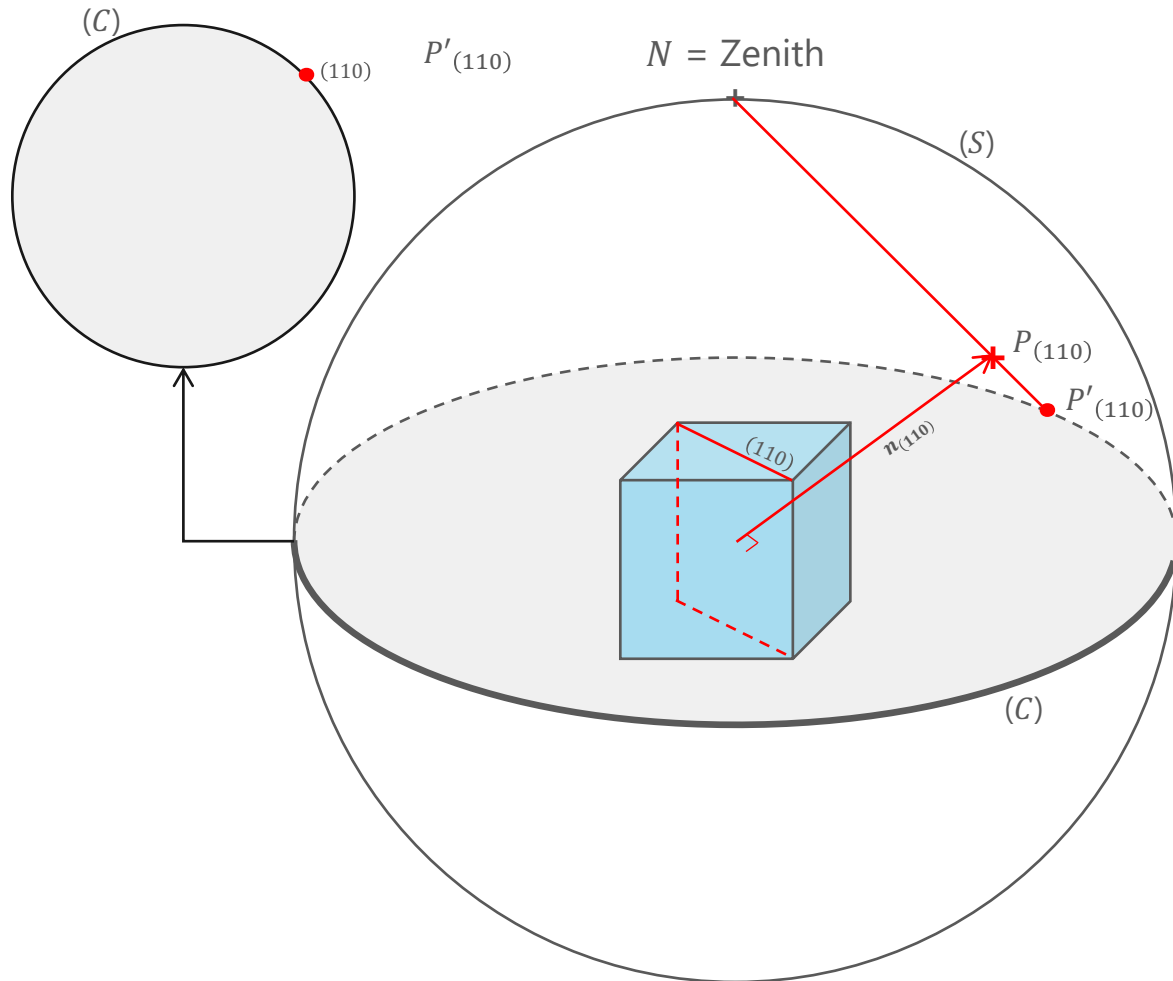
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- ❖ Project point $P_{(110)}$ onto the equatorial plane (C) along a straight line from the North pole (N) of the sphere.

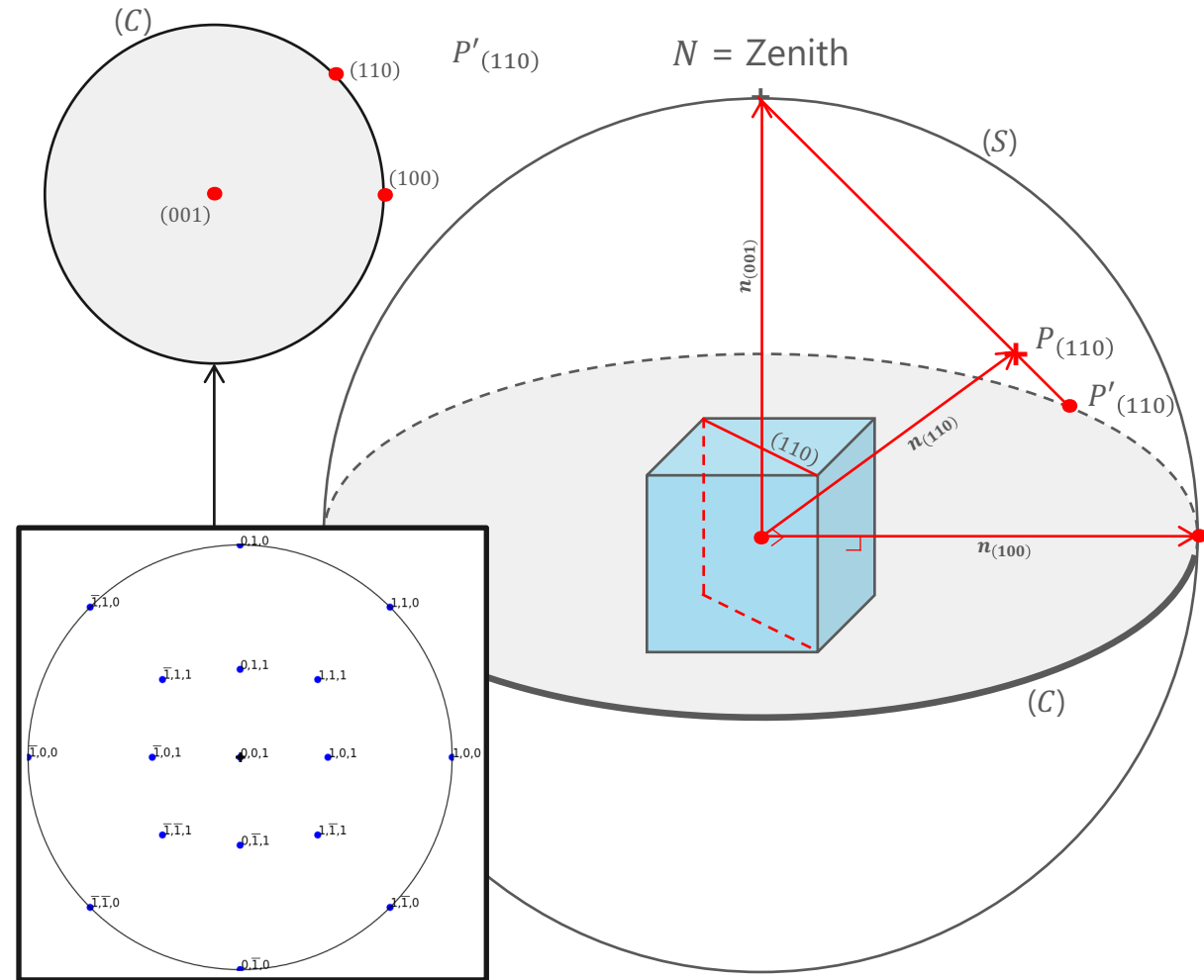
↪ The North pole is used as the projection point
 ↪ Any direction in the upper hemisphere is inverted ($\mathbf{n} \rightarrow -\mathbf{n}$) before projection.

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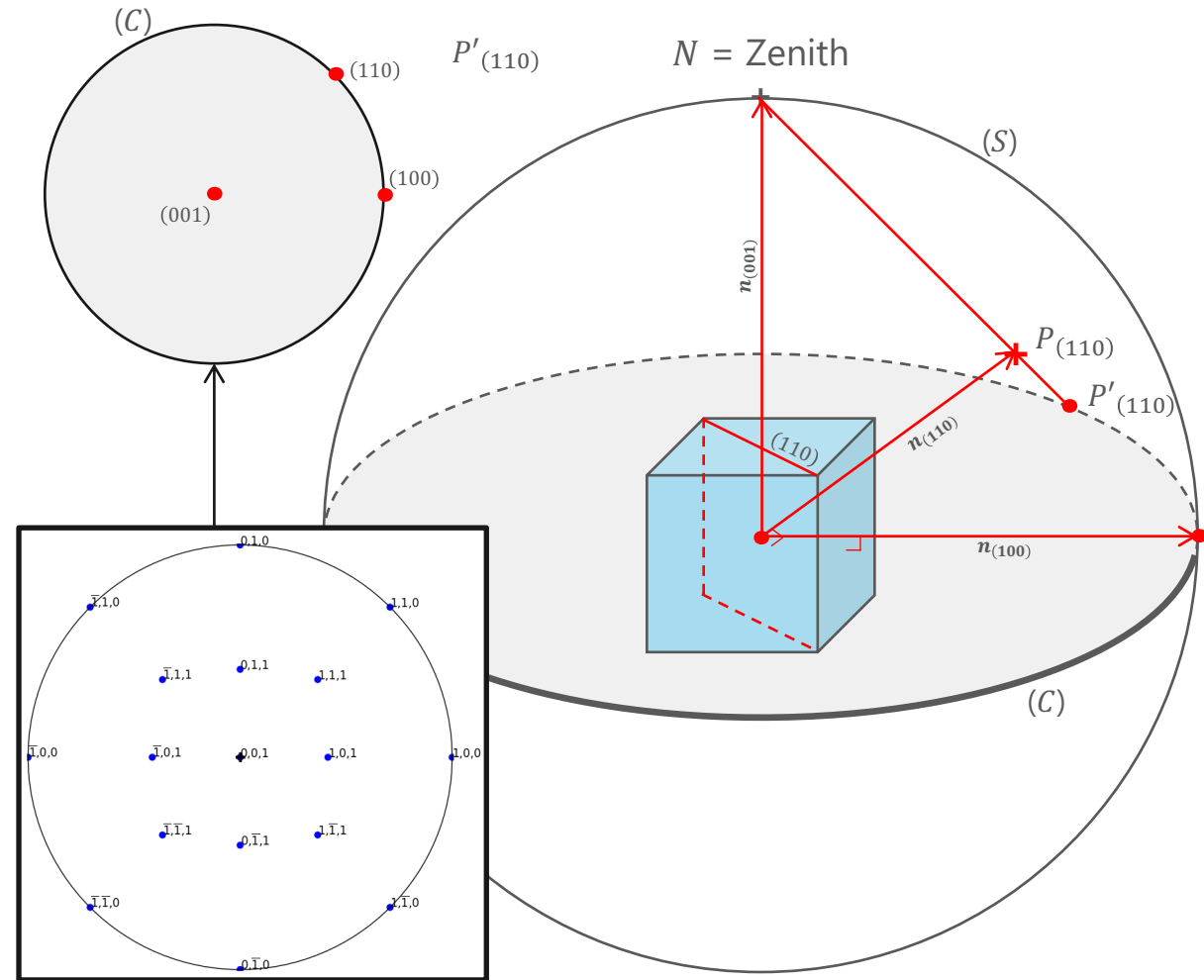
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- ❖ Repeat this process for each crystallographic plane or direction of interest.

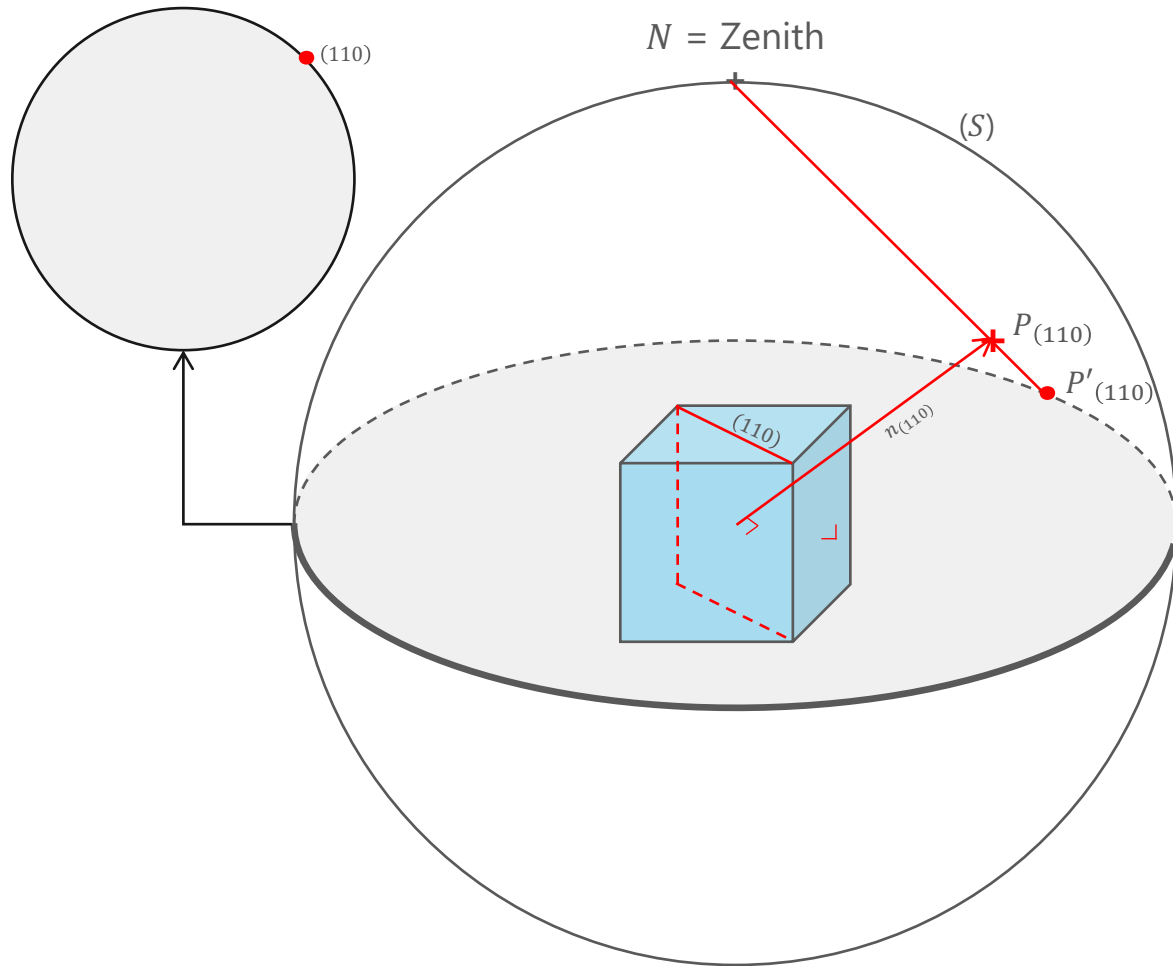
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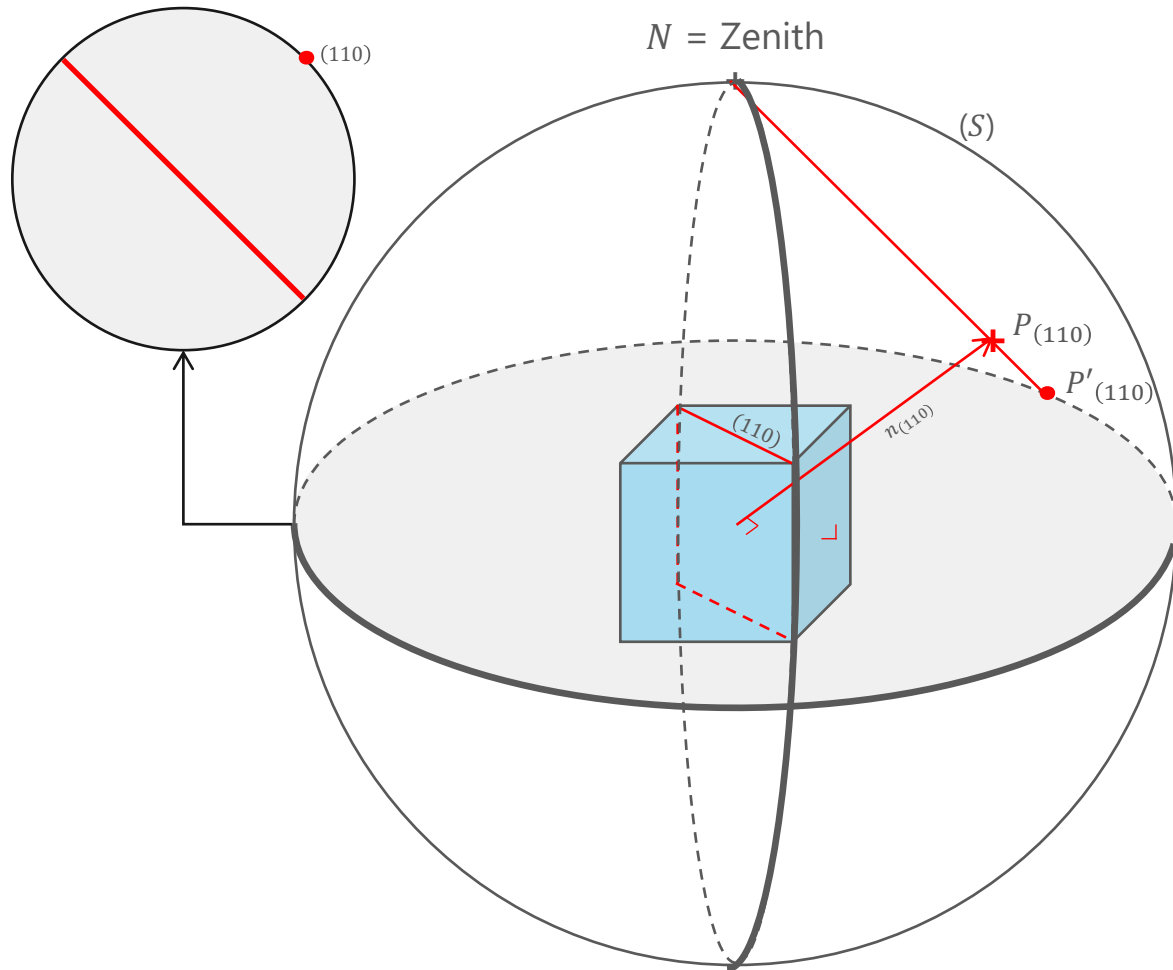
↪ Each crystallographic direction is represented by a single point on the stereographic projection

Stereographic projection of a plane via its trace



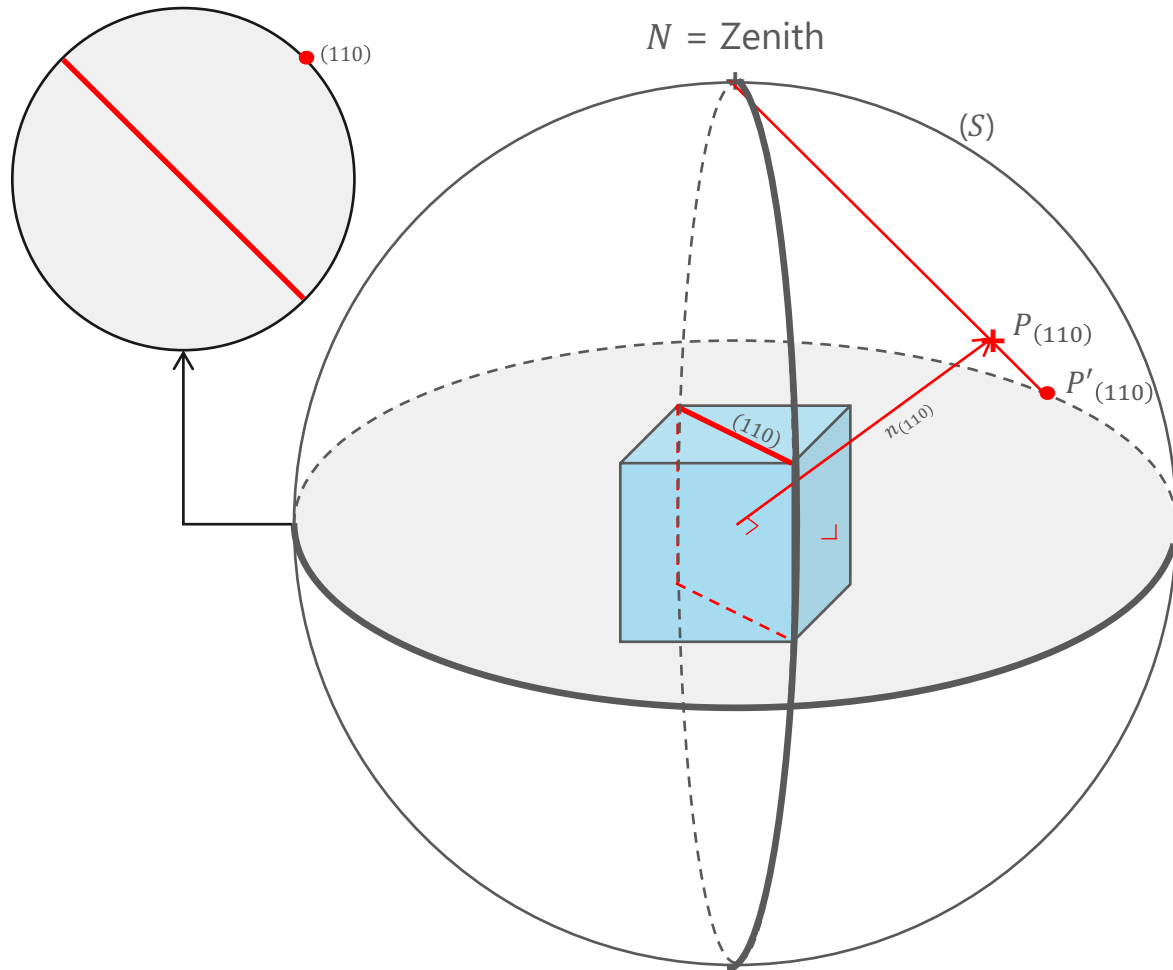
- ❖ Place the crystal at the center of the reference sphere (S) .
- ❖ Identify the (110) crystallographic plane within the crystal.

Stereographic projection of a plane via its trace



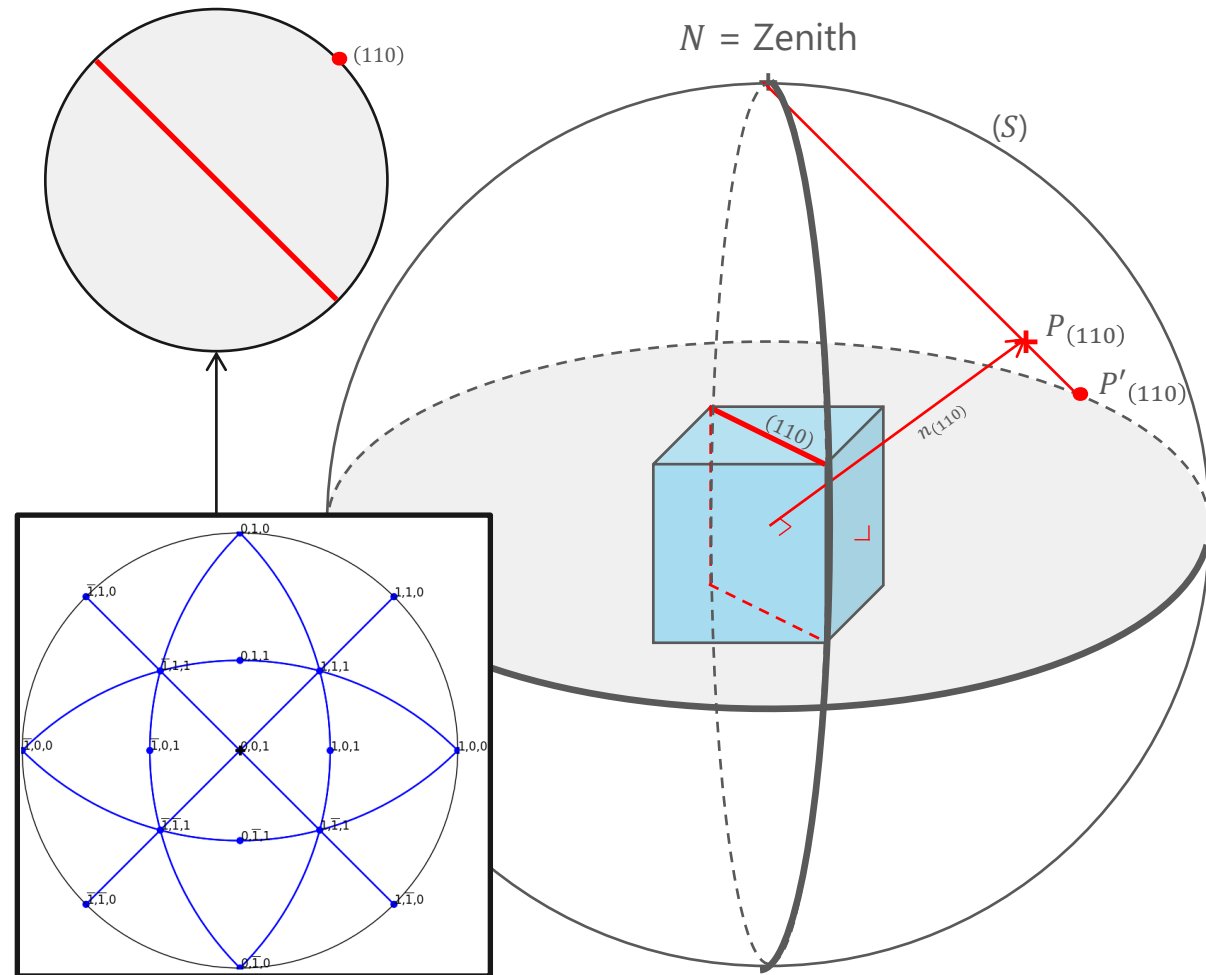
- ❖ Place the crystal at the center of the reference sphere (S).
- ❖ Identify the (110) crystallographic plane within the crystal.
- ❖ Imagine the intersection between this plane and the sphere: this defines a great circle on the sphere's surface.
- ❖ This great circle represents all directions lying within the (110) plane.

Stereographic projection of a plane via its trace



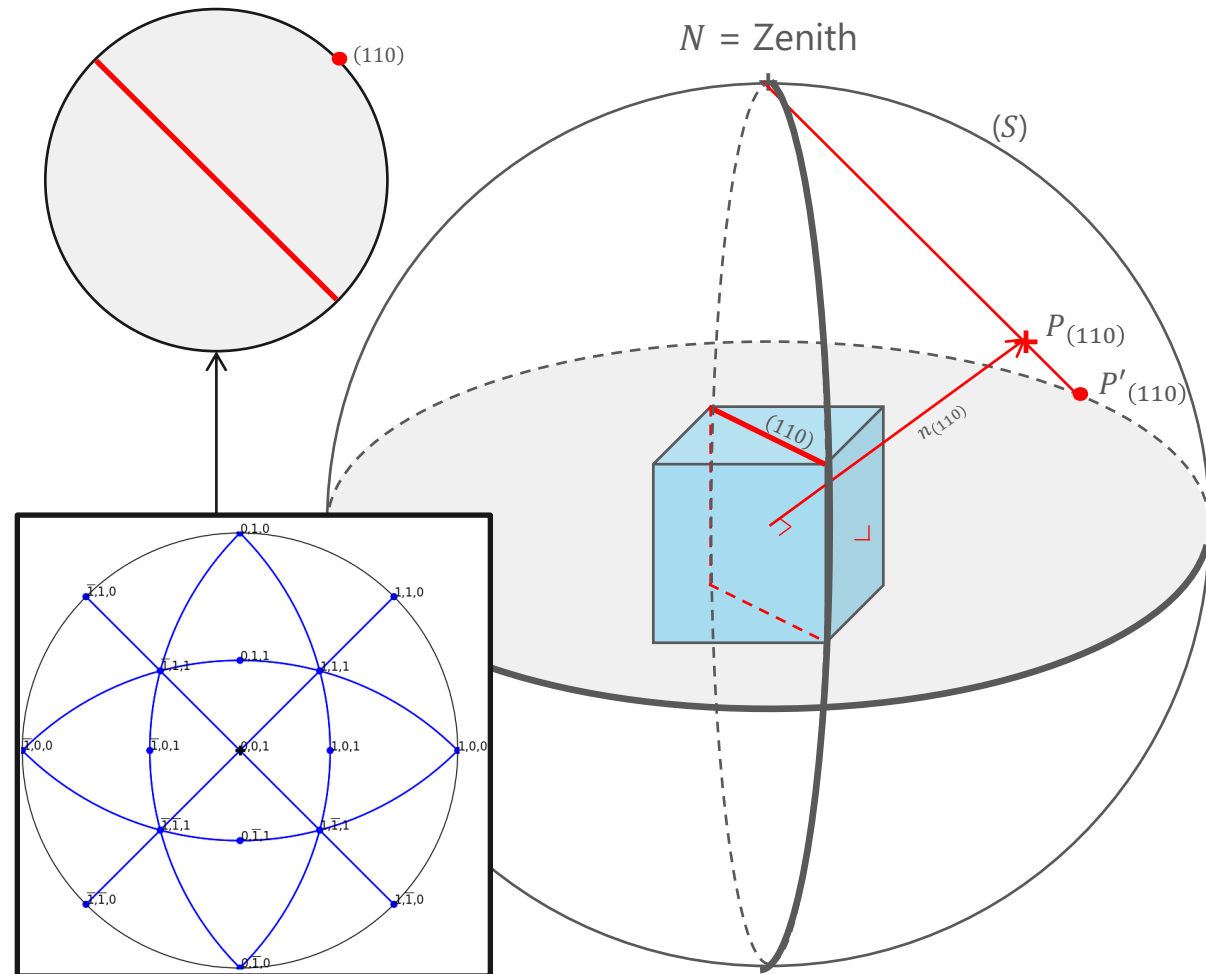
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- ❖ Identify the (110) crystallographic plane within the crystal.
- ❖ Imagine the intersection between this plane and the sphere: this defines a great circle on the sphere's surface.
- ❖ This great circle represents all directions lying within the (110) plane.
- ❖ This great circle intersects the equatorial disk (C) as a straight line or arc of a circle, depending on its orientation.

Stereographic projection of a plane via its trace



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- ❖ Repeat this process for each crystallographic plane

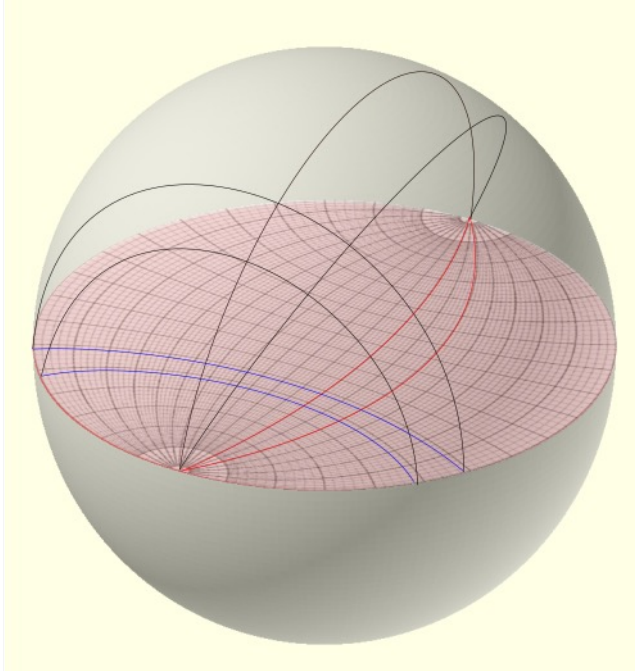
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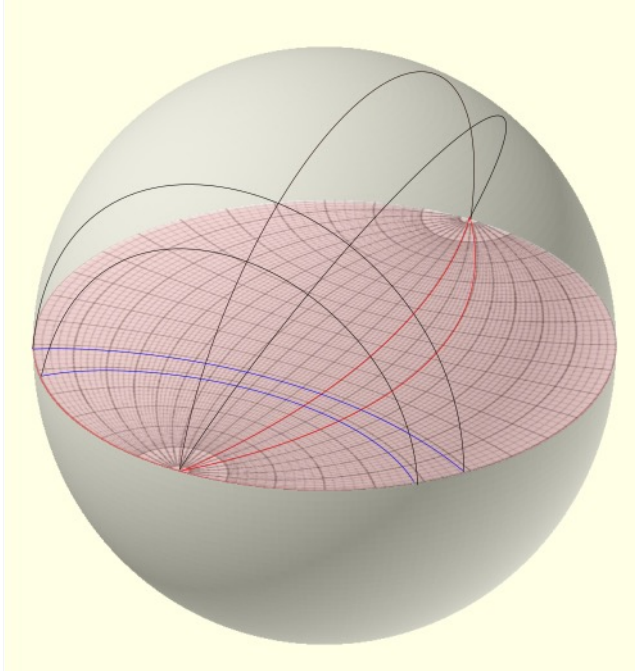
↪ The great circle trace shows all directions lying in the plane on the 2D projection.

The Wulff net



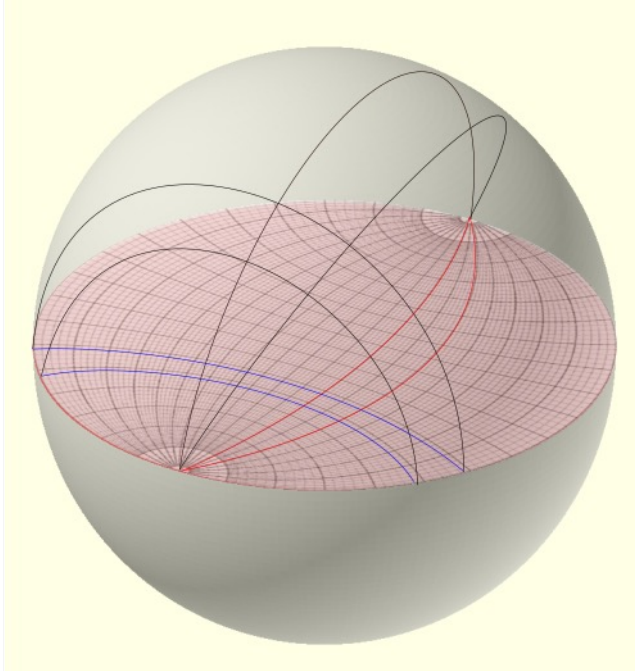
- ❖ The Wulff net is built using a stereographic projection of a reference sphere.

The Wulff net



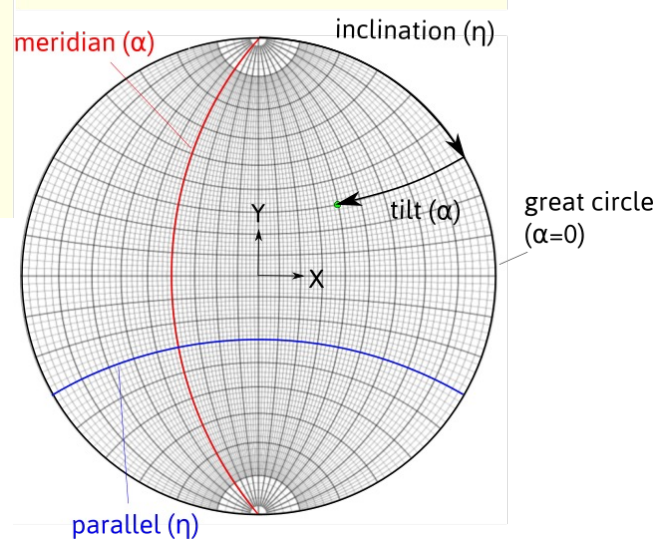
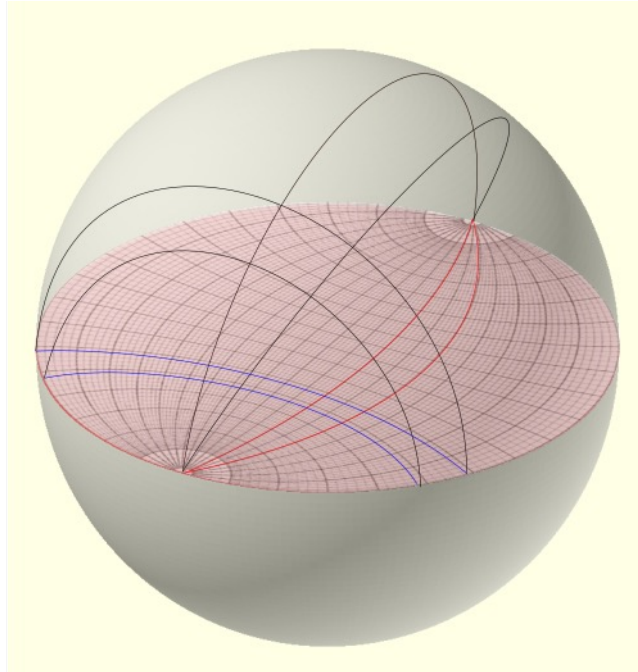
- ❖ The Wulff net is built using a stereographic projection of a reference sphere.
- ❖ It consists of a grid made up of two families of planes:
 - **Meridians:** represent planes with a constant azimuthal (rotation around the vertical axis) angle; they appear as straight lines radiating from the center.
 - **Parallels:** represent planes with a constant inclination angle from the vertical axis; they appear as concentric arcs or circles.

The Wulff net



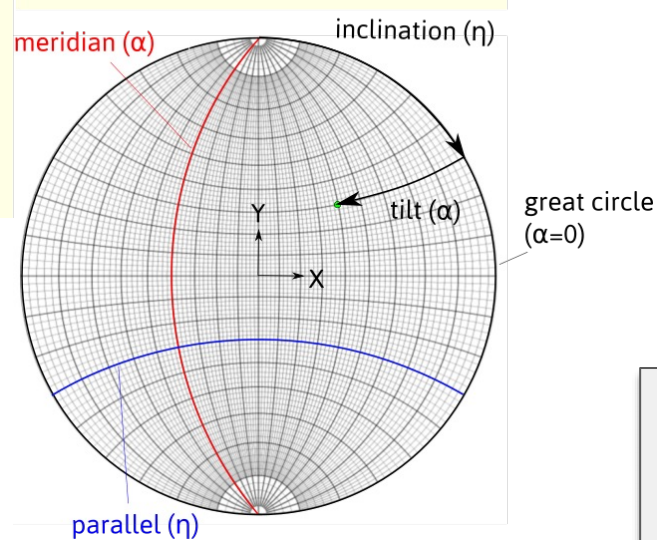
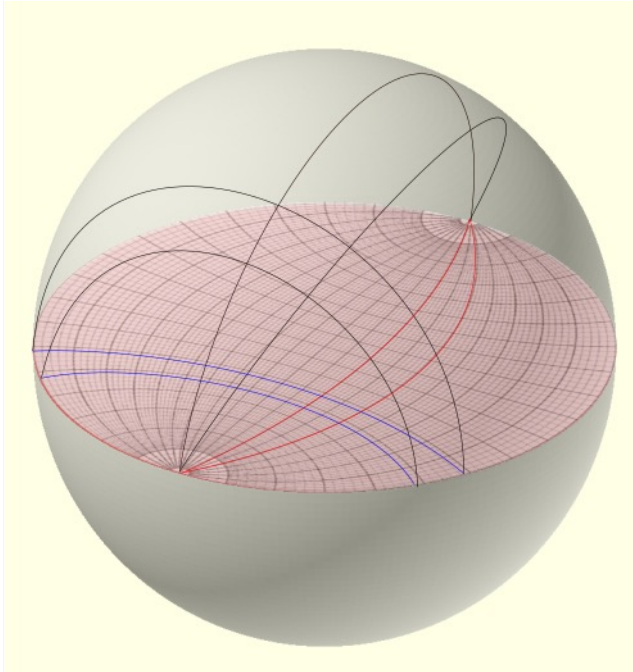
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The Wulff net



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- ❖ The stereographic projection transforms them into the curved grid pattern of the Wulff net.

The Wulff net

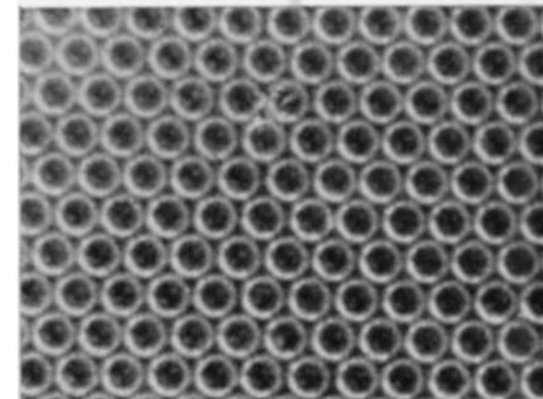
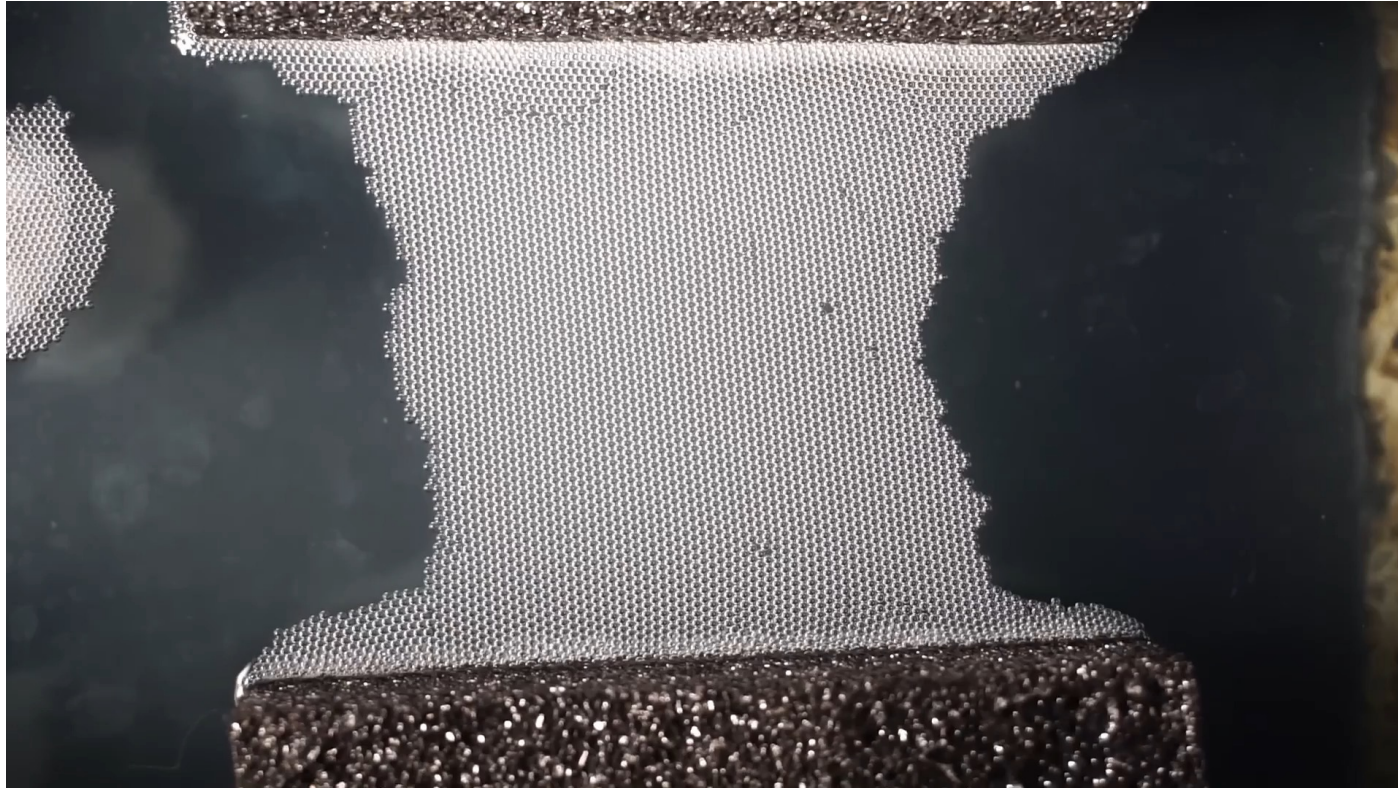


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↪ The Wulff net serves as a reference tool to measure angles and construct stereographic projections accurately.

Dislocations

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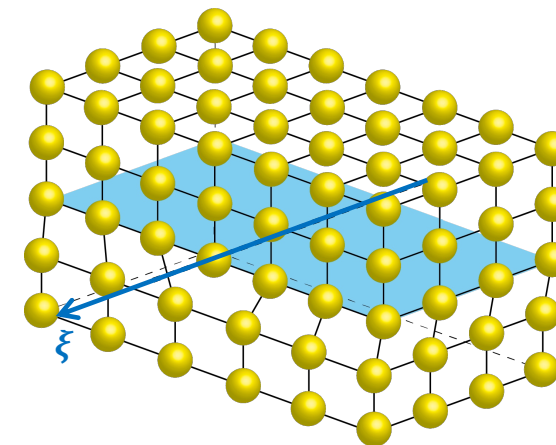
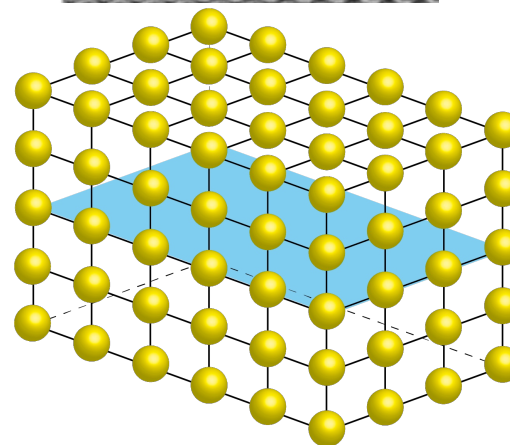
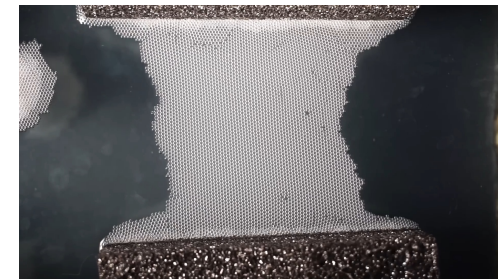
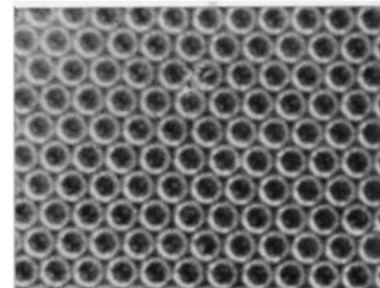


Dislocations

❖ Definition:

- Line defects in a crystalline material where atoms are misaligned.
- They represent a discontinuity in the crystal lattice that allows deformation to occur at lower stresses.
- Characterized by b and ξ .

L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)

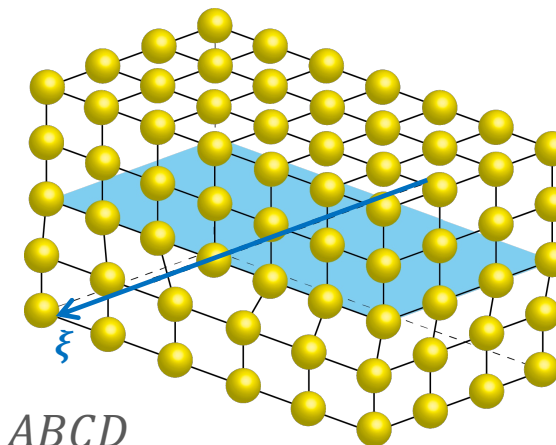
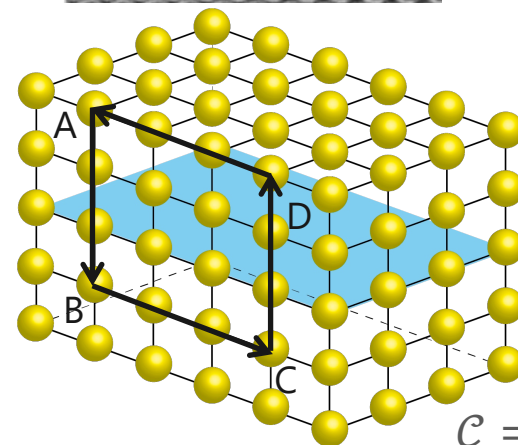
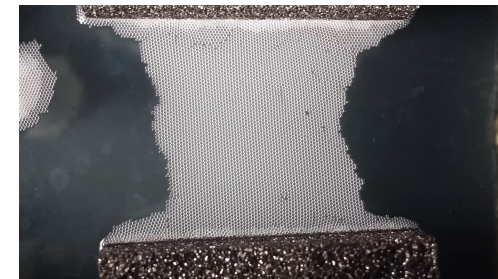
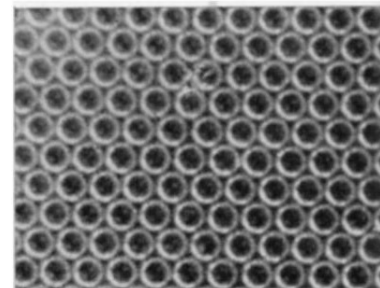


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Dislocations

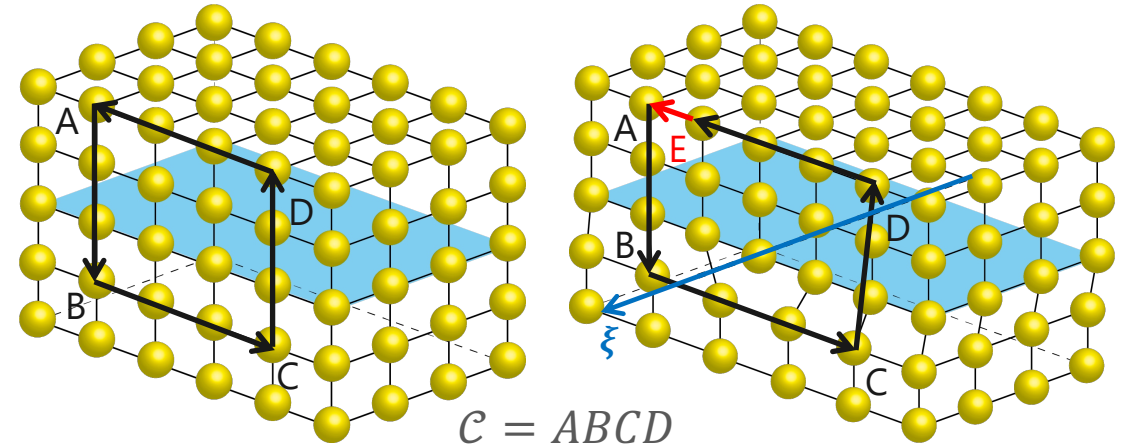
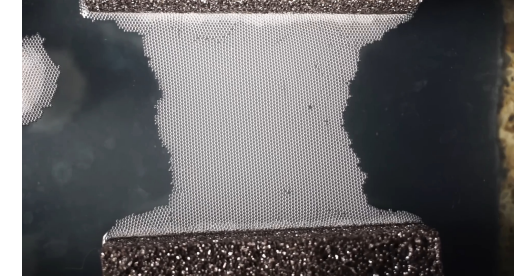
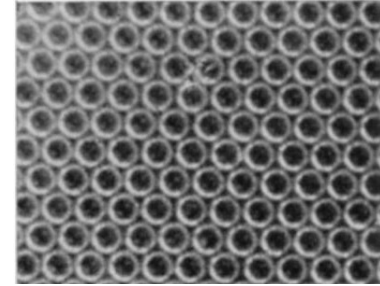
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❖ Types:

- *Edge dislocation*: characterized by an extra half-plane of atoms inserted into the lattice ($\mathbf{b} \perp \xi$).
- *Screw dislocation*: results from a shear distortion, forming a helical ramp in the crystal lattice ($\mathbf{b} \parallel \xi$).
- *Mixed dislocation*: a combination of edge and screw dislocations ($\mathbf{b} \angle \xi$).

L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)

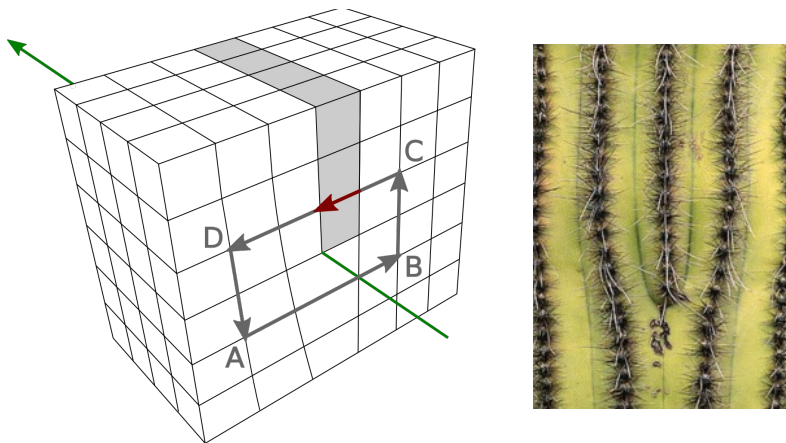


$$EA = \mathbf{b} = \oint_C \mathbf{U}^e d\mathbf{r} \Rightarrow b_i = \oint_C \frac{\partial u_i}{\partial x_j} dx_j$$

Dislocation types

Edge

Characterized by an extra half-plane of atoms inserted into the lattice ($b \perp \xi$).

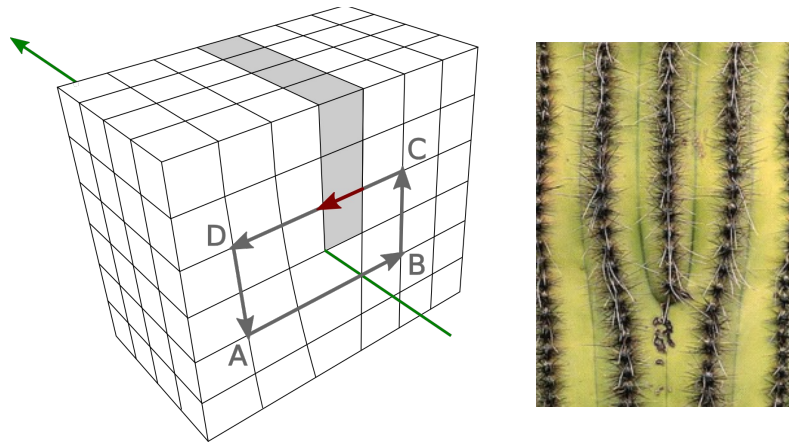


Symbol: \perp

Dislocation types

Edge

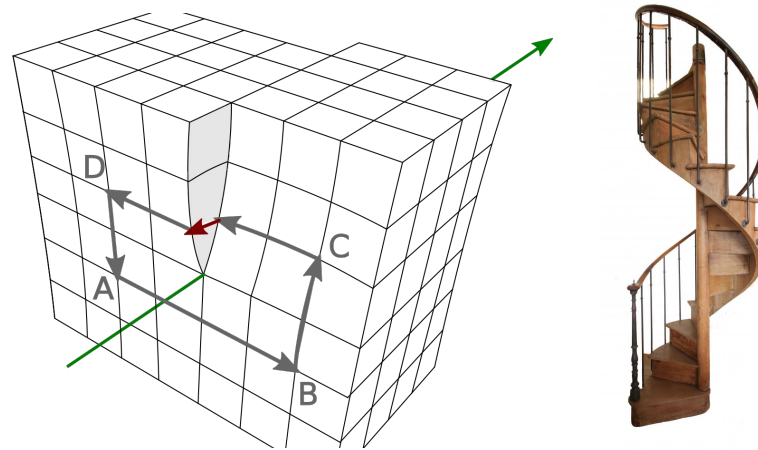
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Screw

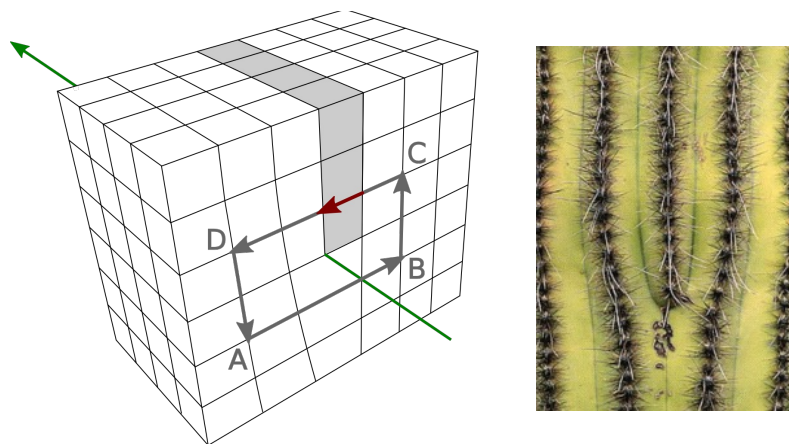
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Dislocation types

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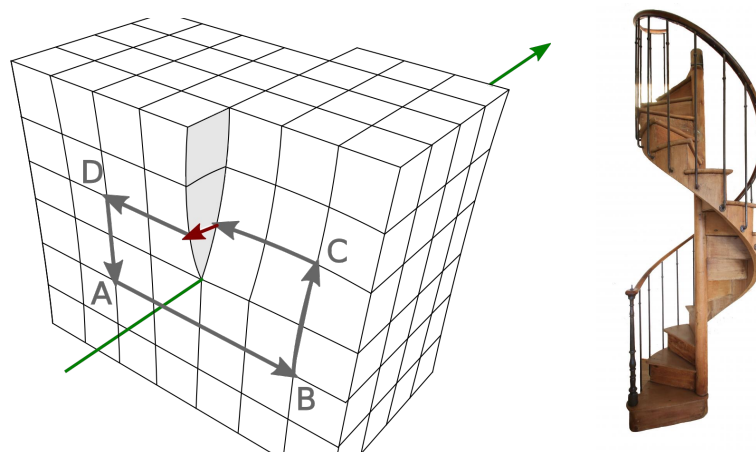
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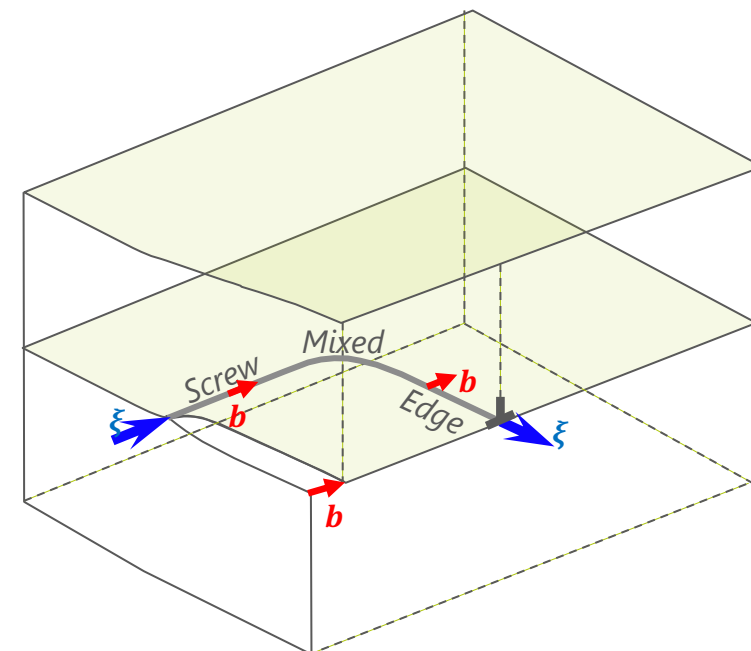
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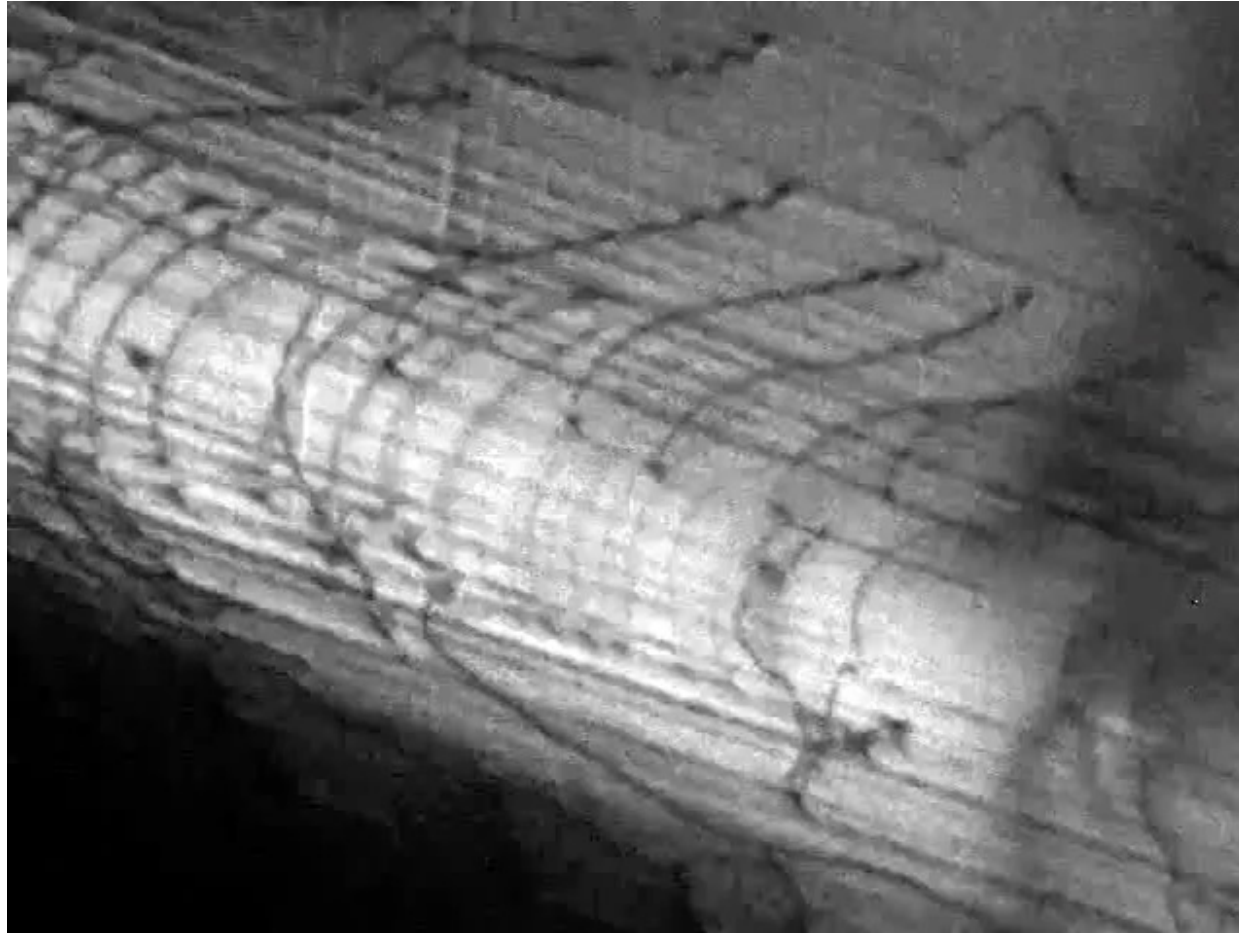
Mixed

A combination of edge and screw dislocations ($b \angle \xi$).



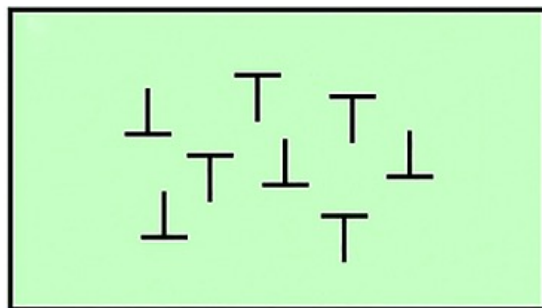
Dislocation glide -in 304 stainless steel

46



Different types of dislocation densities

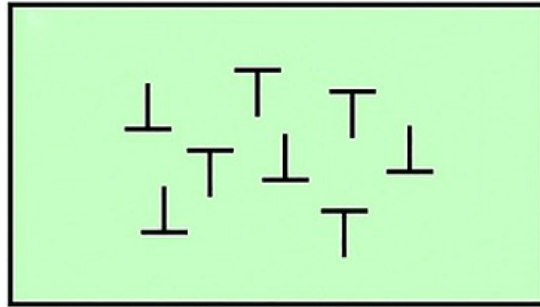
Statistically stored dislocations (SSD)



⇒ No lattice curvature at long range

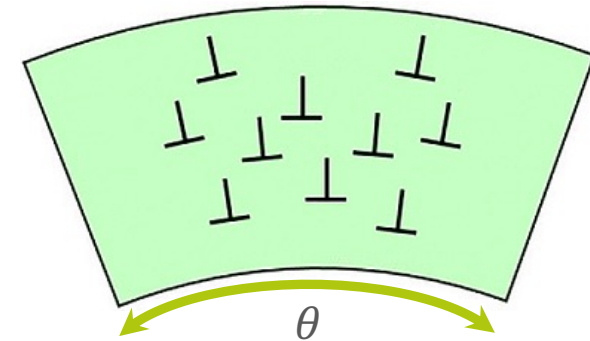
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Geometrically necessary dislocations (GND)

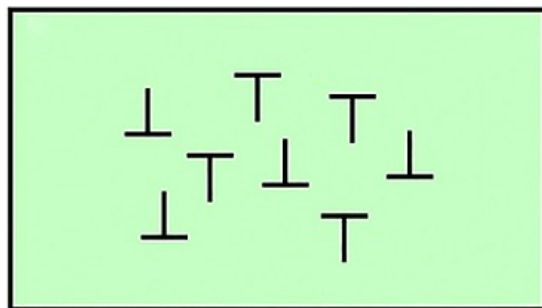


⇒ Lattice curvature at long range

$$\rho_{GND} \propto \frac{\theta}{\|b\| \delta x}$$

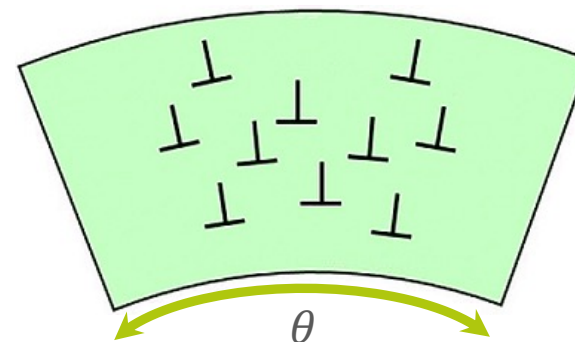
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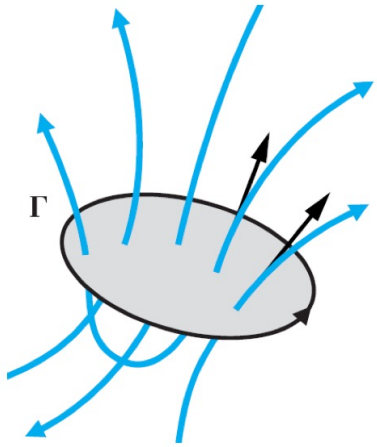
$$\rho_{\text{GND}} \approx \frac{1}{\|\mathbf{b}\|} \|\boldsymbol{\alpha}\|; \|\boldsymbol{\alpha}\| = \sqrt{\alpha_{ij} \cdot \alpha_{ij}}$$

$$\boldsymbol{\alpha} = \text{curl } \boldsymbol{\varepsilon} + \text{tr}(\boldsymbol{\kappa}_e) \cdot \mathbb{I} - \boldsymbol{\kappa}_e^T$$

$$\kappa_{ij} \cong \frac{\Delta \omega_i}{\Delta x_j}; \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

The Nye tensor

50



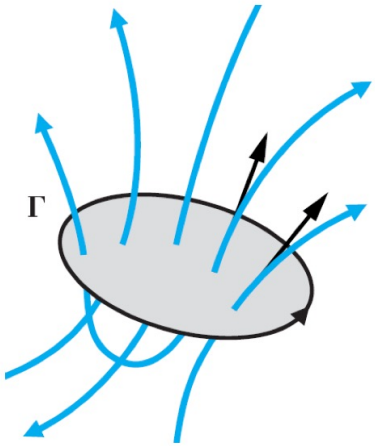
The Nye tensor

- The net Burgers vector on the closed circuit Γ limiting the surface S

$$\mathbf{B} = \left(\oint_{\Gamma} \mathbf{u}(\mathbf{x}) \mathbf{n} dS \right) \mathbf{b} = \oint_{\Gamma} \boldsymbol{\alpha} \mathbf{n} dS$$

Where

$$\boldsymbol{\alpha} = \mathbf{b} \otimes \mathbf{u}(\mathbf{x})$$



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Edge dislocations (1 of 2 types)

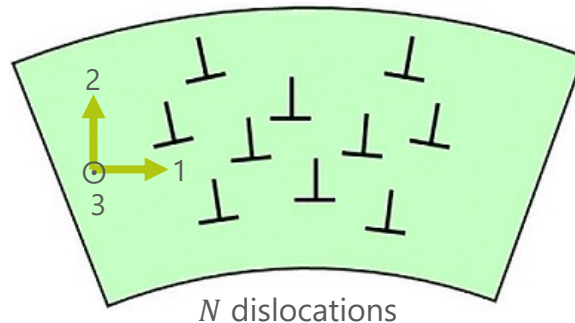
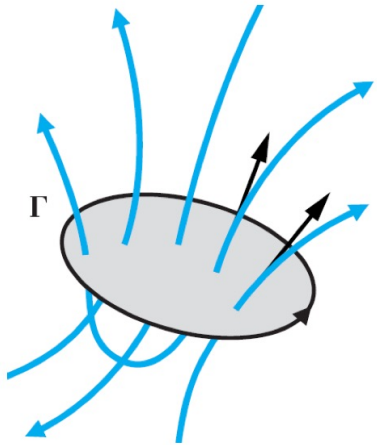
$$\mathbf{B}_{edge} = N \mathbf{b} \mathbf{e}_1 = \boldsymbol{\alpha} \mathbf{e}_3 S$$

$$\boldsymbol{\alpha} = \frac{N}{S} \mathbf{b} \otimes \mathbf{u}(\mathbf{x}) = \frac{N}{S} \mathbf{b} \mathbf{e}_1 \otimes \mathbf{e}_3 = \rho_{GND} \mathbf{e}_1 \otimes \mathbf{e}_3$$

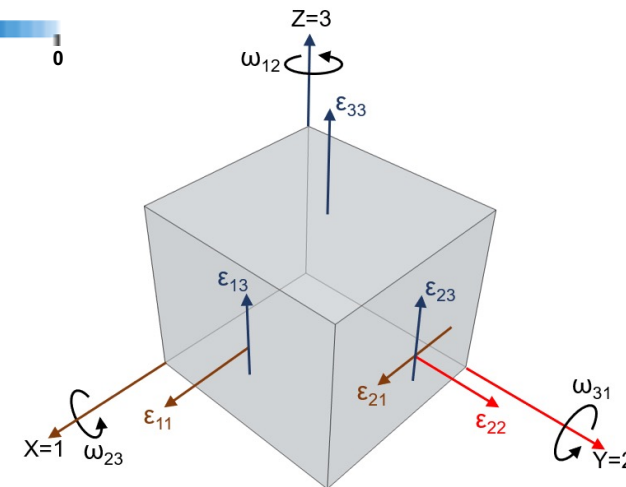
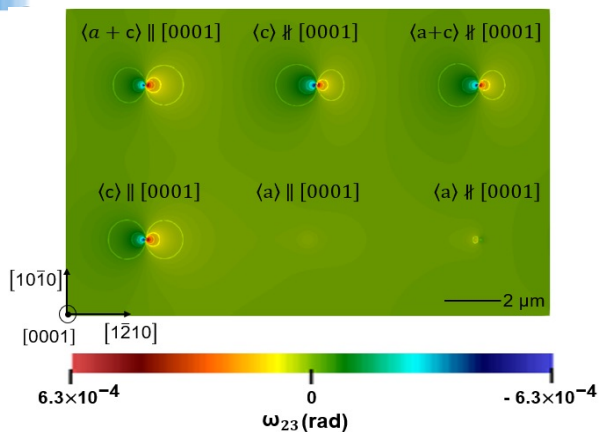
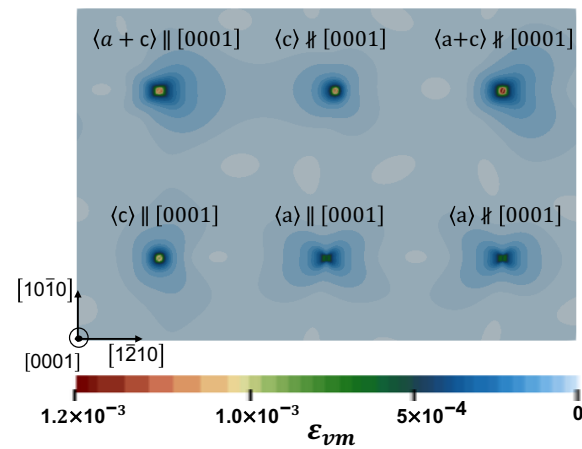
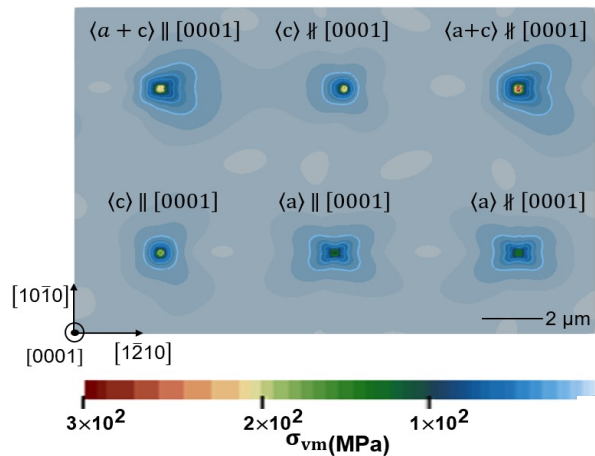
Screw dislocations

$$\mathbf{B}_{screw} = N \mathbf{b} \mathbf{e}_3 = \boldsymbol{\alpha} \mathbf{e}_3 S$$

$$\boldsymbol{\alpha} = \frac{N}{S} \mathbf{b} \otimes \mathbf{u}(\mathbf{x}) = \frac{N}{S} \mathbf{b} \mathbf{e}_3 \otimes \mathbf{e}_3 = \rho_{GND} \mathbf{e}_3 \otimes \mathbf{e}_3$$

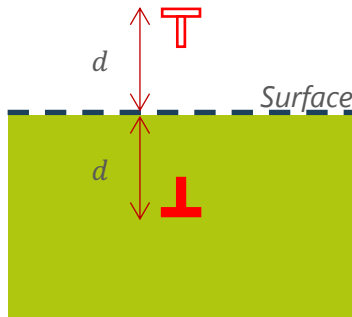


Elastic fields of dislocations



Dislocations induce elastic fields, which are key to their detection and analysis.

Effect of free surfaces on dislocations

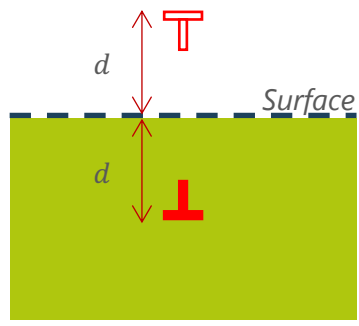


❖ **Surface presence alters the elastic field of a dislocation loop.**

⇒ An image force appears to satisfy the zero-traction boundary condition at the surface.

⇒ Physically, it corresponds to the attraction between the real dislocation and a virtual dislocation of opposite Burgers vector ($-b$) placed symmetrically on the other side of the surface.

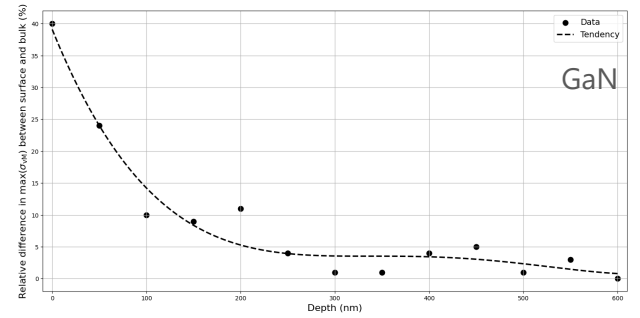
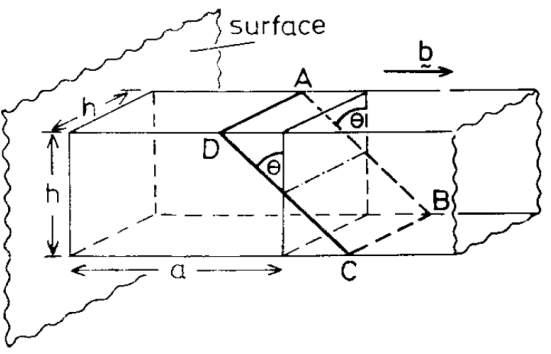
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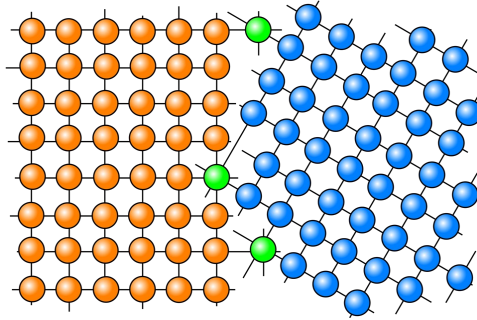
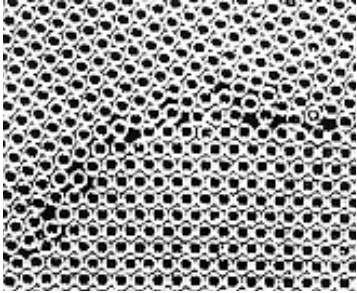
❖ **Surface effects are short-ranged:**

- Significant influence when depth $a \leq 2h - 3h$
- a : distance from loop center to surface; h : loop size (length of one side of rectangular loop)
- For $a > 3h$, the dislocation behaves as in an infinite medium.

↪ The behavior of dislocations is influenced by the presence of free surfaces

Grain boundaries

L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)

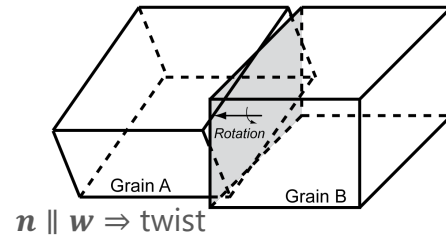
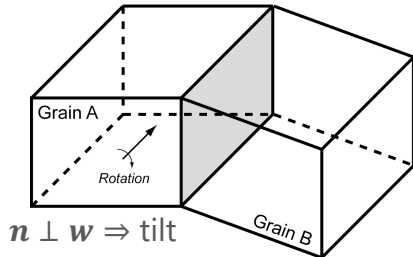
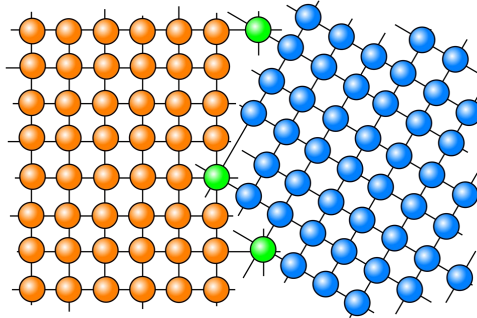
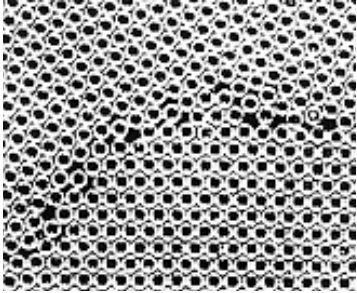


❖ Definition:

- GBs are interfaces where crystals of different orientations meet in a polycrystalline material.
- Five parameters to describe a GB
 - ✓ 1 disorientation angle (θ) \rightarrow 1 parameter
 - ✓ Unitary rotation vector (\mathbf{w}) \rightarrow 2 parameters
 - ✓ Unitary vector for the GB plane normal (\mathbf{n}) \rightarrow 2 parameters

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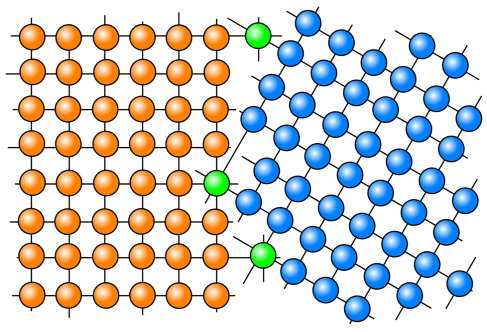
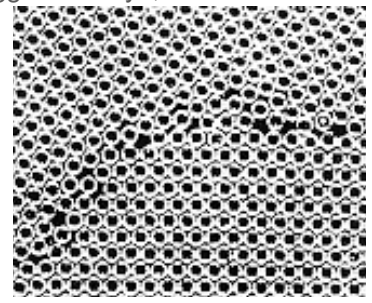
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❖ Types:

- *Tilt GB*: $\mathbf{n} \perp \mathbf{w}$
- *Twist GB*: $\mathbf{n} \parallel \mathbf{w}$
- *Mixed GB*: combination of tilt and twist components.

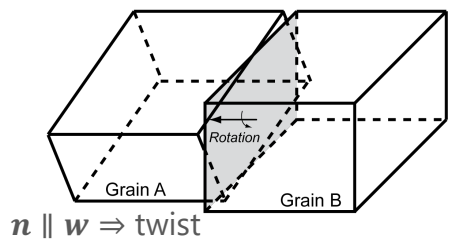
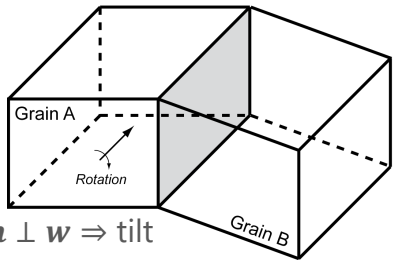
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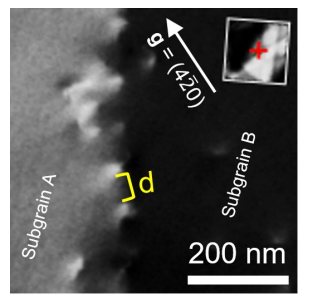
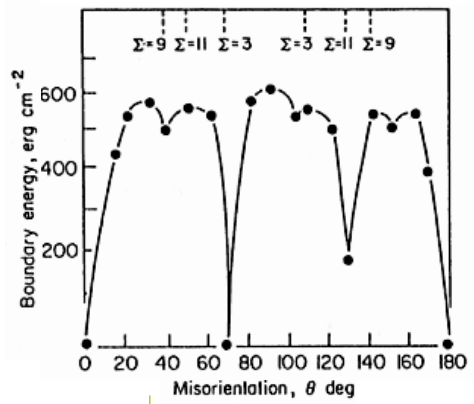
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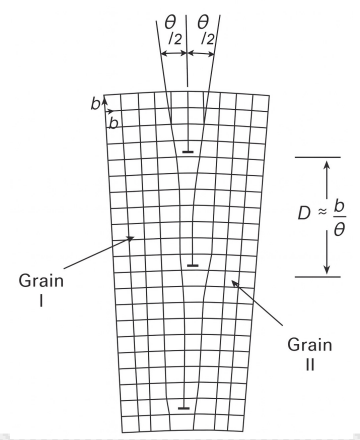
❖ Types:

- Tilt GB: $\mathbf{n} \perp \mathbf{w}$
- Twist GB: $\mathbf{n} \parallel \mathbf{w}$
- Mixed GB: combination of tilt and twist components.
- Low-Angle GB (LAGB): $\theta \ll 15^\circ$

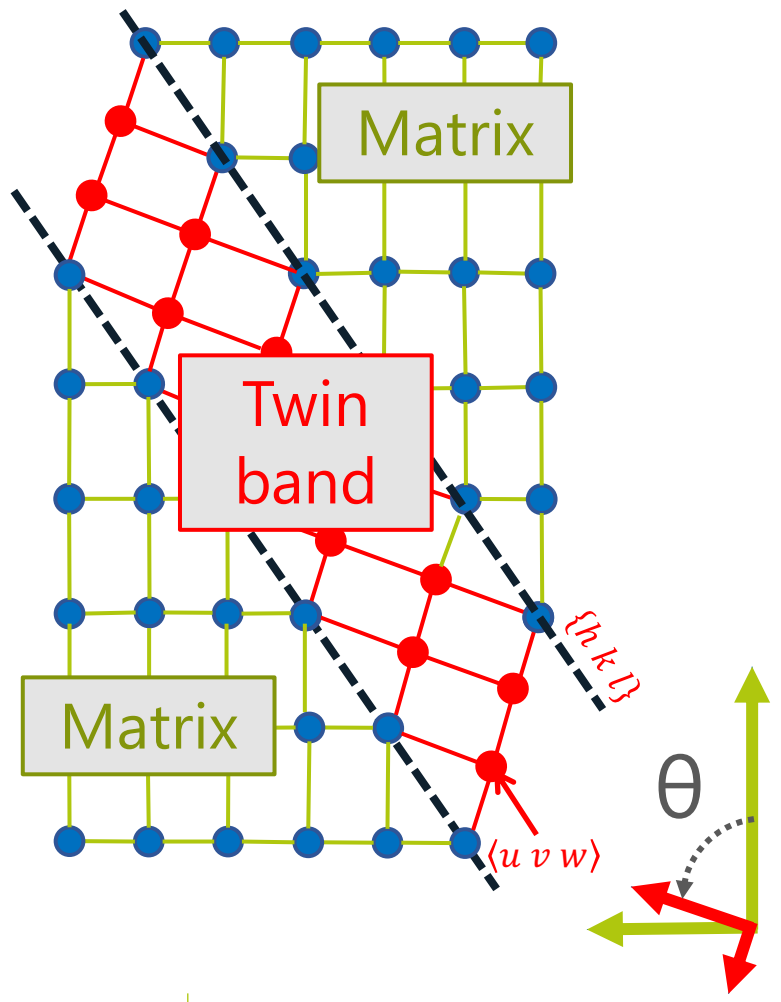
$$\theta = 2 \sin^{-1} \left(\frac{|b|}{2D} \right)$$
- High-Angle GB (HAGB): $\theta \gg 15^\circ$



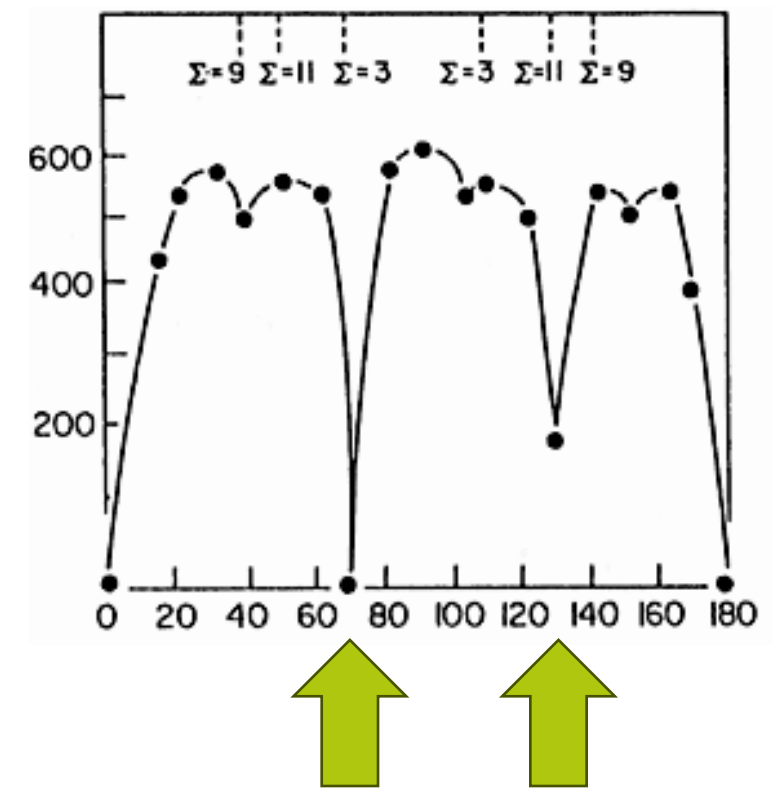
$\triangleright D = 60 \pm 5.52 \text{ nm}$
 $\triangleright \mathbf{b} = \frac{a}{2} \langle 110 \rangle$
 $\Rightarrow \theta = 0.24 \pm 0.02^\circ$



Focus on the twinning



- ❖ Identical crystalline structure
- ❖ Mirror reflection
- ❖ System: $\langle u v w \rangle \{h k l\}$
- ❖ Low-energy HAGB
- ❖ Tilted by a specific angle (θ)



Punctual defects

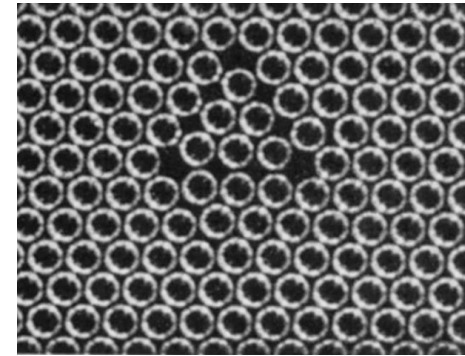
❖ Definition:

- Point defects are atomic-scale imperfections localized at or around a single lattice site in a crystalline material.

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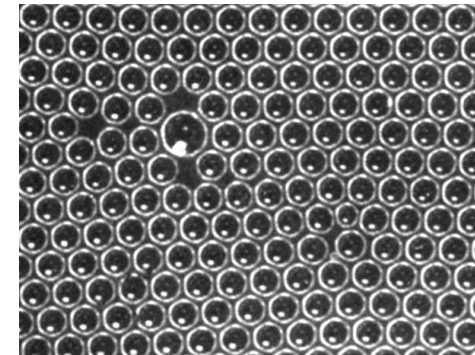
- *Vacancies*: Missing atoms in the lattice.
- *Interstitials*: Extra atoms positioned in the interstitial sites between the regular lattice points.
- *Substitutional defects*: Foreign atoms replacing host atoms in the lattice.

Vacancy



L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)

Interstitial



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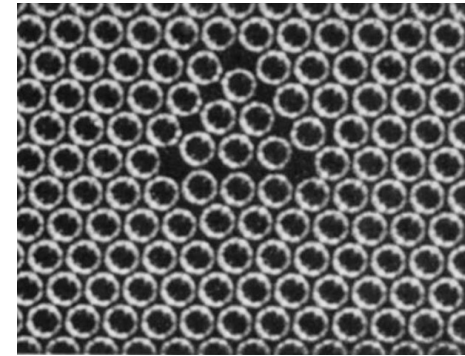
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❖ Diffusion:

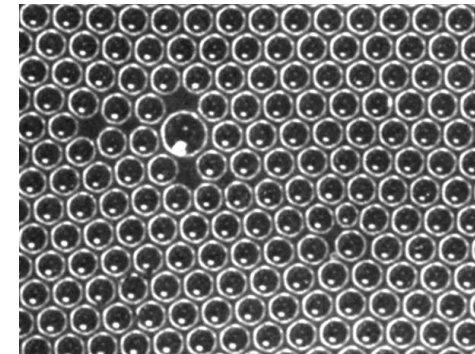
- They enable atomic diffusion, which is governed by Fick's Laws...

Vacancy



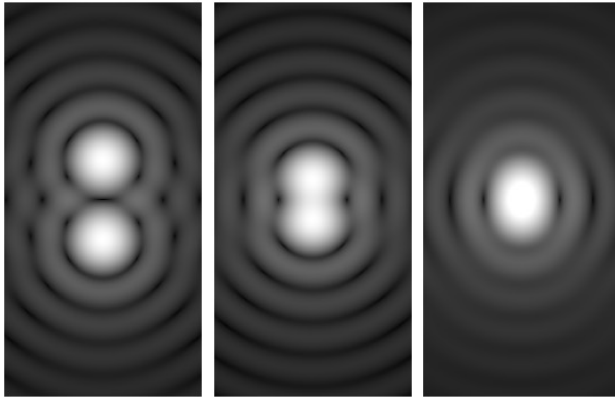
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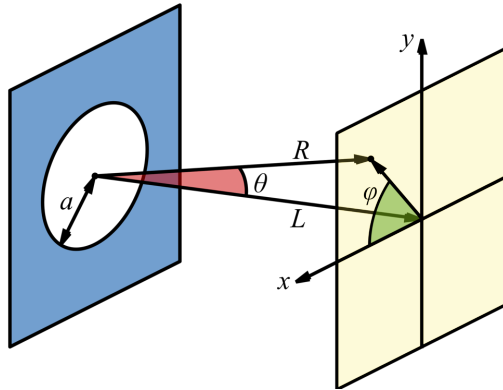
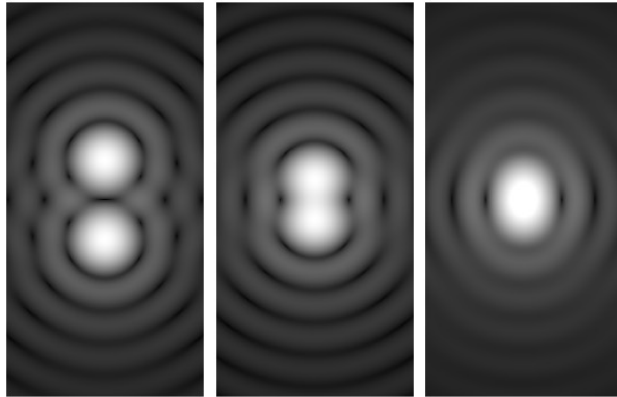
Limit of resolution



❖ Airy disk and Rayleigh criterion

- It refers to the ability to distinguish between very close details.

Limit of resolution

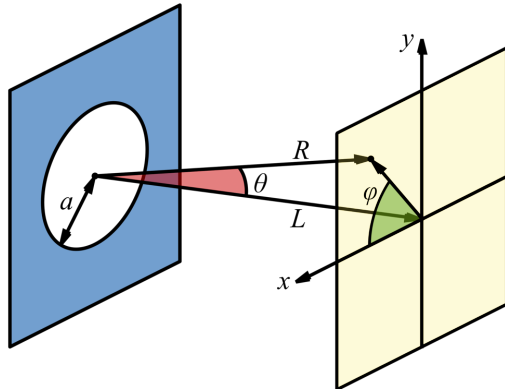
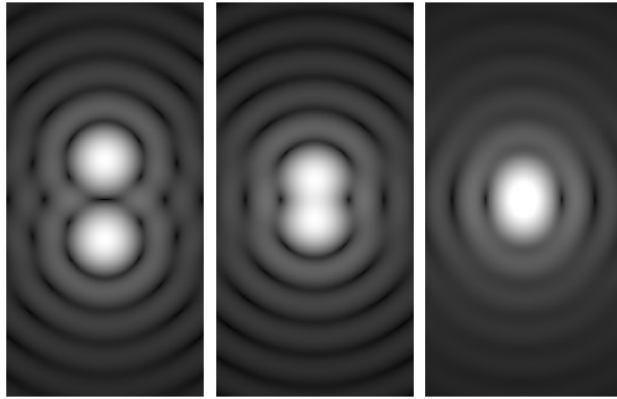


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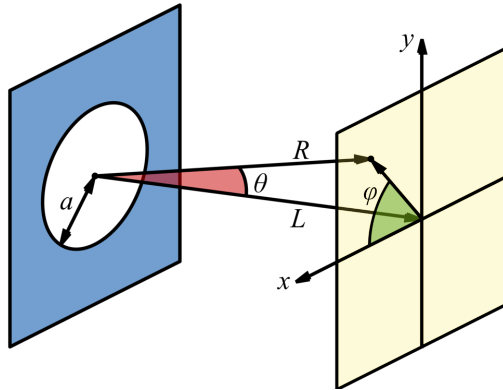
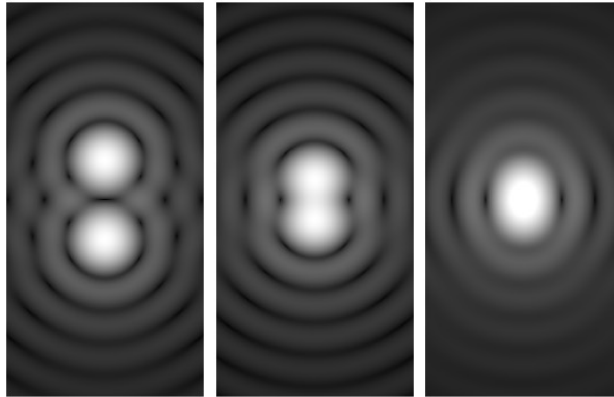
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$$\sin \theta = 0.61 \frac{\lambda}{a} \sim \theta$$

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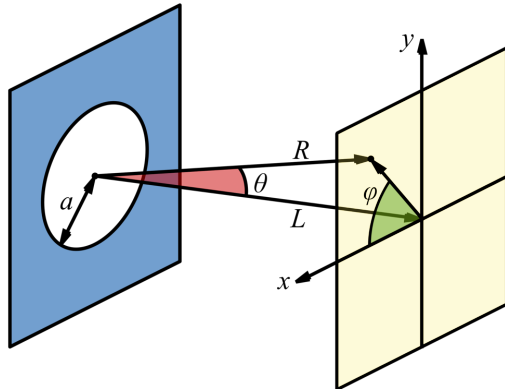
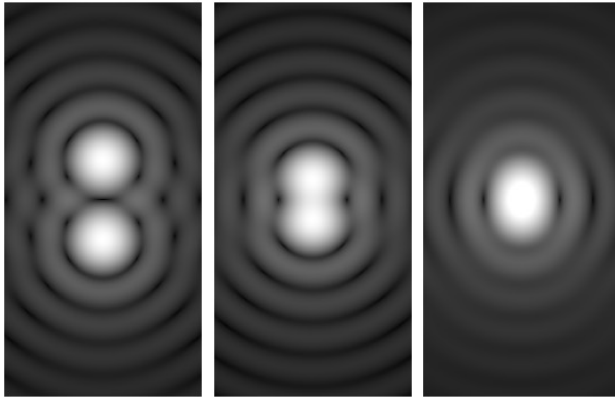
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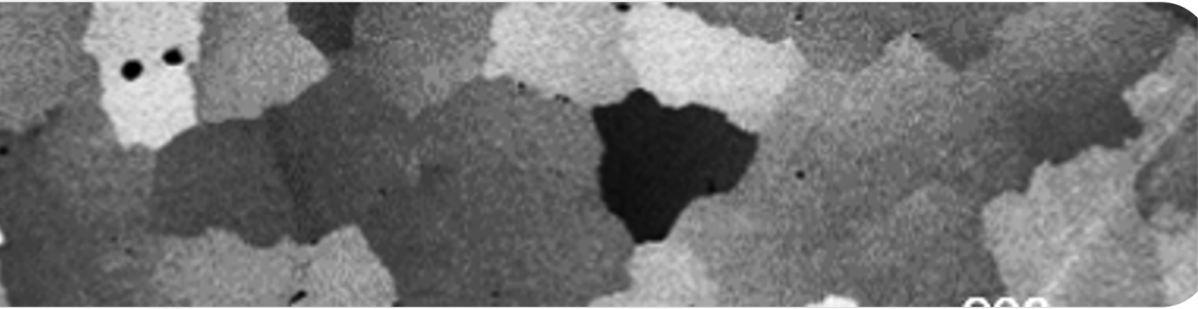
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↪ Two objects cannot be resolved if they are closer than one wavelength λ .



Characterization by X-rays diffraction

Meso- Micro-meter scale

“Radiations studied by M. Röntgen”

❖ On the evening of November 8, 1895, in Würzburg, Germany:

- Röntgen made a screen fluoresce near a Crookes tube inside a black box.
- Then, he played with various objects revealing their transparency.



Wilhelm RÖNTGEN
(1845 – 1923) 🇩🇪
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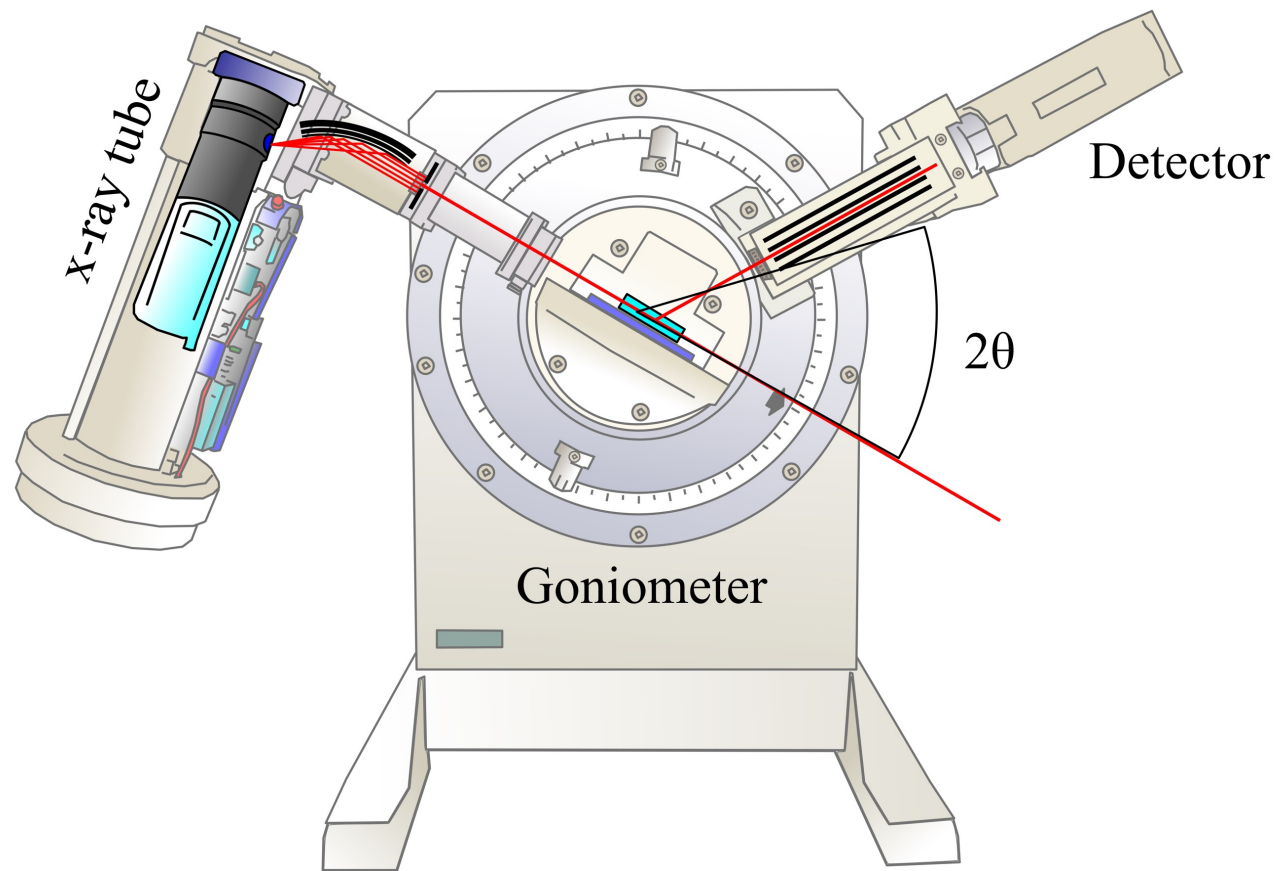
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↪ Since Röntgen did not understand their nature, he called these rays, X-rays!

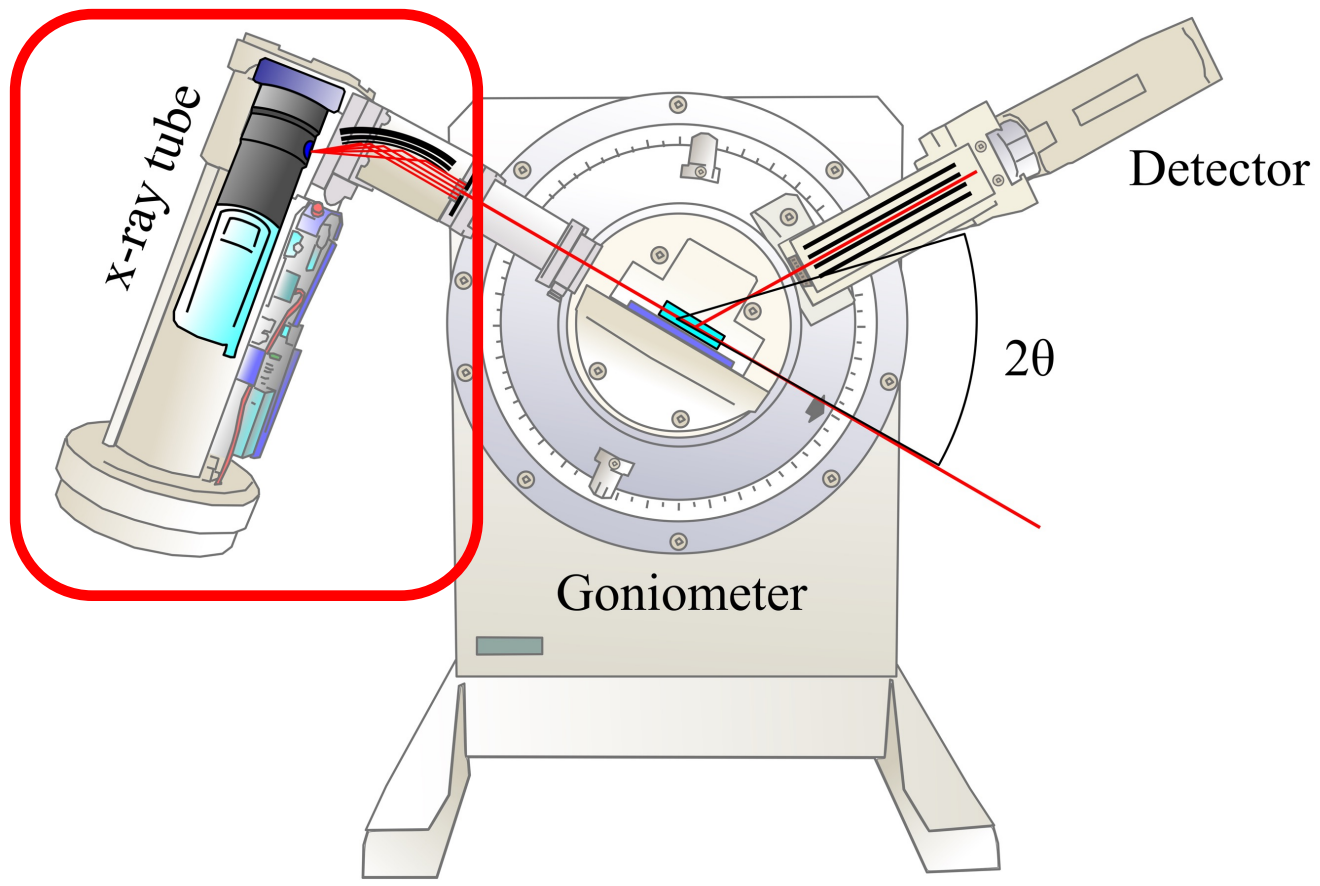
The experimental setup: the diffractometer

The diffractometer (Bragg-Brentano configuration)

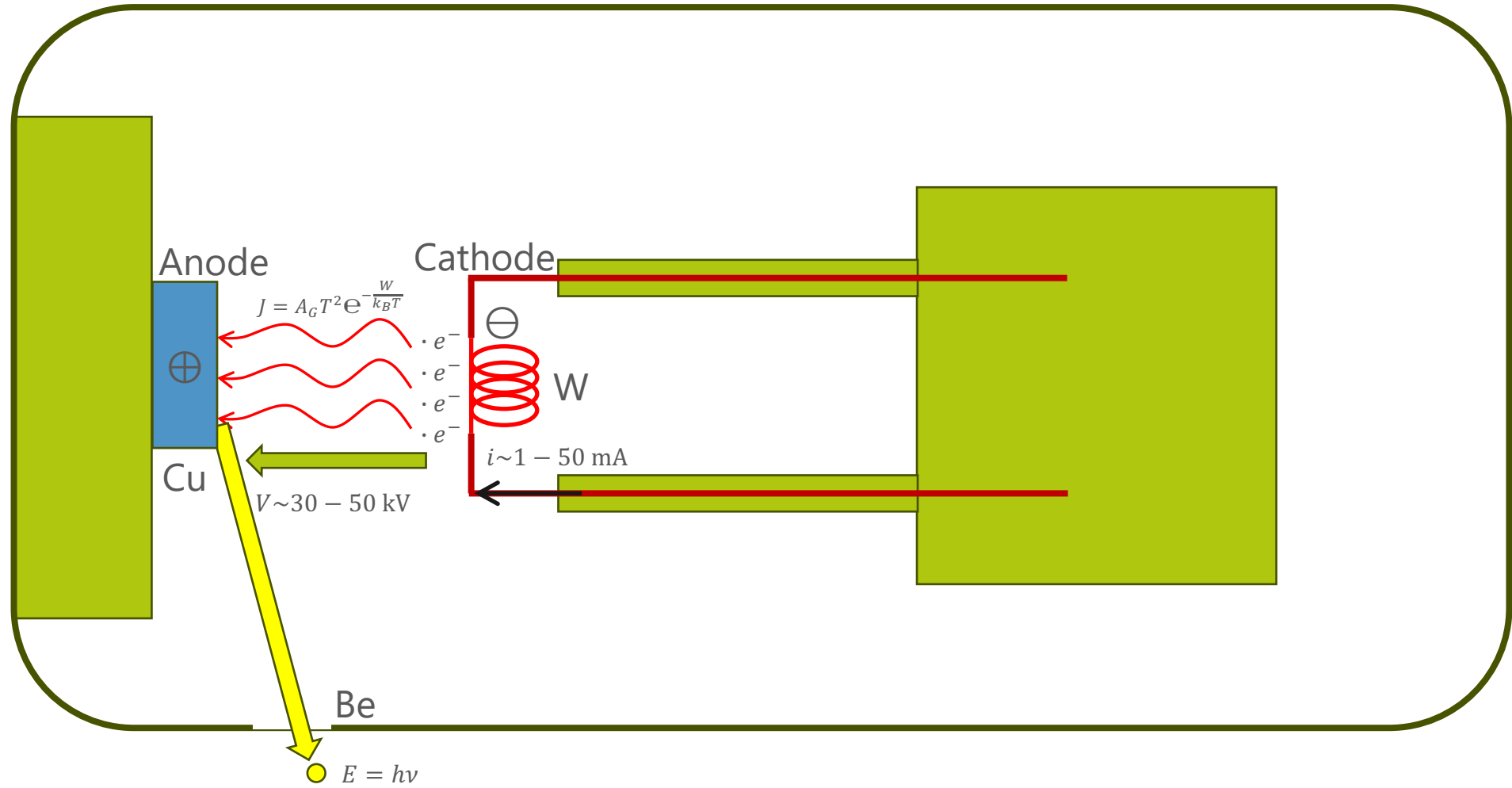


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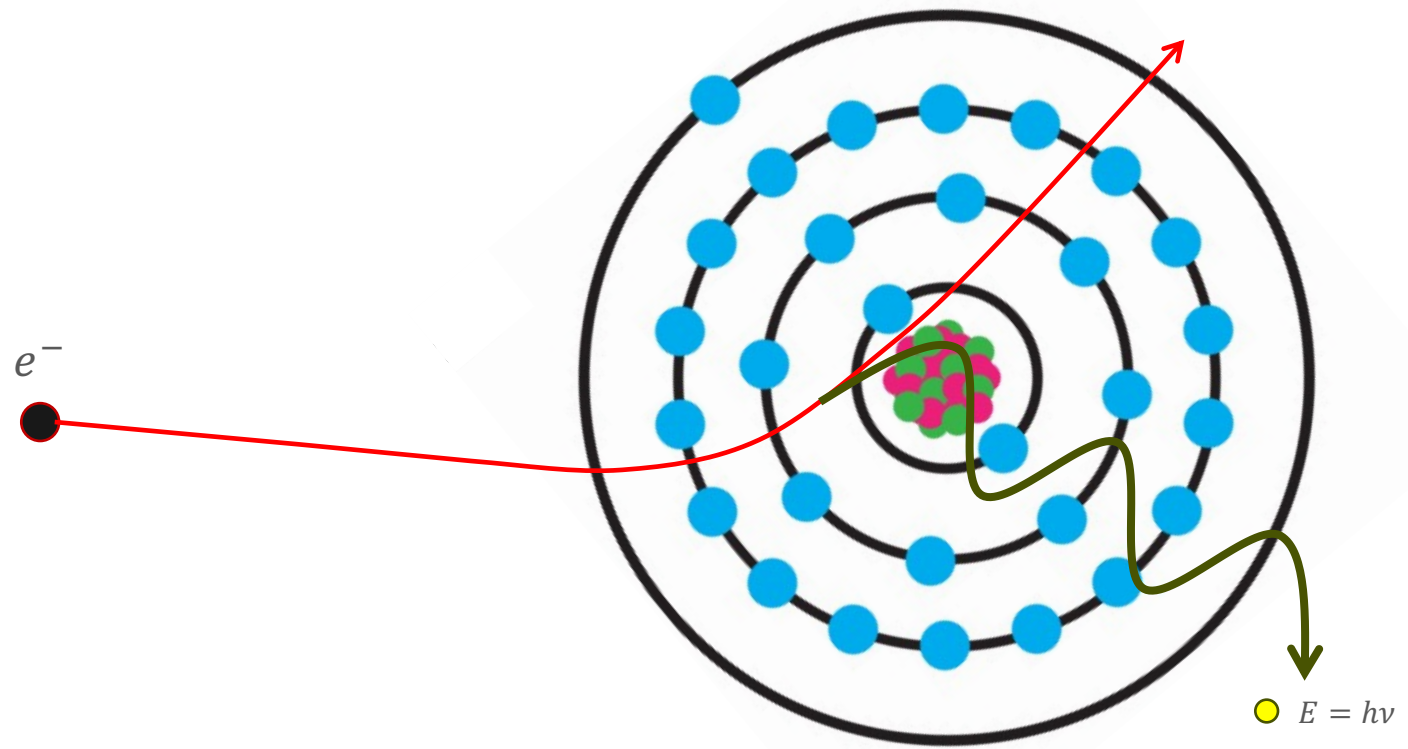


X-ray production



X-ray production

Bremsstrahlung radiation



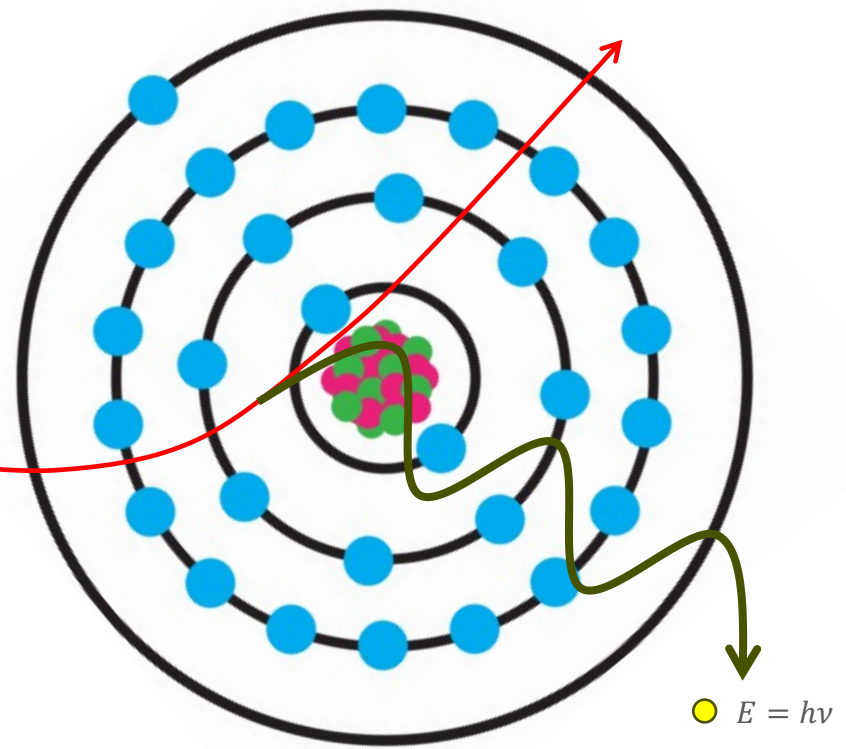
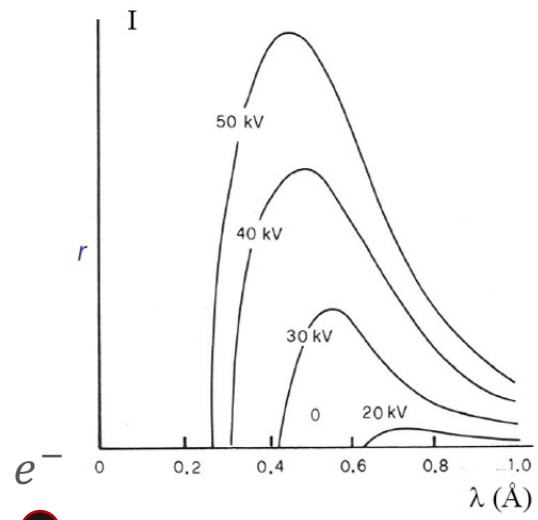
❖ Definition:

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❖ Mechanism:

- Electrons lose kinetic energy when interacting with atomic nuclei.
- Lost energy is emitted as X-ray photons.

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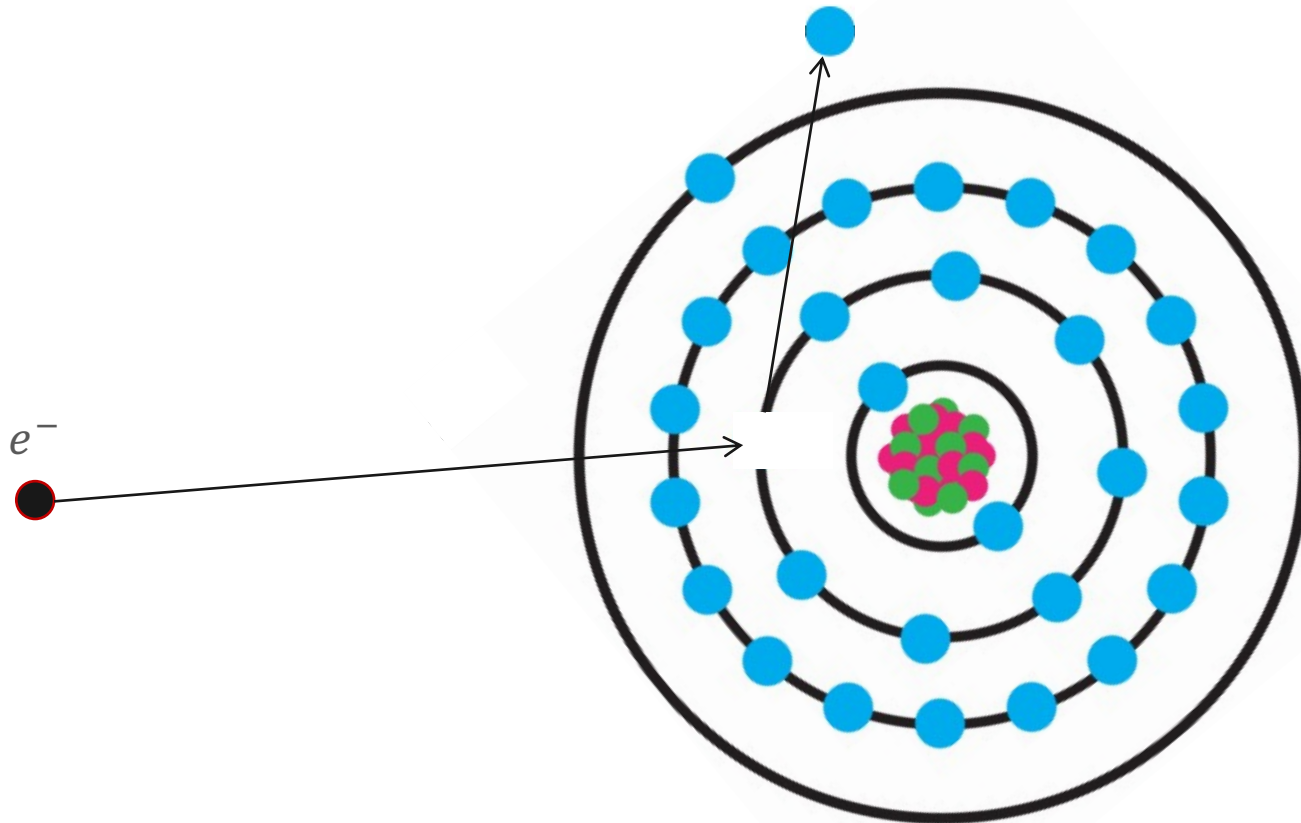
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- Lost energy is emitted as X-ray photons.

❖ Characteristic:

- *Continuous*: due to gradual deceleration of electrons.
- Higher for heavier nuclei ($\propto Z^2$).

↪ Bremsstrahlung radiation is the primary contributor to the background in X-ray emission spectra.

X-ray production



Characteristic radiation

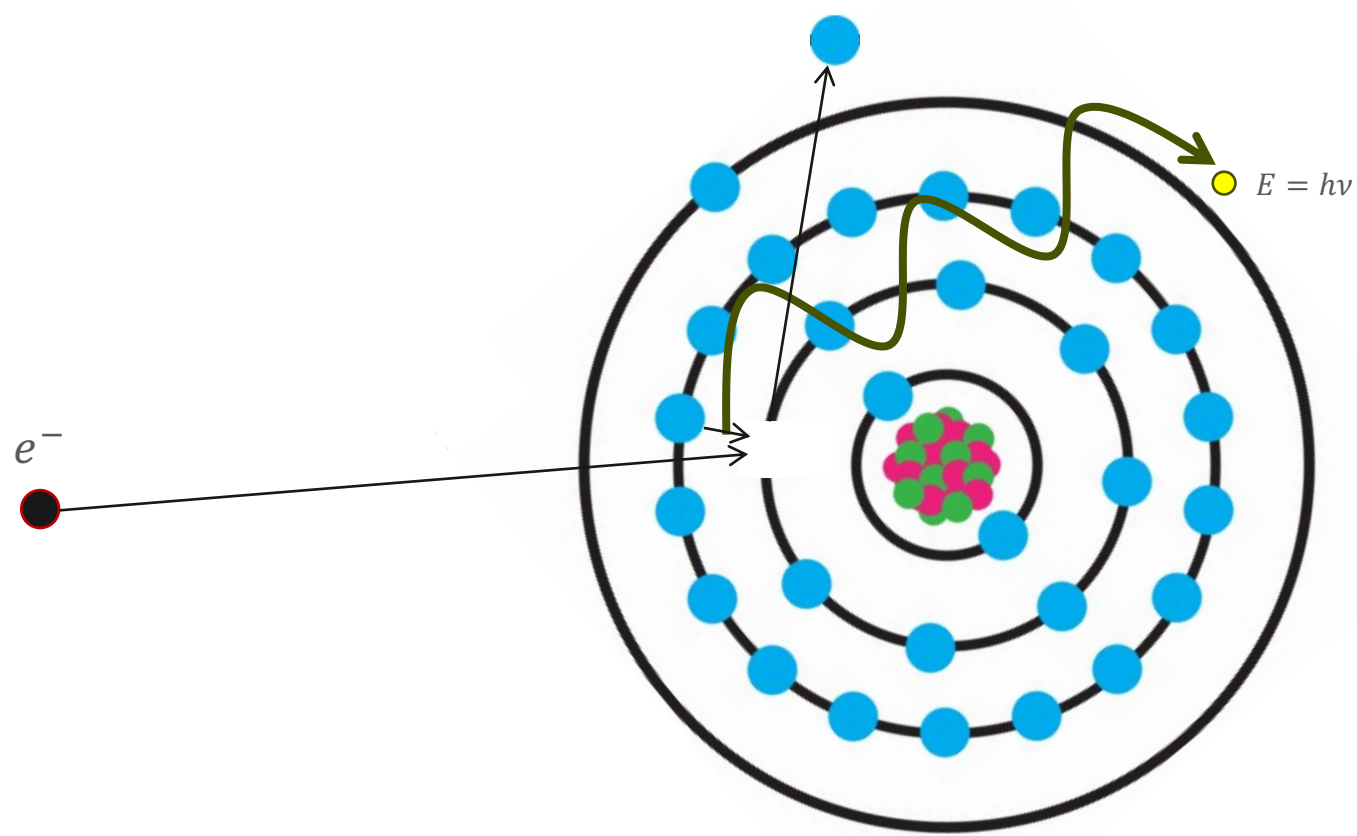
❖ **Definition:**

- X-rays emitted as an outer electron fills an inner-shell vacancy.

○ **Mechanism:**

- The incident electron removes an inner-shell electron.

X-ray production



Characteristic radiation

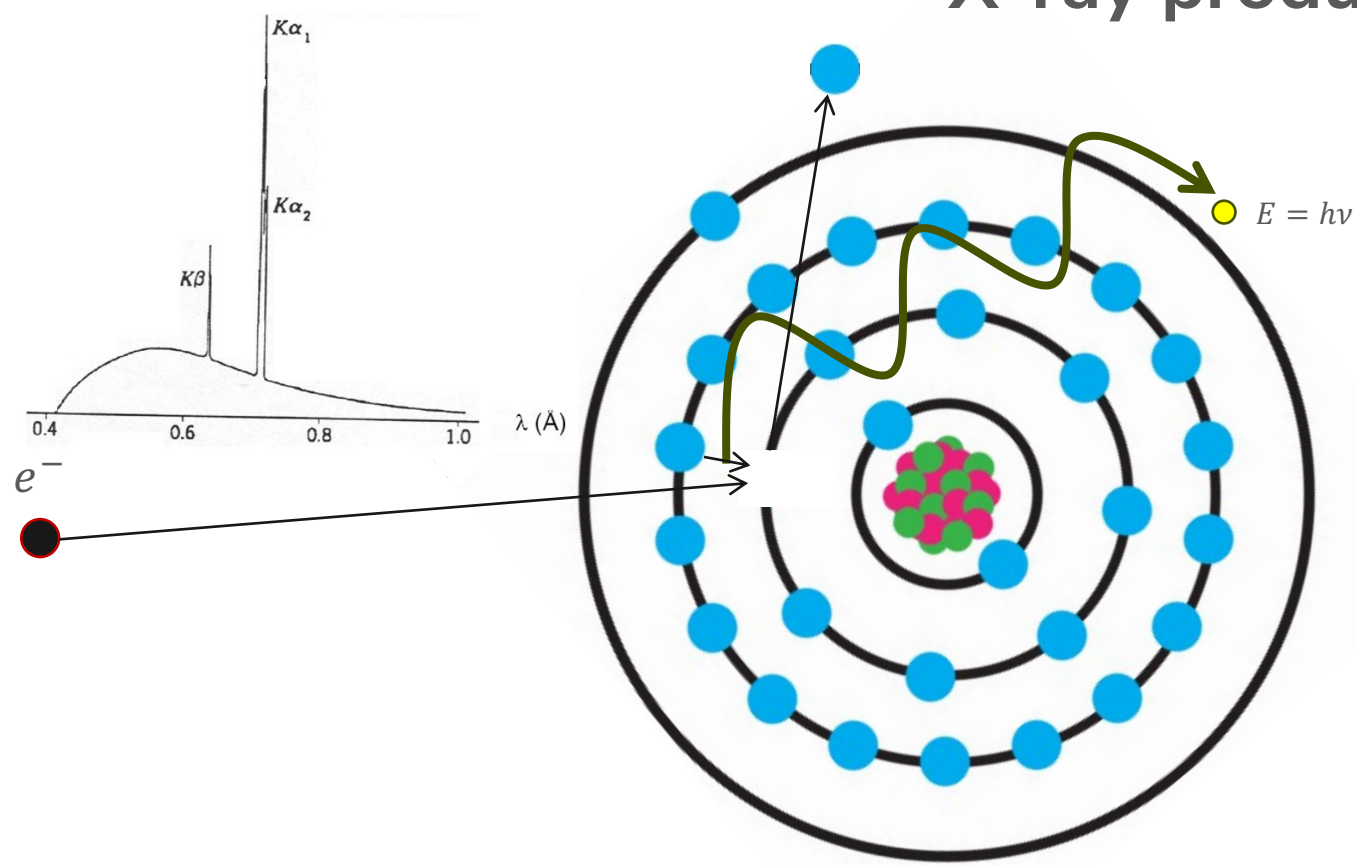
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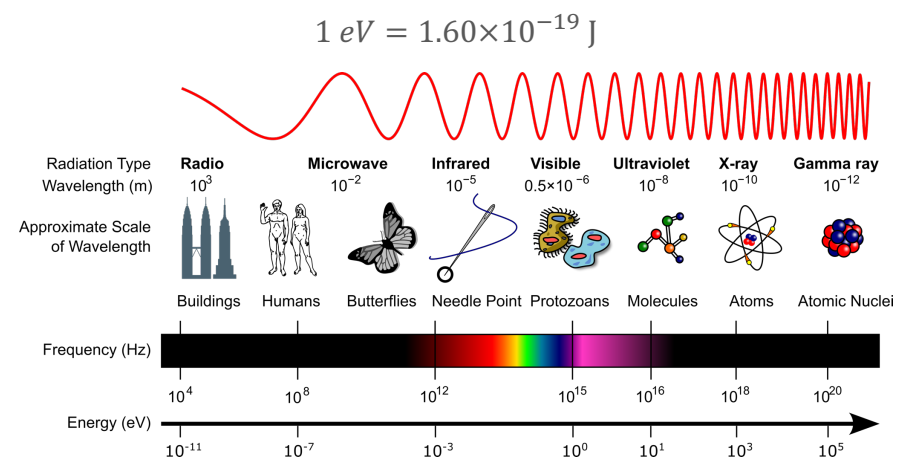
- Distinct peaks
- Element-specific

↳ Characteristic X-rays enable elemental analysis, aiding in materials characterization.

What are x-rays?

Electromagnetic waves

- ❖ X-rays are electromagnetic waves. If they have enough energy ($E_{rad} \sim 10 \text{ keV}$), some electrons can be removed from atoms or molecules, thereby ionizing them ($E_{ion} \sim 10 \text{ eV}$).



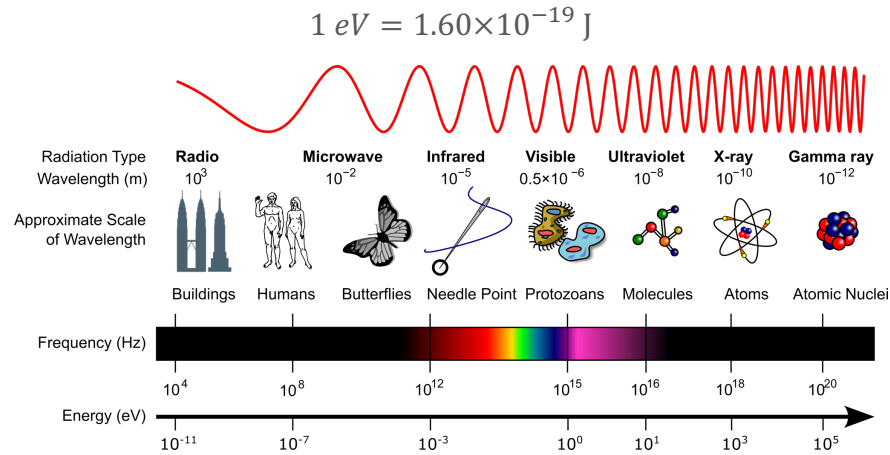
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where h : Planck's constant; ν : frequency; ω : pulsation

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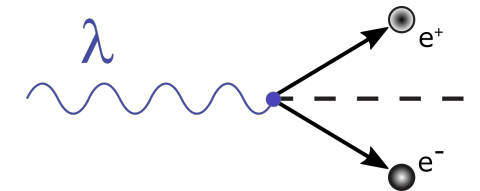
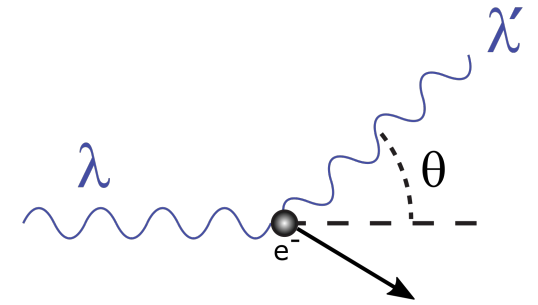
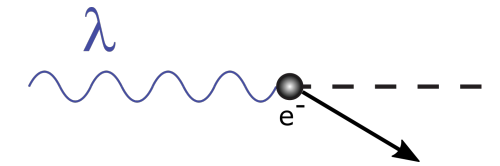


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Photon-matter interaction

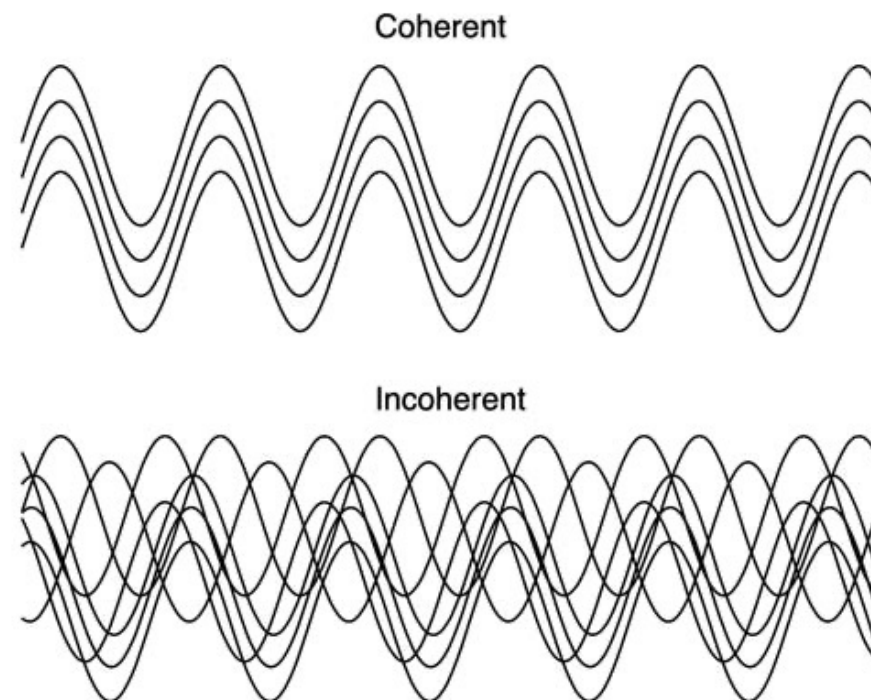
- Photoabsorption effect (10 eV – 100 keV):** A photon transfers all its energy to an electron, ejecting it if the energy exceeds the electron's binding energy.
- Compton scattering (10 keV – 1 MeV):** An incident photon collides with a free or loosely bound electron, losing energy and changing direction. The electron gains this energy and is ejected with a new direction and velocity.
- Pair production ($> 1 \text{ MeV}$):** A high-energy photon interacts with an atom's nucleus and converts into an electron-positron pair.



Coherency

❖ Definition:

- Coherence describes the stability of an X-ray wave's phase in time and space, crucial for producing clear interference patterns.



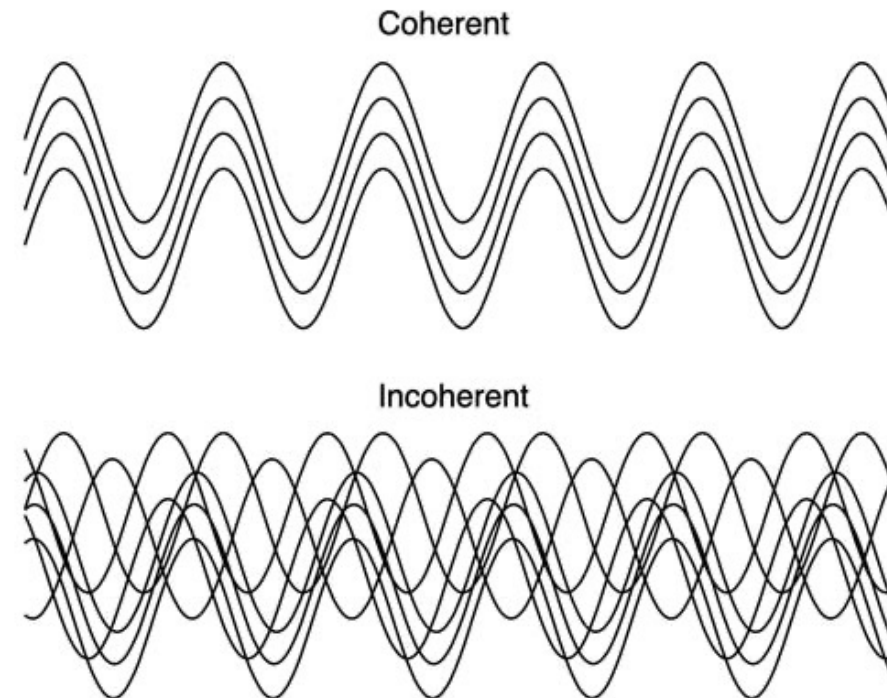
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❖ Types of coherence:

- Temporal coherence (energy stability)
 - ✓ Determines the ability of X-rays to interfere over time.
 - ✓ Related to the spectral width: narrow bandwidth → High temporal coherence.
 - ✓ Expressed by the coherence length:
$$l_c = \frac{c}{\Delta f}$$
 - ✓ Important for monochromatic X-ray sources in diffraction.



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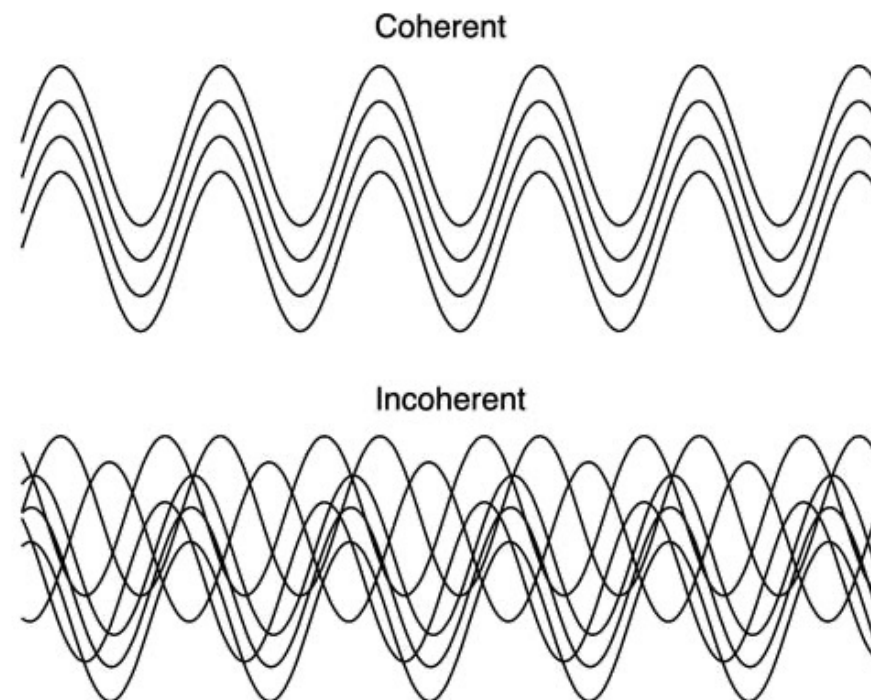
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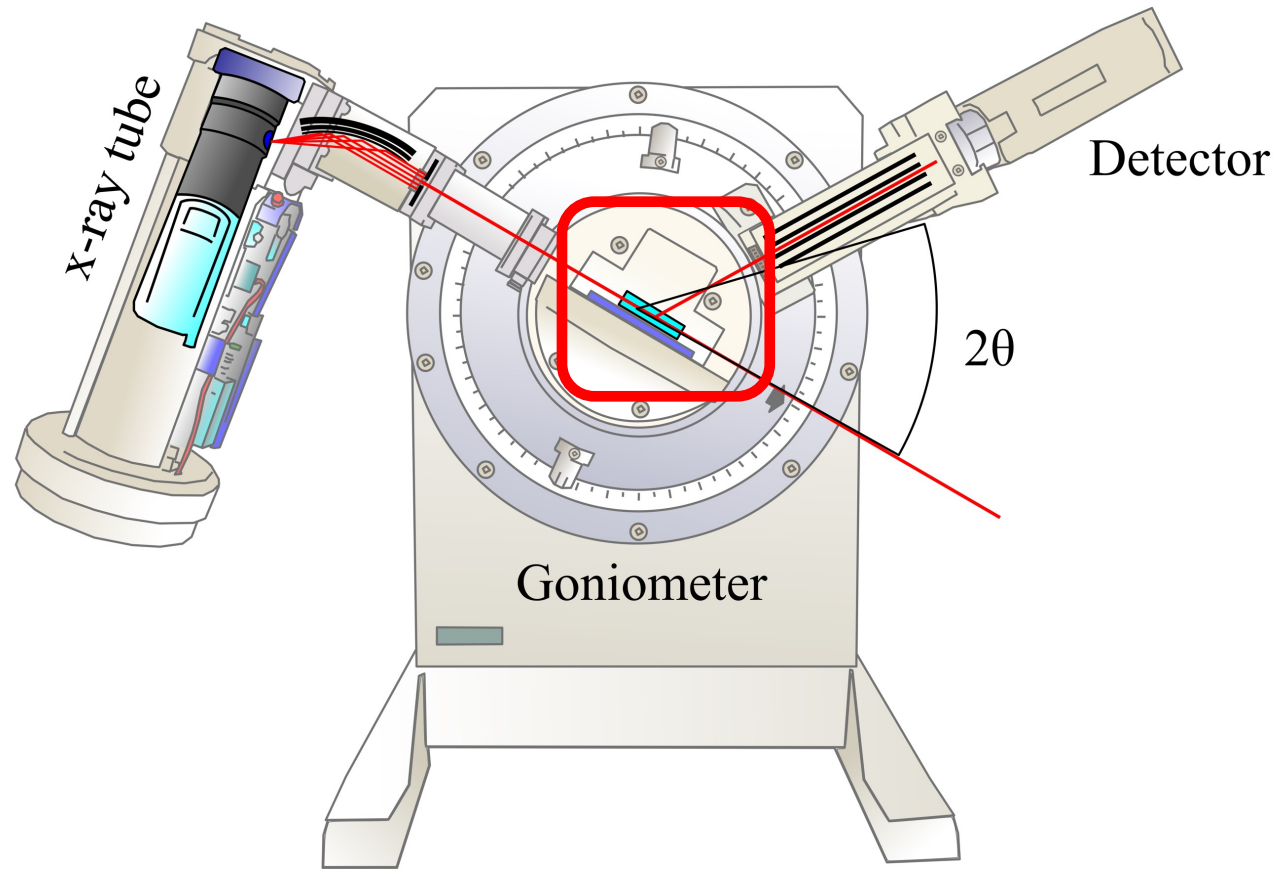
$$l_c = \frac{c}{\Delta f}$$

- ✓ Important for monochromatic X-ray sources in diffraction.
- Spatial coherence (wavefront uniformity)
 - ✓ Measures phase correlation across different points in the X-ray beam.
 - ✓ Affects the sharpness of diffraction spots in XRD.
 - ✓ High spatial coherence is achieved with small, collimated sources (e.g., synchrotron X-rays).



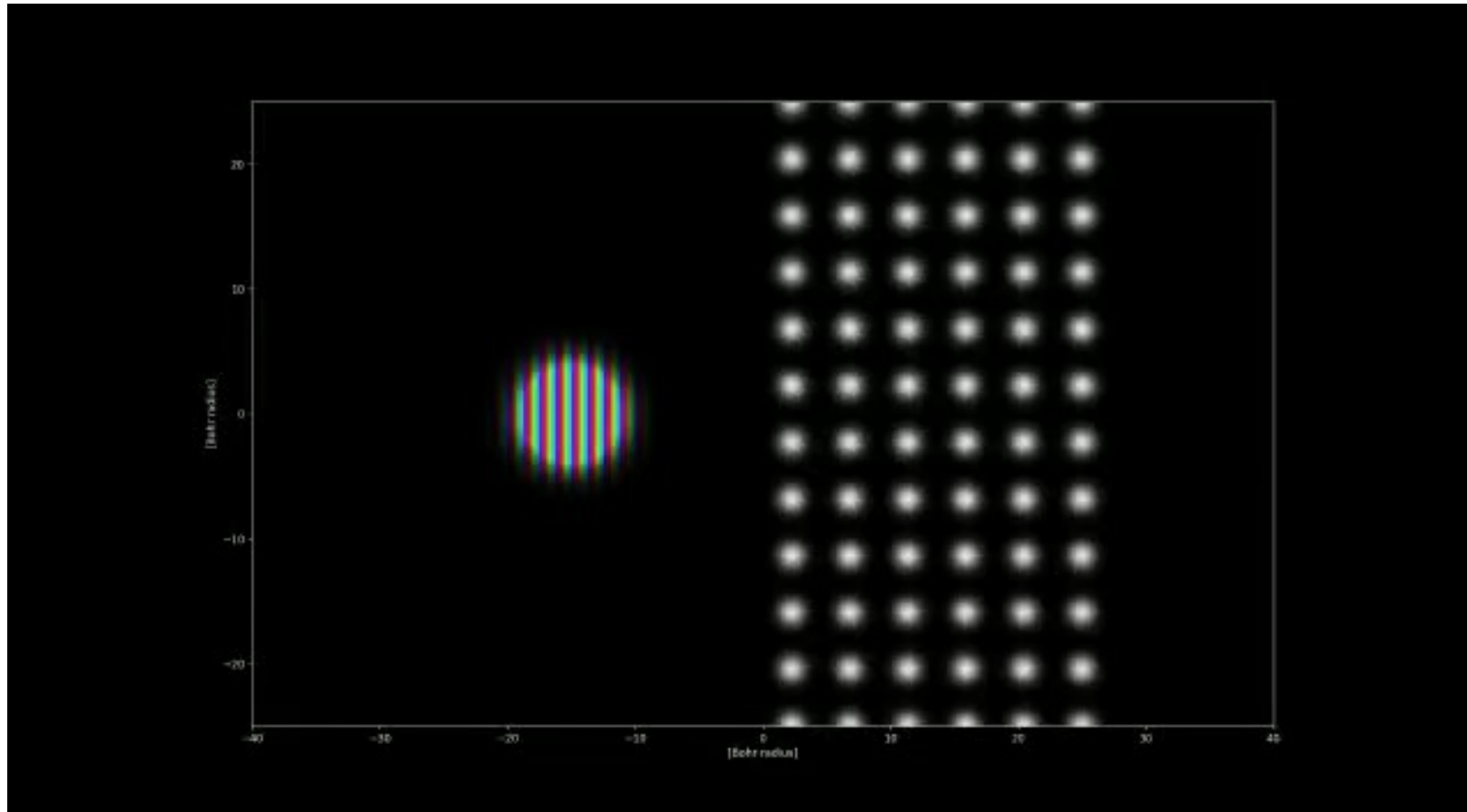
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The diffraction

85

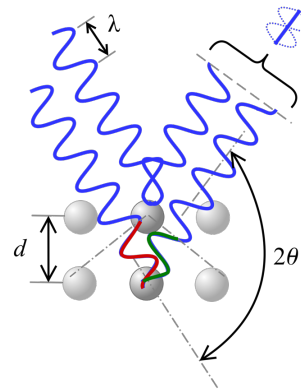
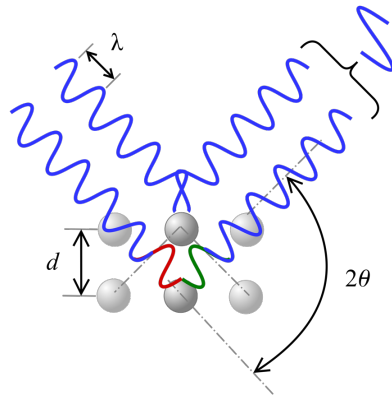


Interaction X-ray matter

❖ Liquids or amorphous materials:

- X-rays are scattered in many directions, leading to broad, diffuse halos.

Interaction X-ray matter



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❖ In crystalline materials:

- X-rays produce sharp and intense peaks of radiation at certain angles

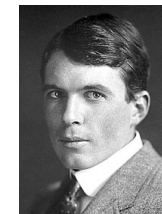
↪ This suggests that X-rays interact in a structured manner with the internal atomic planes of the crystal.



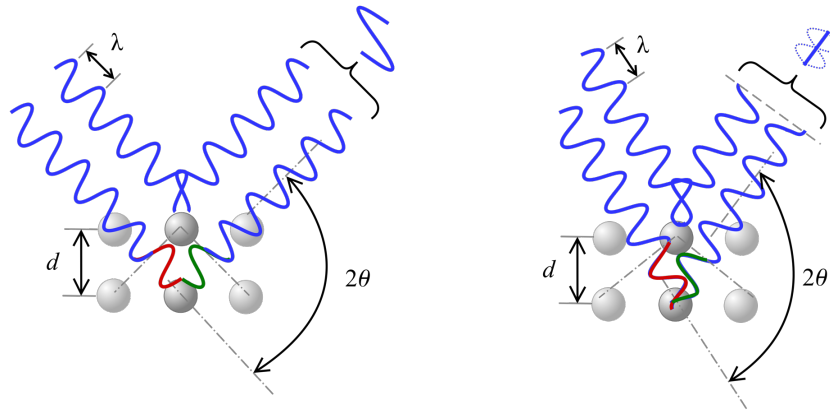
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(1862 – 1940) 

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2/2 Nobel Prize (1915)



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$$n\lambda = 2d \sin \theta$$

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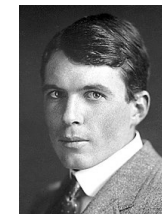
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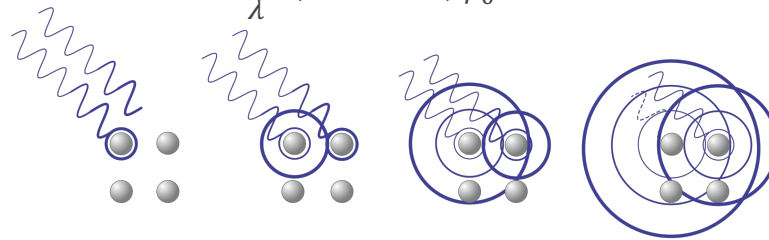


Interaction X-ray matter

- ❖ X-ray electromagnetic waves propagate in the direction of \mathbf{u} and oscillate in time and space:

$$A_0 \cos(2\pi \mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0) \rightarrow \Re\{\psi(\mathbf{r}, t) = A_0 \exp[-i\omega t] \exp[i(2\pi \mathbf{k} \cdot \mathbf{r} + \varphi_0)]\}$$

$$\mathbf{k} = \frac{1}{\lambda} \mathbf{u}; \quad \omega = 2\pi\nu; \quad \varphi_0 = 0$$

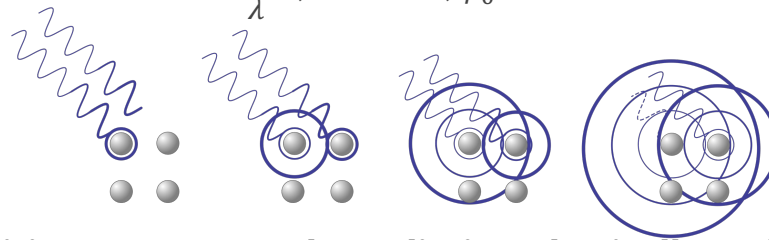


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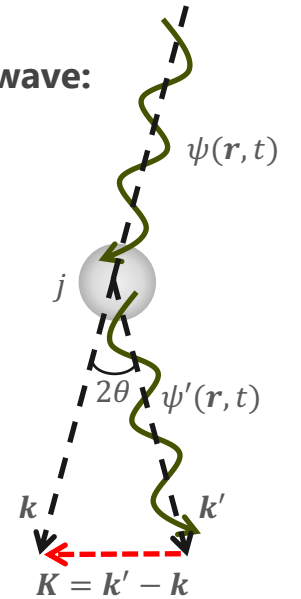
- ❖ For one atom j , located at position \mathbf{r}_j , scatters the radiation elastically and is assumed to emit a plane wave:

$$\text{Elastic} \Rightarrow \|\mathbf{k}\| = \|\mathbf{k}' = \frac{1}{\lambda} \mathbf{u}'\|$$

$$\psi'_j(\mathbf{r}, t) = A_0 f_j \exp[-i\omega t] \exp[\phi_j]$$

$$\phi_j = -2\pi\mathbf{k} \cdot \mathbf{r}_j - 2\pi\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}_j)$$

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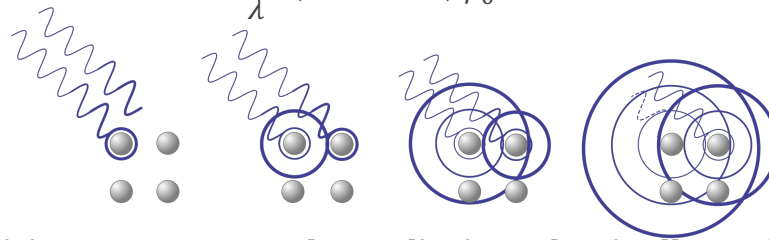


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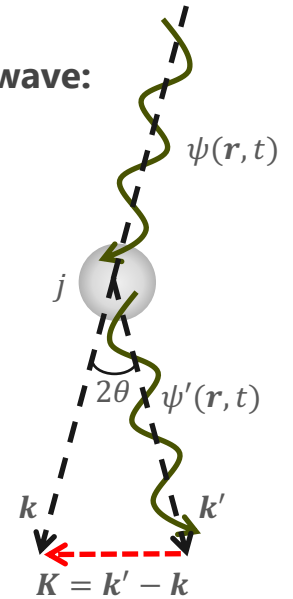
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- ❖ For a given unit cell of the crystal composed of n atoms:

$$\psi'(\mathbf{r}, t) = \sum_{j=1}^n \psi'_j(\mathbf{r}, t) = A_0 \exp[-i(\omega t + 2\pi\mathbf{k}' \cdot \mathbf{r})] F(\mathbf{K})$$

$$\text{Structure factor} \Rightarrow F(\mathbf{K}) = \sum_{j=1}^n f_j \exp[2\pi i\mathbf{K} \cdot \mathbf{r}_j]$$

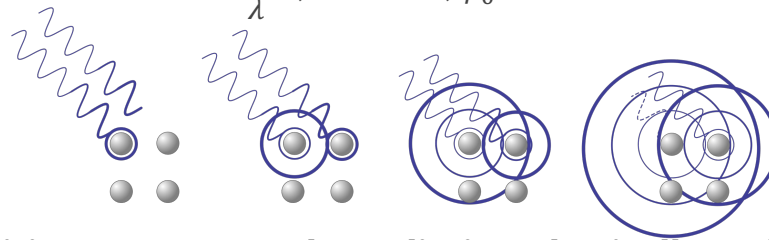


Interaction X-ray matter

- ❖ X-ray electromagnetic waves propagate in the direction of \mathbf{u} and oscillate in time and space:

$$A_0 \cos(2\pi\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0) \rightarrow \Re\{\psi(\mathbf{r}, t) = A_0 \exp[-i\omega t] \exp[i(2\pi\mathbf{k} \cdot \mathbf{r} + \varphi_0)]\}$$

$$\mathbf{k} = \frac{1}{\lambda} \mathbf{u}; \quad \omega = 2\pi\nu; \quad \varphi_0 = 0$$



- ❖ For one atom j , located at position \mathbf{r}_j , scatters the radiation elastically and is assumed to emit a plane wave:

$$\text{Elastic} \Rightarrow \|\mathbf{k}\| = \|\mathbf{k}' = \frac{1}{\lambda} \mathbf{u}'\|$$

$$\psi'_j(\mathbf{r}, t) = A_0 f_j \exp[-i\omega t] \exp[\phi_j]$$

$$\phi_j = -2\pi\mathbf{k} \cdot \mathbf{r}_j - 2\pi\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}_j)$$

$$\psi'_j(\mathbf{r}, t) = A_0 f_j \exp[-i(\omega t + 2\pi\mathbf{k}' \cdot \mathbf{r})] \exp[2\pi i\mathbf{K} \cdot \mathbf{r}_j]$$

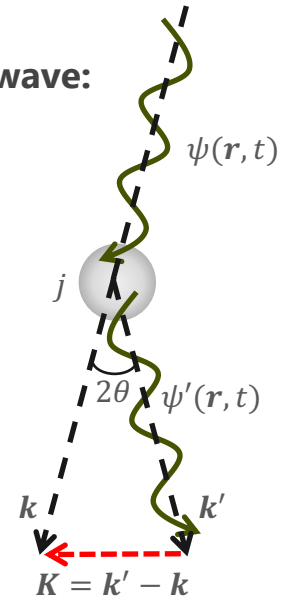
- ❖ For a given unit cell of the crystal composed of n atoms:

$$\psi'(\mathbf{r}, t) = \sum_{j=1}^n \psi'_j(\mathbf{r}, t) = A_0 \exp[-i(\omega t + 2\pi\mathbf{k}' \cdot \mathbf{r})] F(\mathbf{K})$$

$$\text{Structure factor} \Rightarrow F(\mathbf{K}) = \sum_{j=1}^n f_j \exp[2\pi i\mathbf{K} \cdot \mathbf{r}_j]$$

- ❖ Diffracted intensity:

$$I(\mathbf{K}) \propto |\psi'(\mathbf{r}, t)|^2$$



Diffraction condition

- ❖ **The diffracted intensity:**

$$I(\mathbf{K}) \propto |\psi'(\mathbf{r}, t)|^2 ; \psi'(\mathbf{r}, t) = A_0 \exp[-i(\omega t + 2\pi \mathbf{k}' \cdot \mathbf{r})] F(\mathbf{K})$$

- ❖ **The diffracted intensity is maximal when:**

$$\max\{F(\mathbf{K})\} = \max \left\{ \sum_{j=1}^n f_j \exp[2\pi i \mathbf{K} \cdot \mathbf{r}_j] \right\} \Rightarrow \mathbf{K} \cdot \mathbf{r}_j \in \mathbb{Z}, \forall j$$

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❖ **In a crystal:**

- A crystal is composed of several unit cells located at positions \mathbf{R}_m , a Bravais vector
- The atoms inside each unit cell are positioned at \mathbf{r}_j

$$\begin{aligned} \mathbf{r}_{jm} &= \mathbf{R}_m + \mathbf{r}_j \\ \mathbf{R}_m &= u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3 \end{aligned}$$

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- The total diffracted wave:

$$\psi'(\mathbf{r}, t) \propto \sum_j f_j \exp[2\pi i \mathbf{K} \cdot \mathbf{r}_j] \sum_m \exp[2\pi i \mathbf{K} \cdot \mathbf{R}_m]$$

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- The Laue condition:

$$\exp[2\pi i \mathbf{K} \cdot \mathbf{R}_m] \neq 0 \Rightarrow \mathbf{K} \cdot \mathbf{R}_m \in \mathbb{Z} \Rightarrow \mathbf{K} \cdot \mathbf{a}_i \in \mathbb{Z}$$

Therefore, under the diffraction condition, \mathbf{K} is identified with a vector \mathbf{G} whose basis is defined as:

$$\begin{aligned} \mathbf{G} &= h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3 \\ \mathbf{b}_i &= 2\pi \frac{\mathbf{a}_j \times \mathbf{a}_k}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)} \Rightarrow \mathbf{b}_i \cdot \mathbf{a}_i = 2\pi \delta_{ij} \end{aligned}$$

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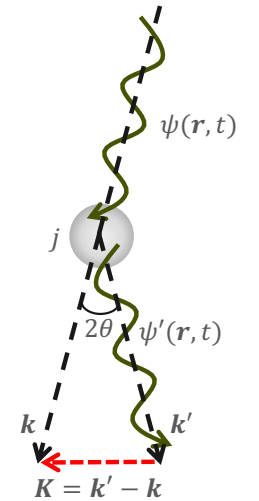
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The Bragg's law

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$$\hookrightarrow n\lambda = 2d \sin \theta$$



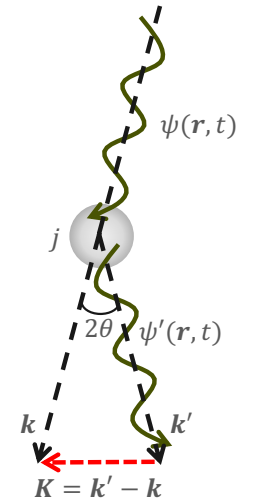
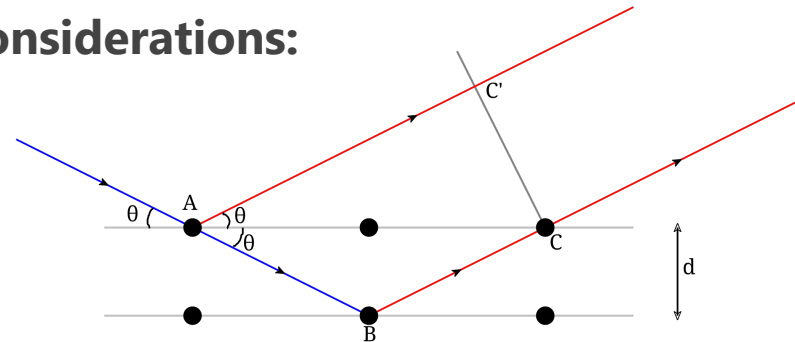
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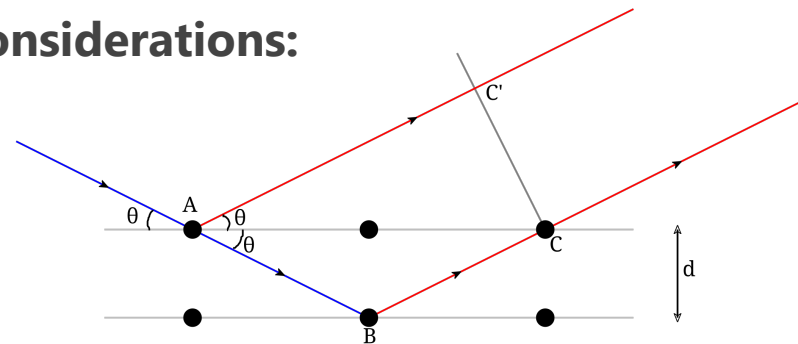
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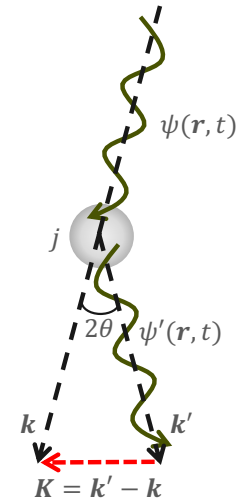
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❖ From geometrical considerations:



- The path difference $\Rightarrow (AB) - (BC) - (AC') = \frac{d}{\sin \theta} + \frac{d}{\sin \theta} - \frac{2d}{\tan \theta} \cos \theta$
- Constructive diffraction occurs when both waves are in phase $\Rightarrow n\lambda = (AB) - (BC) - (AC')$

$$\hookrightarrow n\lambda = 2d \sin \theta$$



The Bragg's law: extra

❖ $\|G\|$:

$$G = hb_1 + kb_2 + lb_3$$

$$b_i = 2\pi \frac{a_j \times a_k}{a_1 \cdot (a_2 \times a_3)} \Rightarrow b_i \cdot a_i = 2\pi \delta_{ij}$$

$$\|G\|^2 = G \cdot G = (hb_1 + kb_2 + lb_3) \cdot (hb_1 + kb_2 + lb_3) = h^2 b_1 \cdot b_1 + k^2 b_2 \cdot b_2 + l^2 b_3 \cdot b_3 + 2hk b_1 \cdot b_2 + 2hl b_1 \cdot b_3 + 2kl b_2 \cdot b_3$$

Let's suppose a orthogonal unit cell:

$$b_i = 2\pi \frac{a_j \times a_k}{a_1 \cdot (a_2 \times a_3)} \Rightarrow b_1 = 2\pi \frac{bc}{abc} a_1 = \frac{2\pi}{a} a_1; b_2 = \frac{2\pi}{b} a_2; b_3 = \frac{2\pi}{c} a_3$$

$$\Rightarrow \|G\|^2 = \left(\frac{2\pi h}{a}\right)^2 + \left(\frac{2\pi k}{b}\right)^2 + \left(\frac{2\pi l}{c}\right)^2 = (2\pi)^2 \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)$$

$$\Rightarrow \|G\| = \frac{2\pi}{d_{hkl}}$$

❖ Bragg's law demonstration from Laue conditions:

$$\Rightarrow K = G \Leftrightarrow k' - k = G \Rightarrow (k' - k)^2 = G^2 \Leftrightarrow G^2 = k'^2 + k^2 - 2k' \cdot k$$

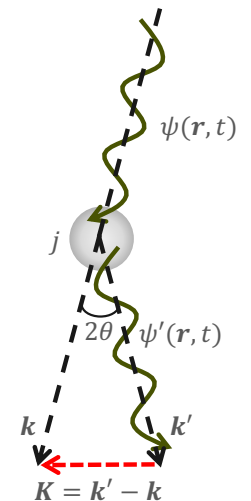
$$\text{Elastic scattering} \Rightarrow \|k'\| = \|k\| = k$$

$$\Rightarrow G^2 = 2k^2 - 2k^2 \cos 2\theta = 2k^2(1 - \cos 2\theta) = 4k^2 \sin^2 \theta$$

$$\text{But } \|G\| = \frac{2\pi}{d_{hkl}}$$

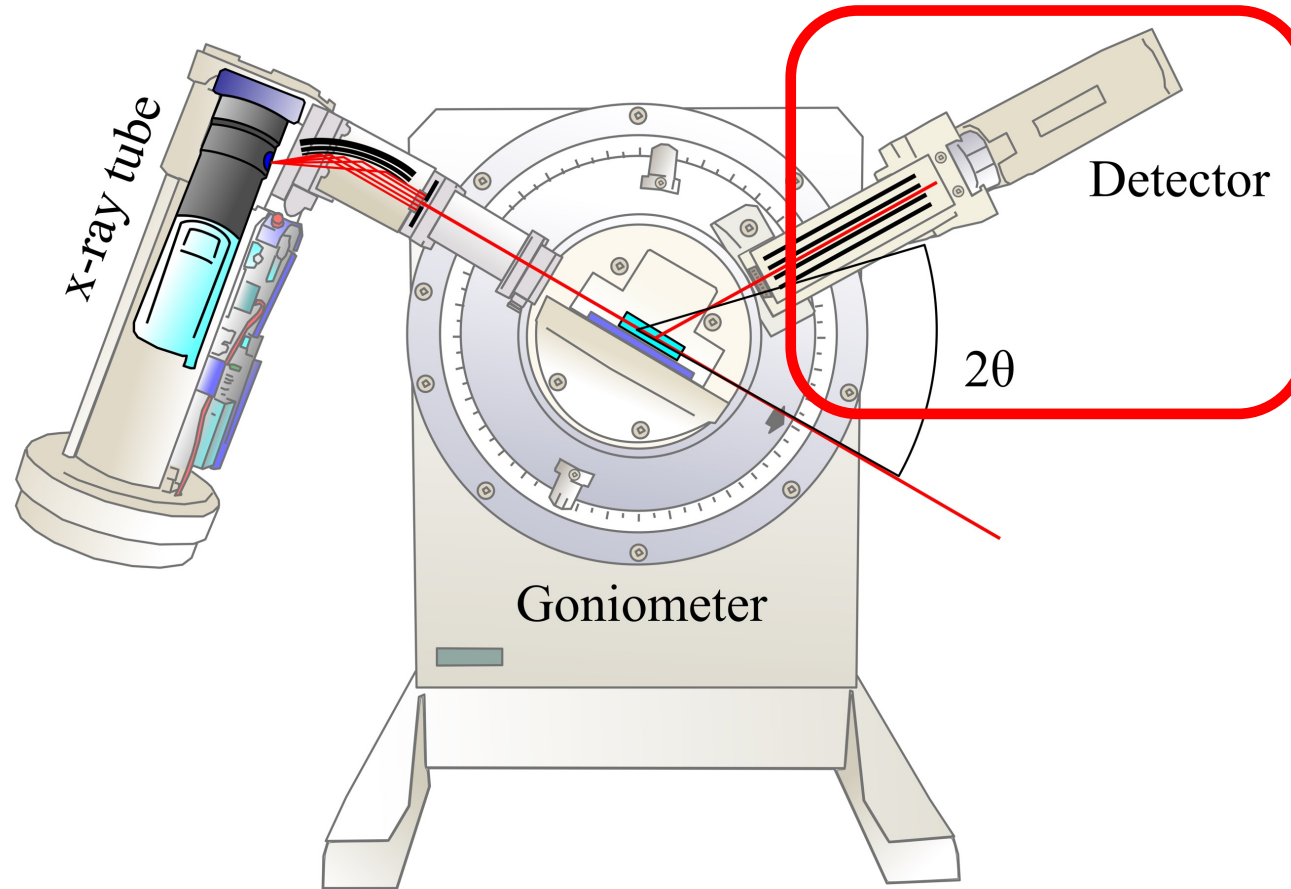
$$\Rightarrow \frac{2\pi}{d_{hkl}} = 4k^2 \sin^2 \theta = 4 \left(\frac{2\pi}{\lambda}\right)^2 \sin^2 \theta$$

$$\Rightarrow 2d_{hkl} \sin \theta = \lambda$$



The experimental setup: the diffractometer

The diffractometer (Bragg-Brentano configuration)

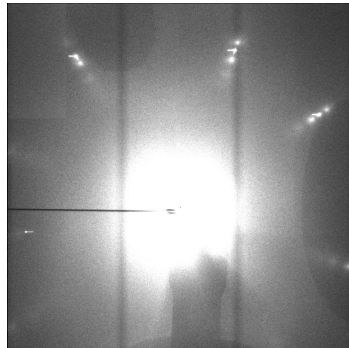


X-ray diffraction techniques

Polychromatic beam

$$n\lambda = 2d \sin \theta$$

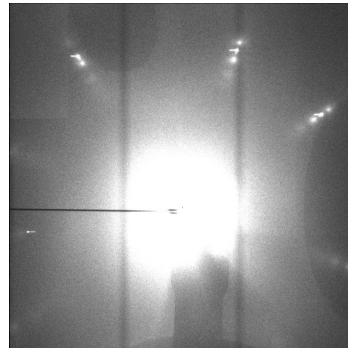
- ❖ Several wave lengths [$\lambda_1; \lambda_2$]
 - ❖ Sample: one monocrystal
 - ❖ In the Bragg's law:
 - Fixed: d, θ
 - Varied: λ
- ⇒ **Isolated spots corresponding to the $\{\lambda, \theta, d\}$ combinations**



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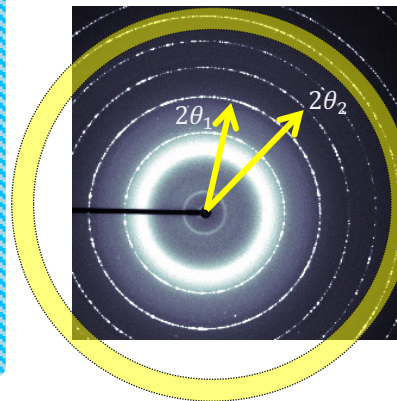
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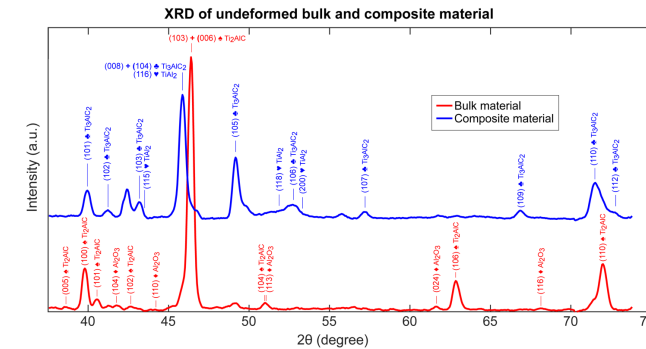
$$n\lambda = 2d \sin \theta$$

Monochromatic beam

- ❖ One unique waves length λ
 - ❖ Sample: powder (random oriented crystals)
 - ❖ In the Bragg's law:
 - Fixed: λ
 - Varied: d, θ
- ⇒ **Concentric rings or peak diffractograms**



$$\int_0^{2\pi} I(r, \varphi) d\varphi$$

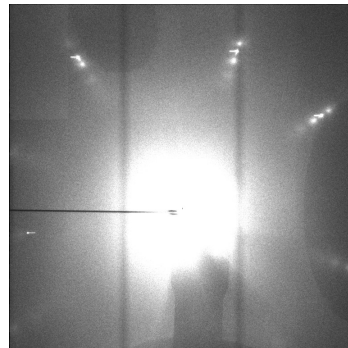


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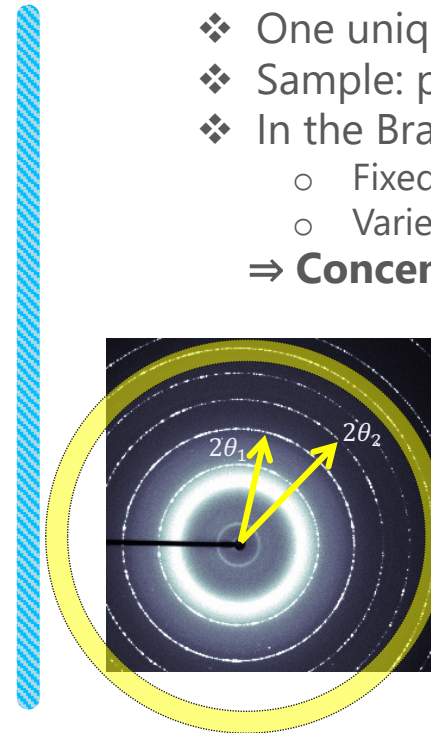
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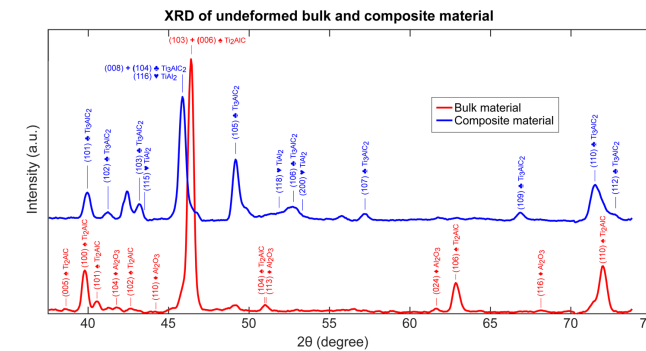


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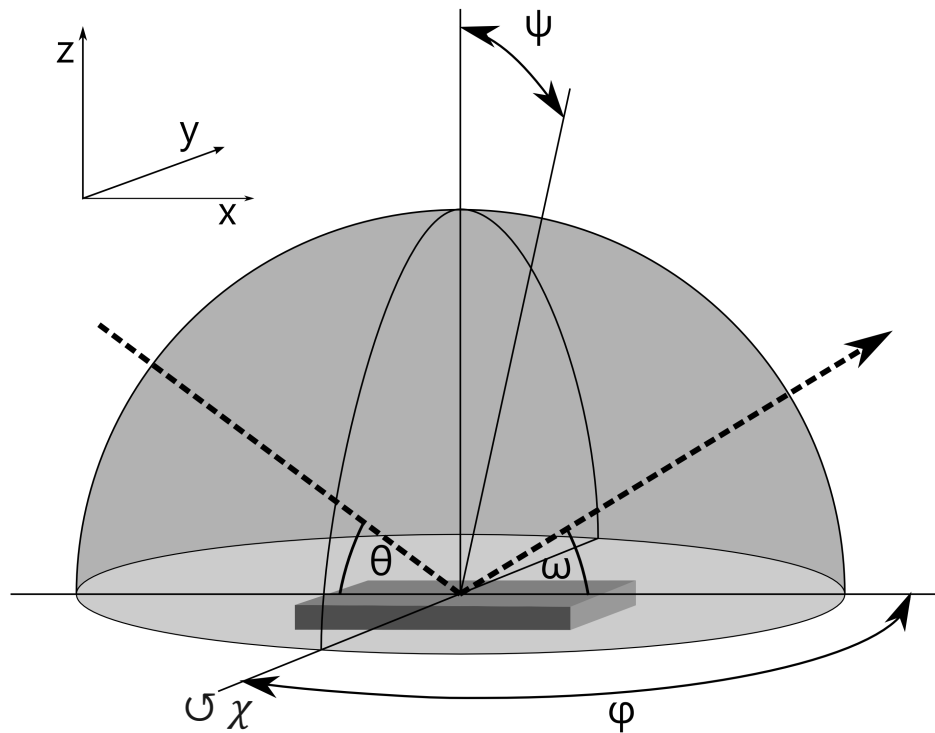


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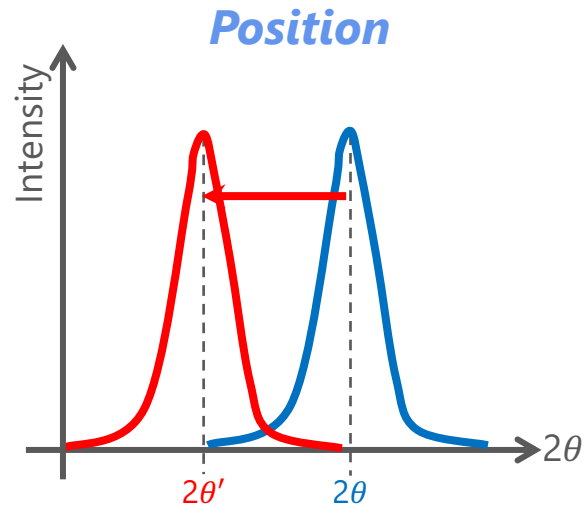
↪ One crystal, many wavelengths – that's Laue.
 ↪ Many crystals, one wavelength – that's powder.

Angle definitions for powder diffraction



- ❖ 2θ : angle between the incident beam and the diffracted beam
- ❖ ω : angle of incidence, i.e. the angle between the incident beam and the sample surface
- ❖ ψ : angle between the sample surface normal and the normal to the diffracting planes
- ❖ χ : tilt angle of the sample, measured from the vertical axis
- ❖ φ : in-plane rotation angle of the sample around the surface normal

Shape of the diffracted peaks



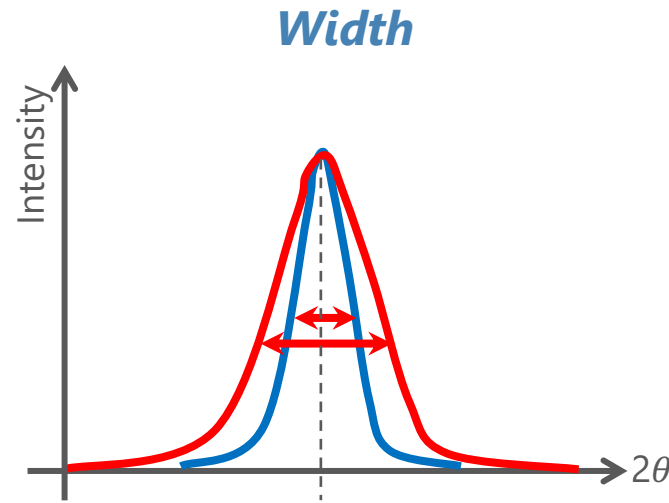
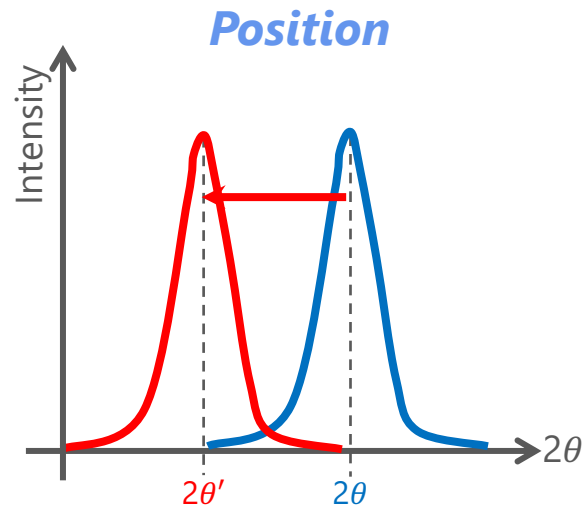
❖ Peak shift:

- $n\lambda = 2d_{hkl} \sin \theta$
 - ✓ $2\theta \searrow \Rightarrow \sin \theta \searrow \Rightarrow d_{hkl} \nearrow \Rightarrow$ macrotraction
 - ✓ $2\theta \nearrow \Rightarrow \sin \theta \nearrow \Rightarrow d_{hkl} \searrow \Rightarrow$ macrocompression

❖ Average elastic strain in the direction normal to the $(h k l)$ planes:

$$\varepsilon_{hkl}^e = \frac{\Delta d_{hkl}}{d_{hkl}^0}$$

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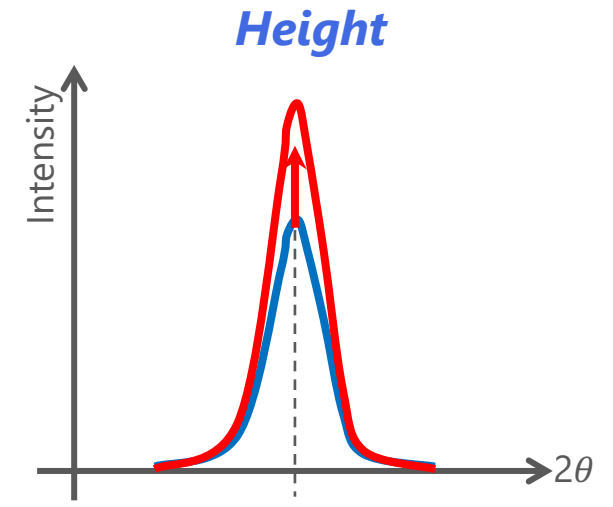
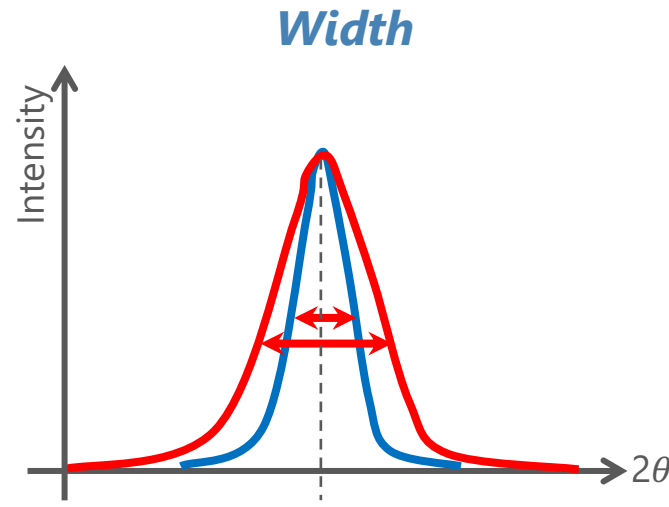
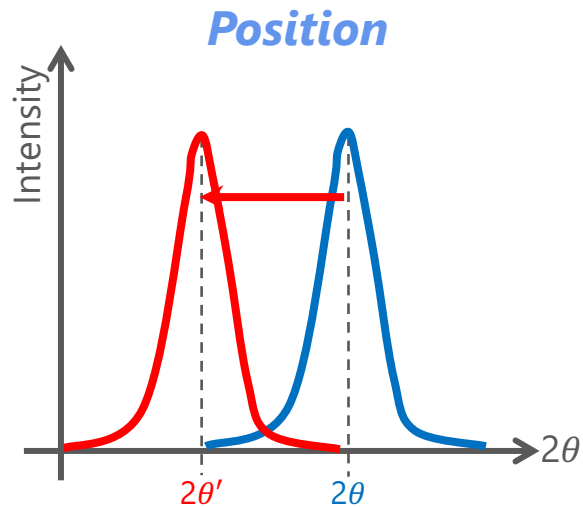
❖ Peak width:

- Crystallite size (L)

$$FWHM \propto \frac{\lambda}{L \cos \theta}$$
- Lattice strain heterogeneities
 - ✓ Dislocation density (ρ)

$$\rho \approx \frac{FWHM^2 \cos^2 \theta}{Kb^2}$$
 - ✓ Microstrain

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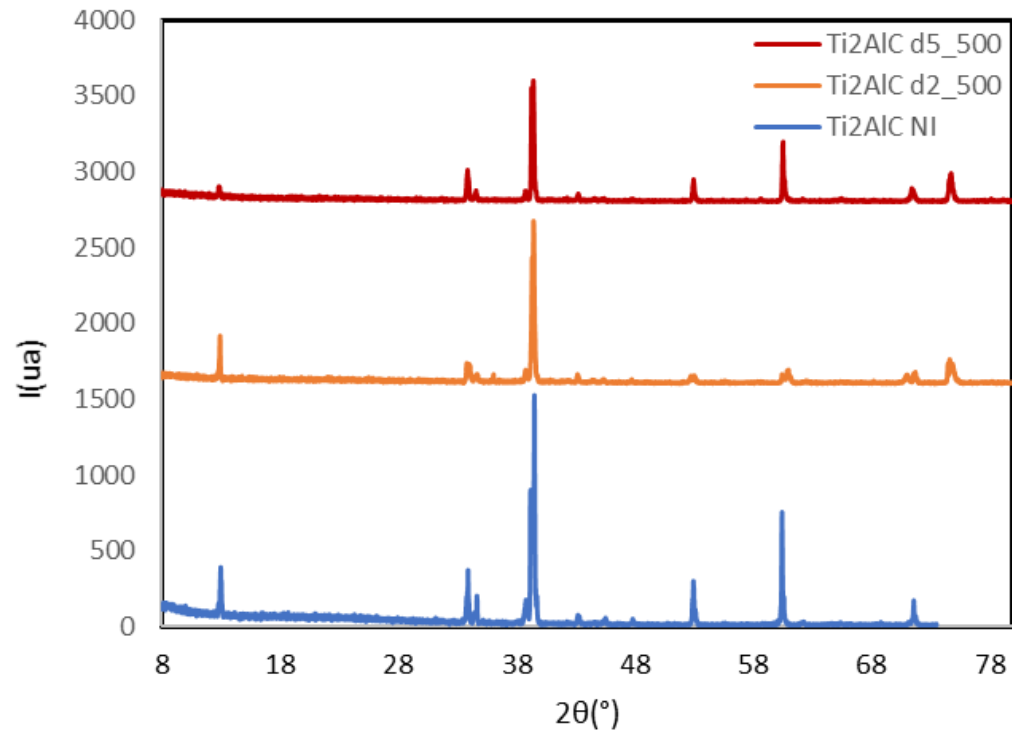
❖ Peak height:

- Structure factor (F)
- Phases
- Texture

Example: Irradiation of Ti_2AlC with Ni^{3+} ions

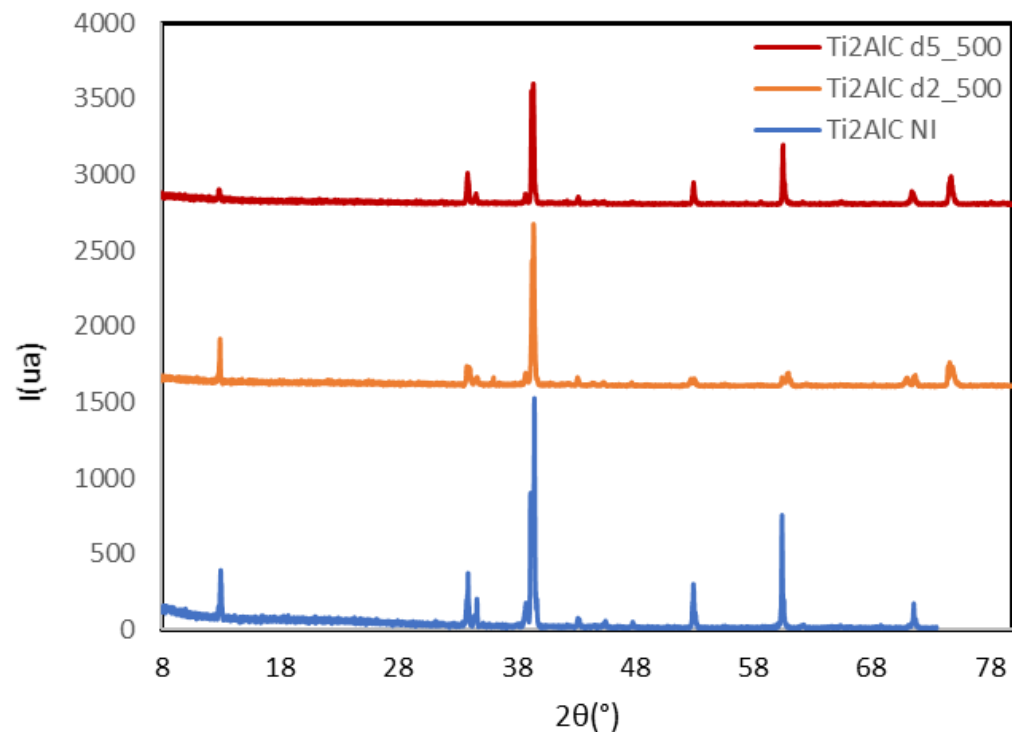
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Irradiation Ni^{3+} (@500°C)	
Fluence 1	$2 \cdot 10^{15} \text{ cm}^{-2}$
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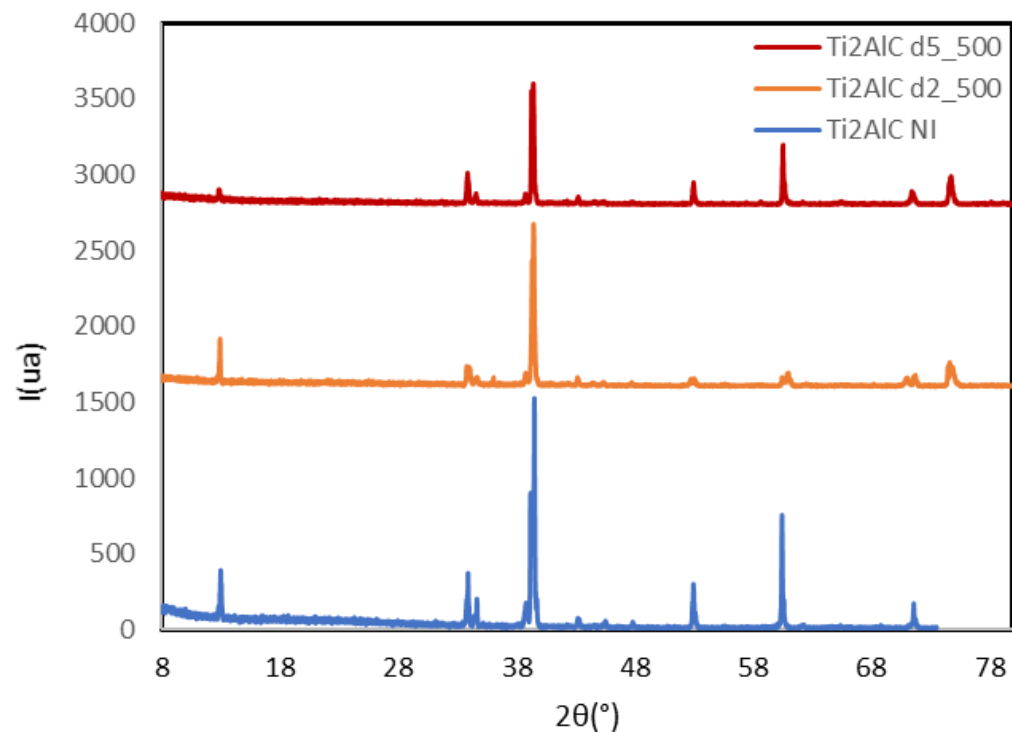


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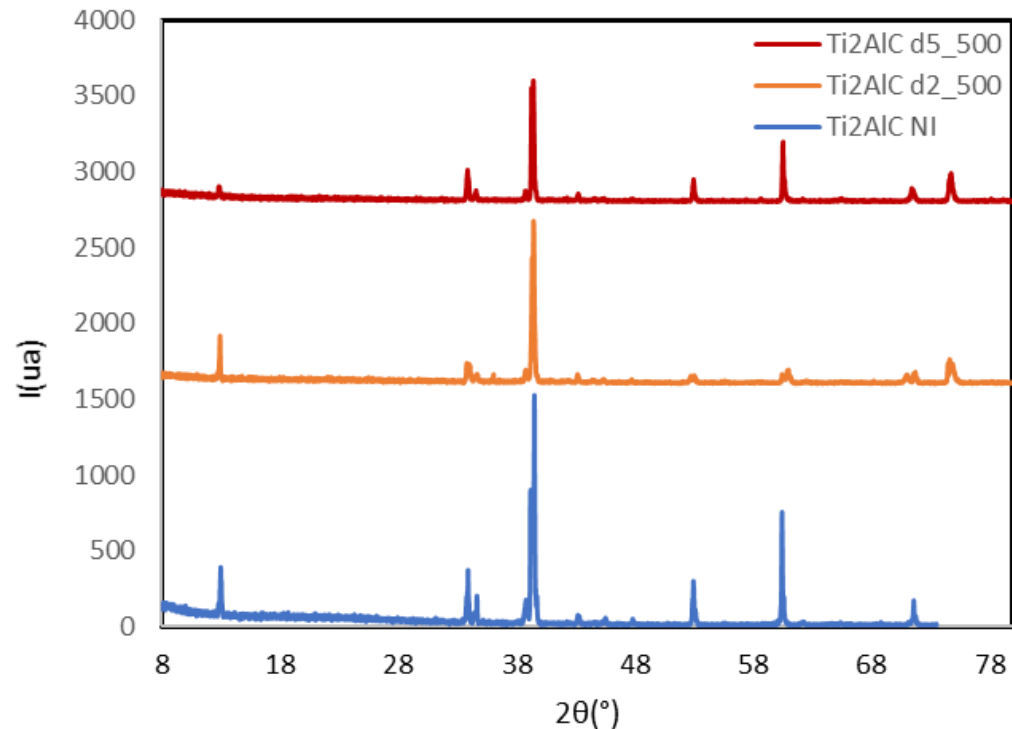
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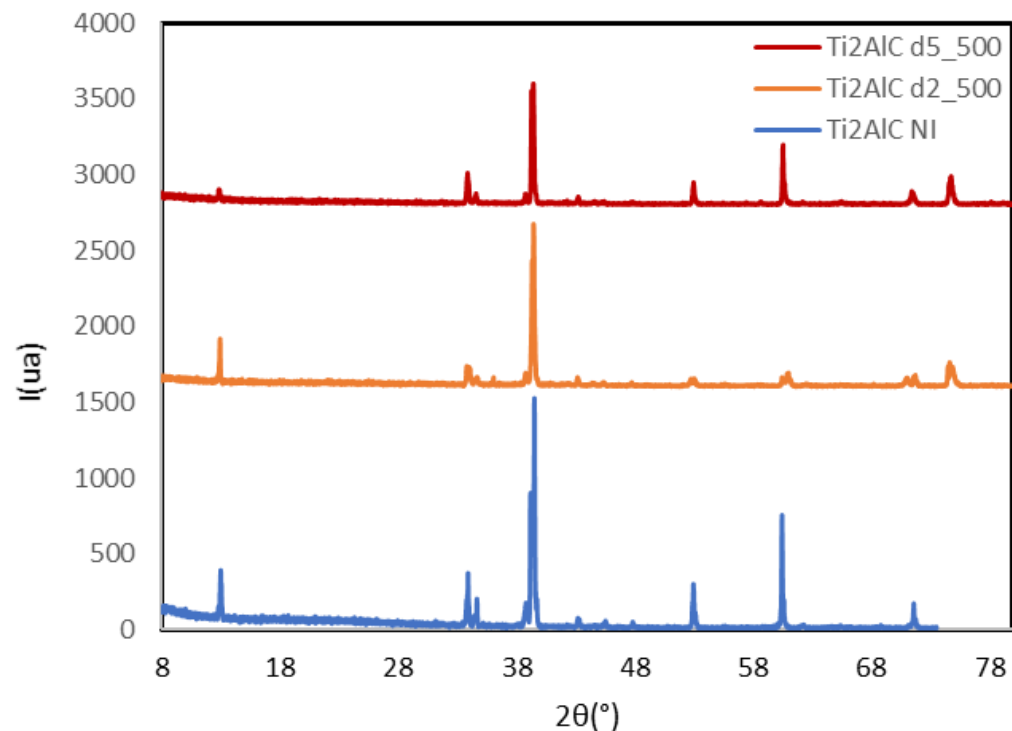
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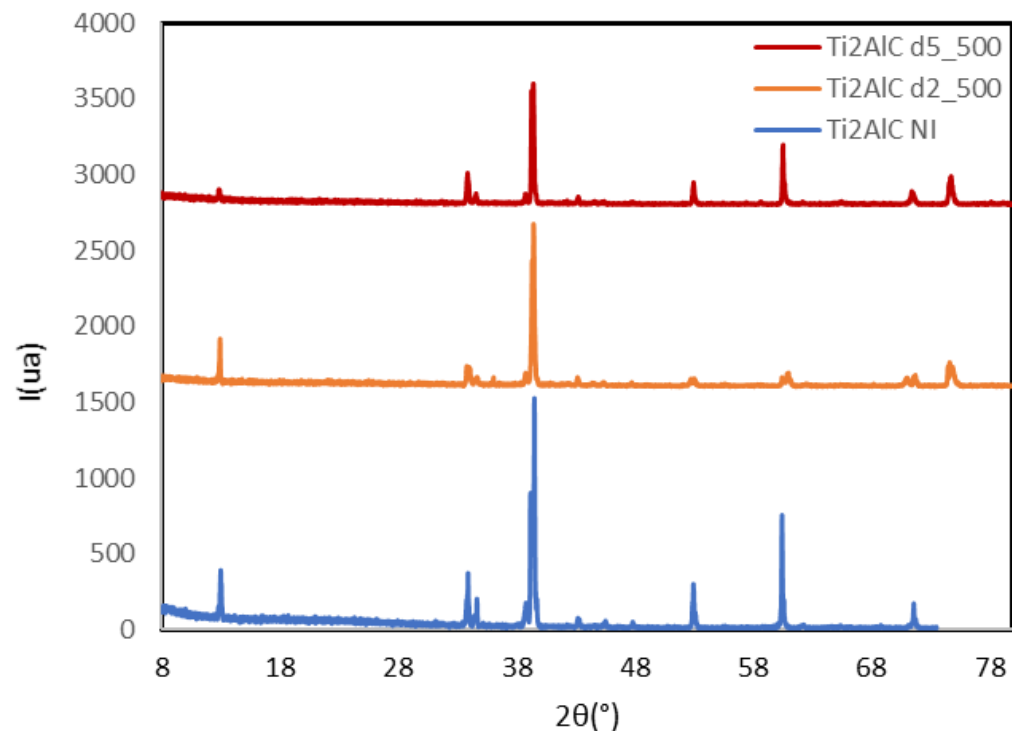
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❖ No clear new phases

- Within this 2θ range, no obvious additional peaks appear, suggesting no major secondary phases were formed (at least above detection limit).

Example: Irradiation of Ti_2AlC with Ni^{3+} ions

Irradiation Ni^{3+} (@500°C)	
Fluence 1	$2 \cdot 10^{15} \text{ cm}^{-2}$
Fluence 2	$5 \cdot 10^{15} \text{ cm}^{-2}$



↳ The structure of Ti_2AlC seems preserved after irradiation, which is significant for radiation tolerance.

❖ Phase identification

- All three diffractograms show sharp peaks typical of crystalline Ti_2AlC .

❖ Comparison of intensities

- The as-received / non-irradiated sample shows strong and well-defined peaks.
- The irradiated samples present similar peak positions but with slightly reduced peak intensities, suggesting a change in crystallinity.

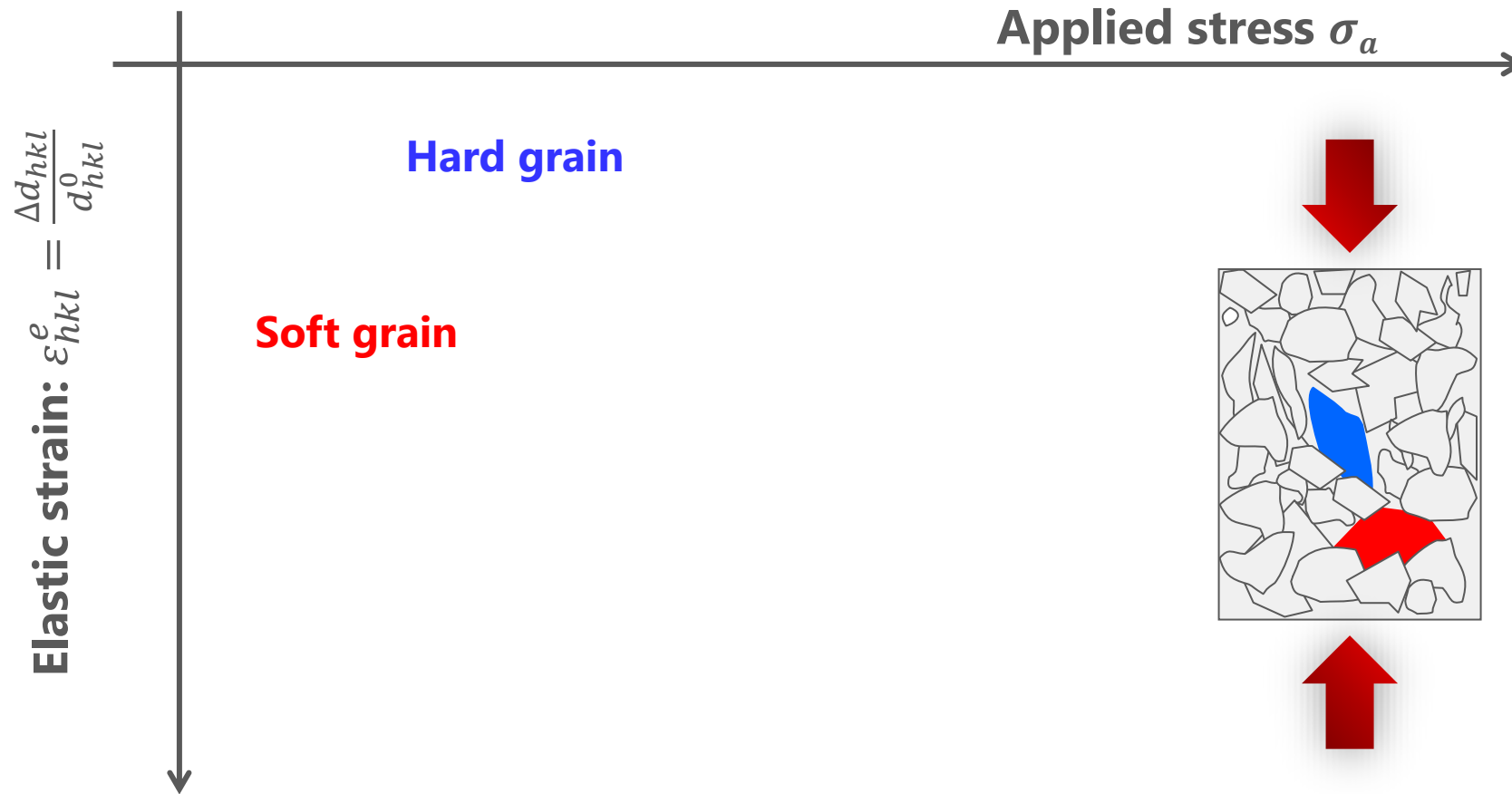
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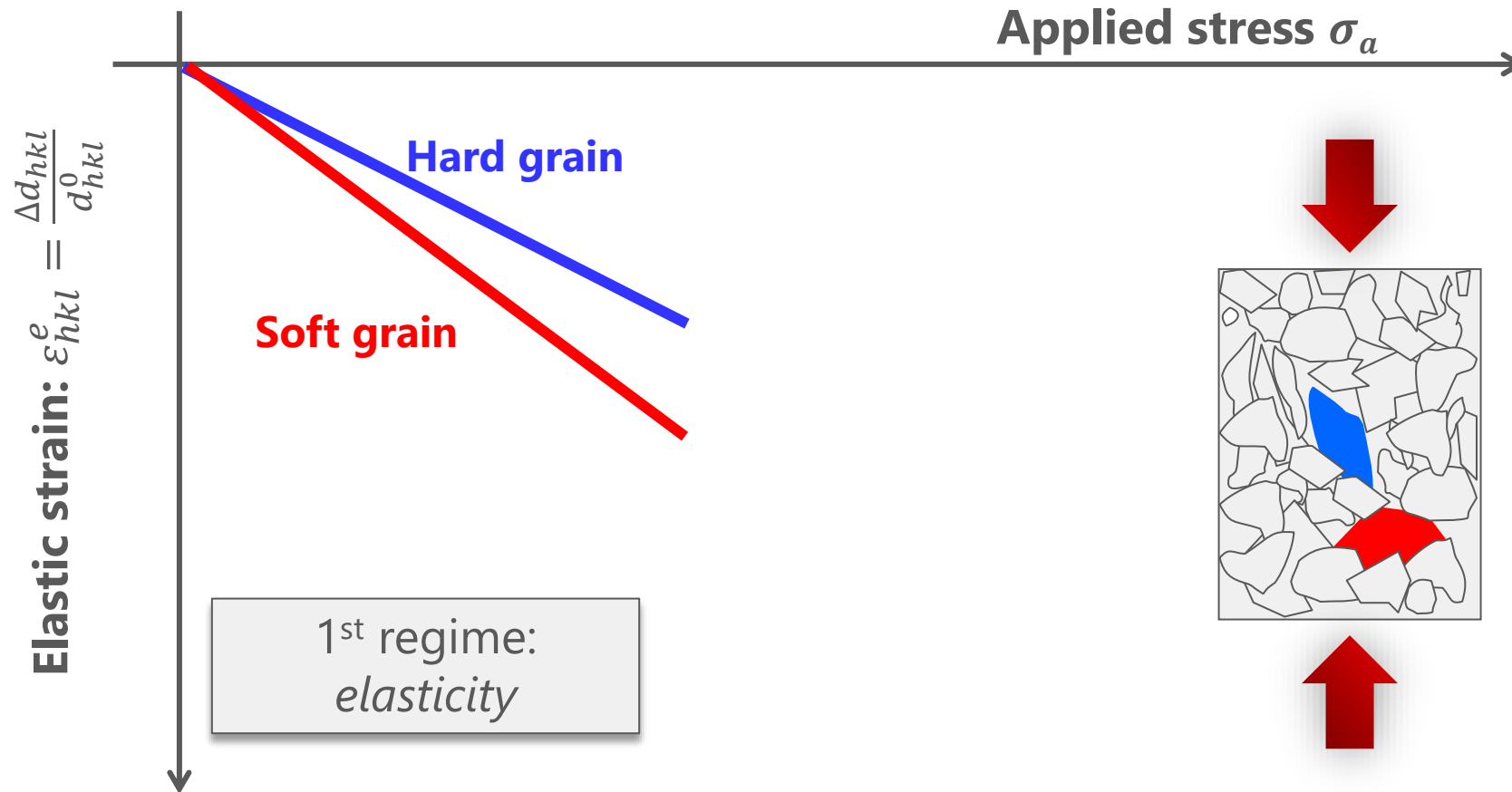
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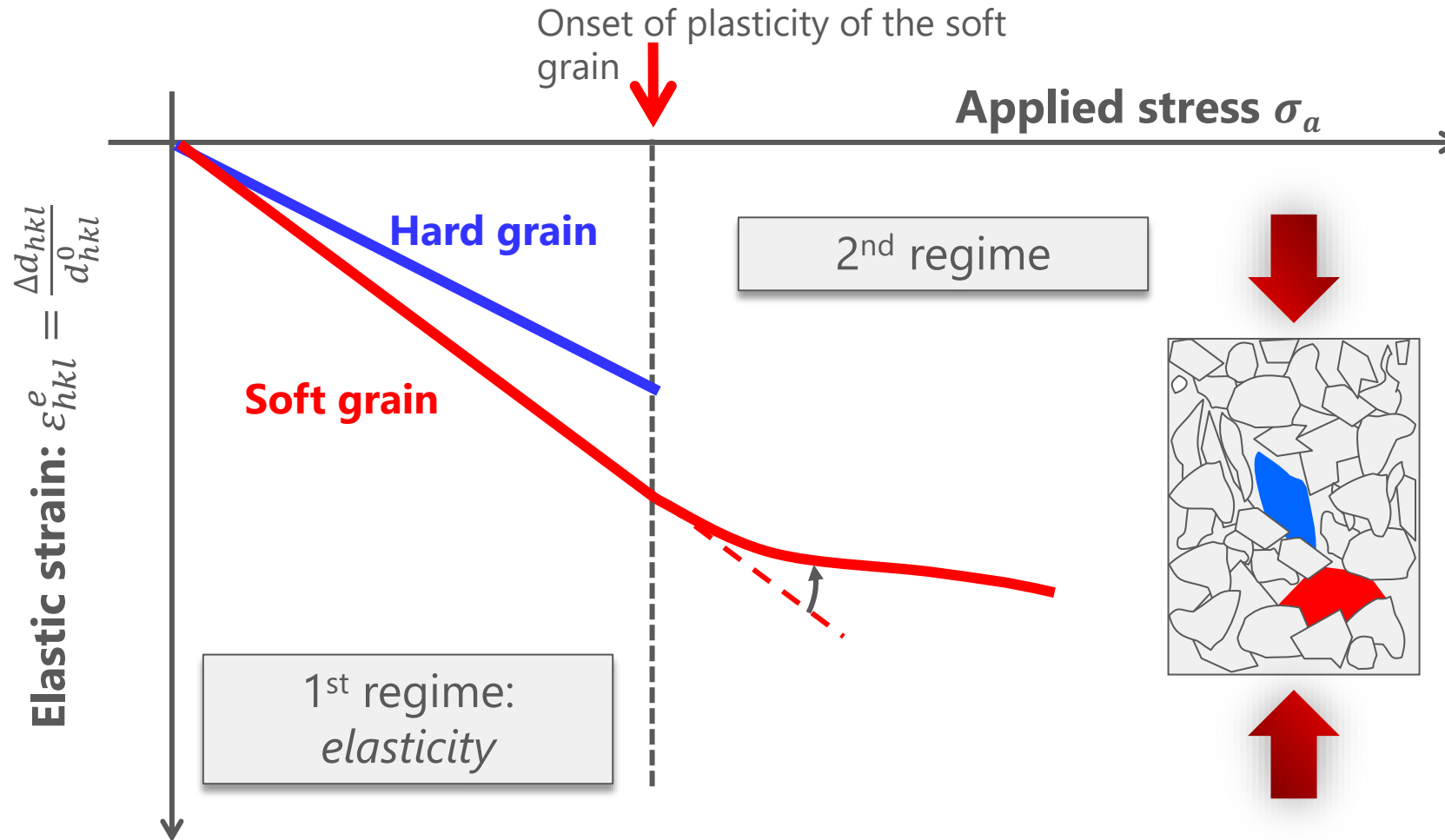
Evolution of the elastic strain



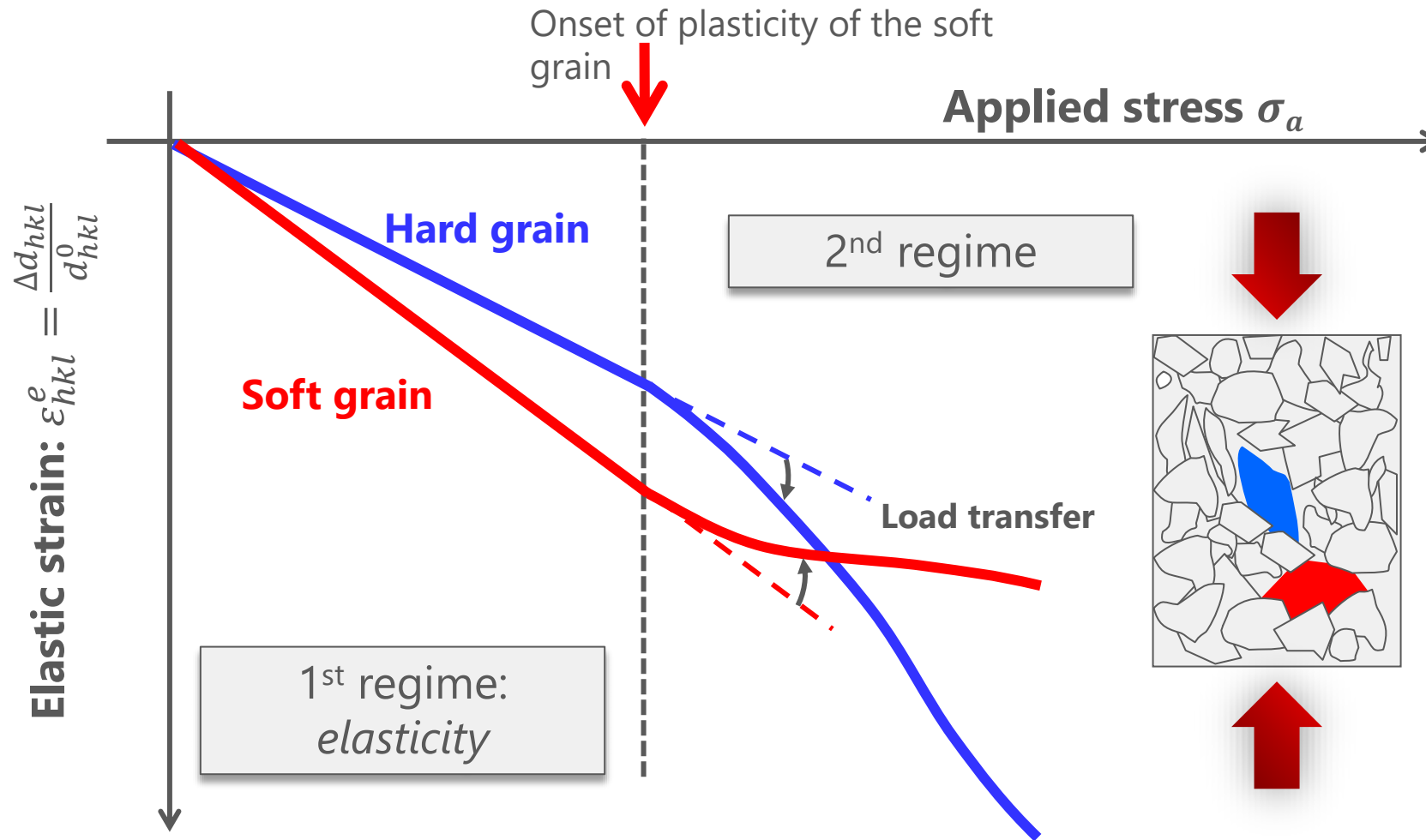
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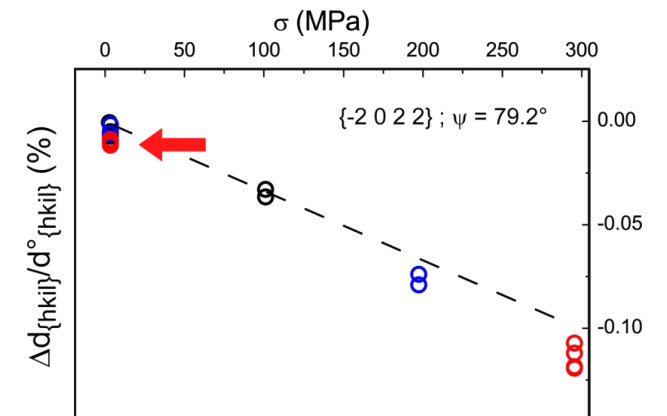
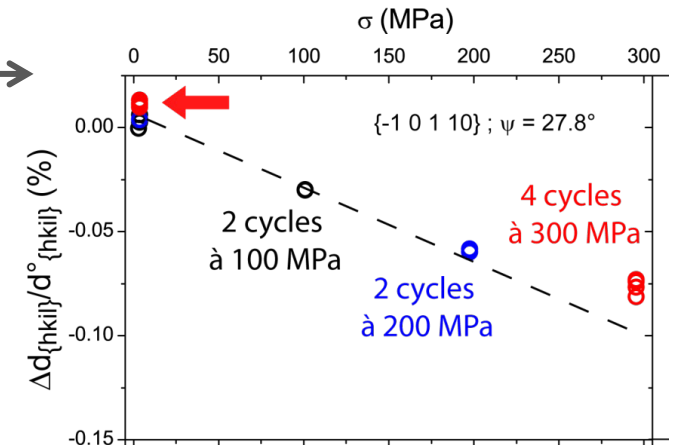
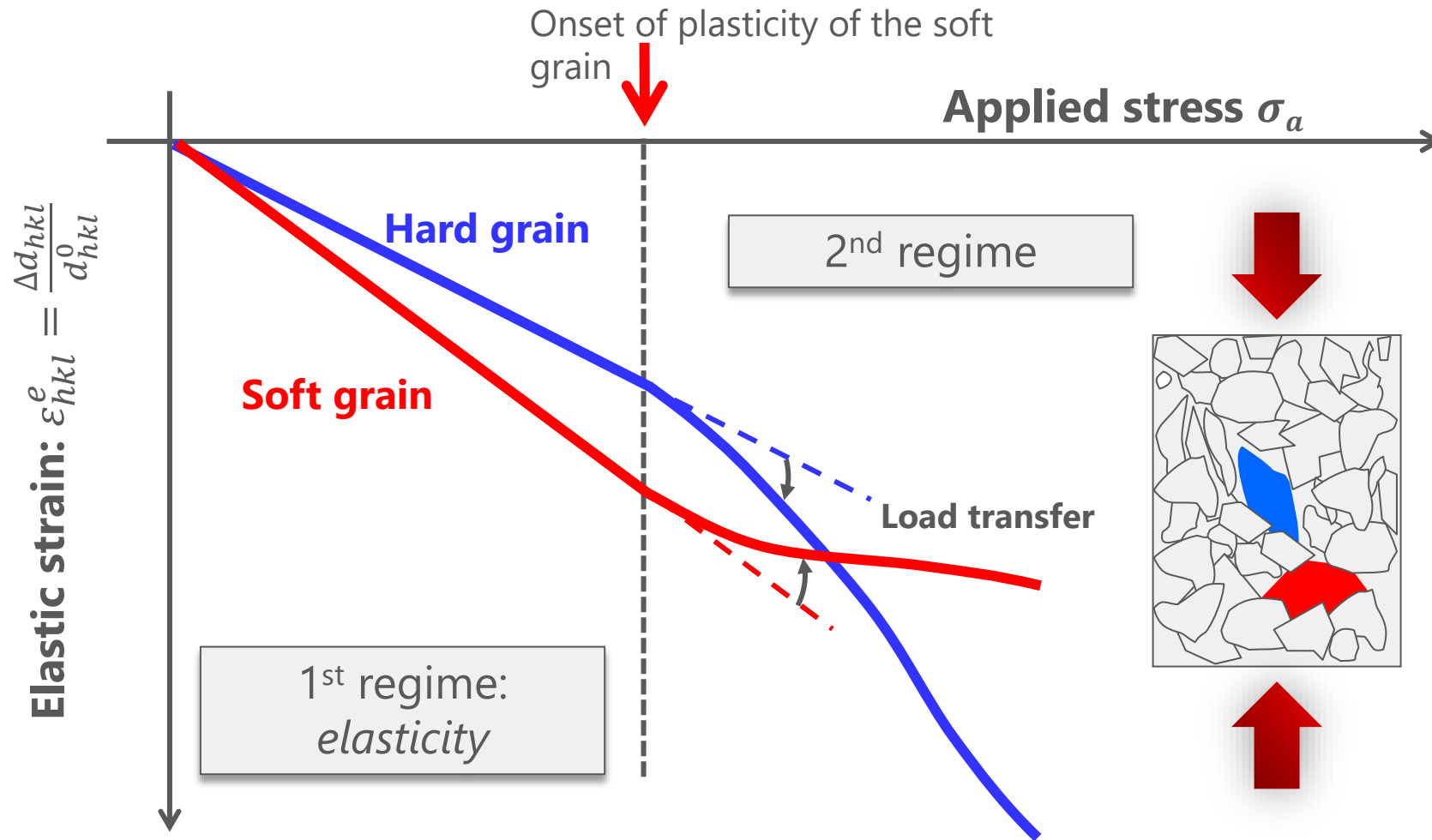
Evolution of the elastic strain



Evolution of the elastic strain



Evolution of the elastic strain



Residual stresses

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Residual stresses are internal stresses that remain within a material after all external forces or thermal gradients have been removed.

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❖ **Strain from diffraction (from the experiment):**

- Elastic strain:

$$\varepsilon_{hkl}^{\psi\varphi} = \frac{d_{hkl}^{\psi\varphi} - d_{hkl}^0}{d_{hkl}^0} = \frac{\Delta d_{hkl}^{\psi\varphi}}{d_{hkl}^0}$$

$\varepsilon_{hkl}^{\psi\varphi}$: strain in the direction defined by ψ ; $d_{hkl}^{\psi\varphi}$: measured interplanar spacing at tilt angle ψ ; d_{hkl}^0 : stress-free interplanar spacing

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- The Bragg's law:

$$\begin{aligned} \lambda &= 2d_{hkl}^{\psi\varphi} \sin \theta_{hkl} \Rightarrow d_{hkl}^{\psi\varphi} = \frac{\lambda}{2 \sin \theta_{hkl}} \Rightarrow \frac{dd_{hkl}^{\psi\varphi}}{d\theta_{hkl}} = -\frac{\lambda \cos \theta_{hkl}}{2 \sin^2 \theta_{hkl}} = -d_{hkl}^{\psi\varphi} \cot \theta_{hkl} \Rightarrow \Delta d_{hkl}^{\psi\varphi} \\ &= -d_{hkl}^{\psi\varphi} \cot \theta_{hkl} \Delta \theta_{hkl} \end{aligned}$$

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$$\varepsilon_{hkl}^{\psi\varphi} = \frac{1+\nu}{E} \sigma_\varphi \sin^2 \psi - \frac{\nu}{E} (\sigma_1 + \sigma_2) \Rightarrow \varepsilon_{hkl}^{\psi\varphi} = f(\sin^2 \psi) = A \cdot \sin^2 \psi + B$$

Where $\sigma_\varphi = \sigma_1 \cos^2 \varphi + \sigma_2 \sin^2 \varphi$

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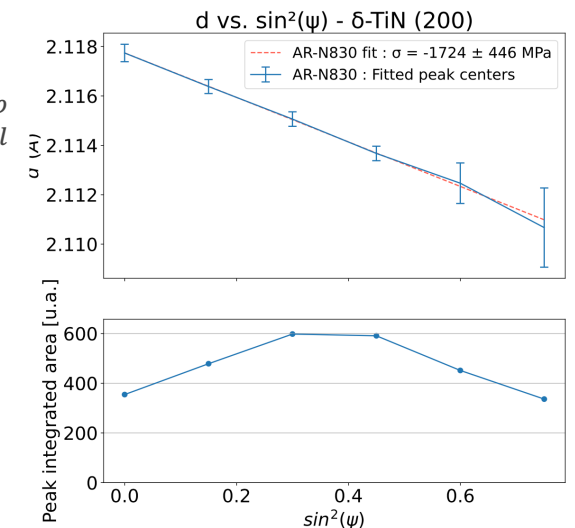
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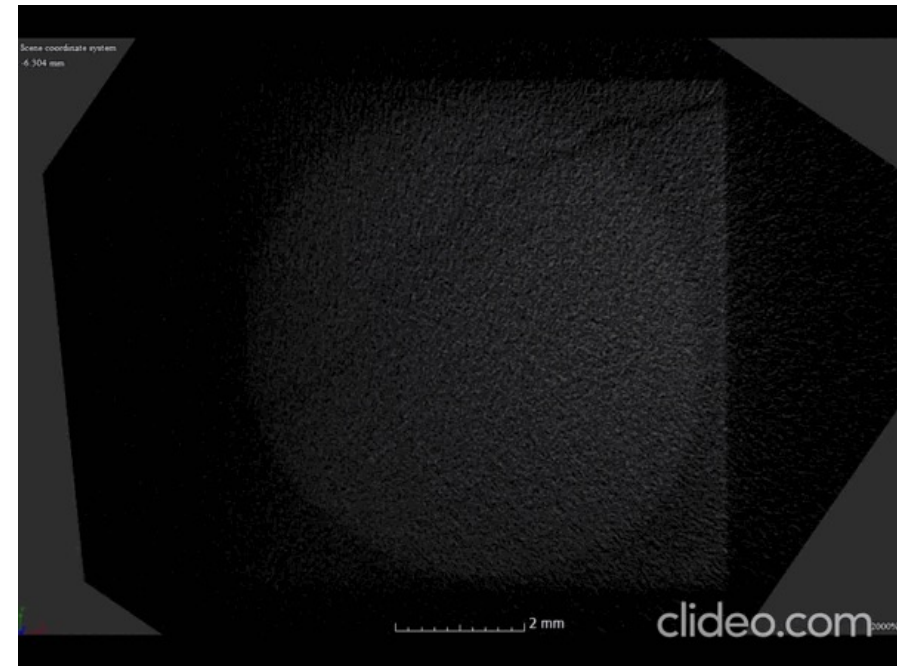
↳ Plotting $\varepsilon_{hkl}^{\psi\varphi} = f(\sin^2 \psi)$ yields a straight line; making residual stress measurable through its slope.

X-ray tomography

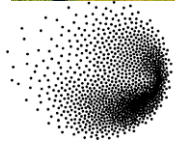
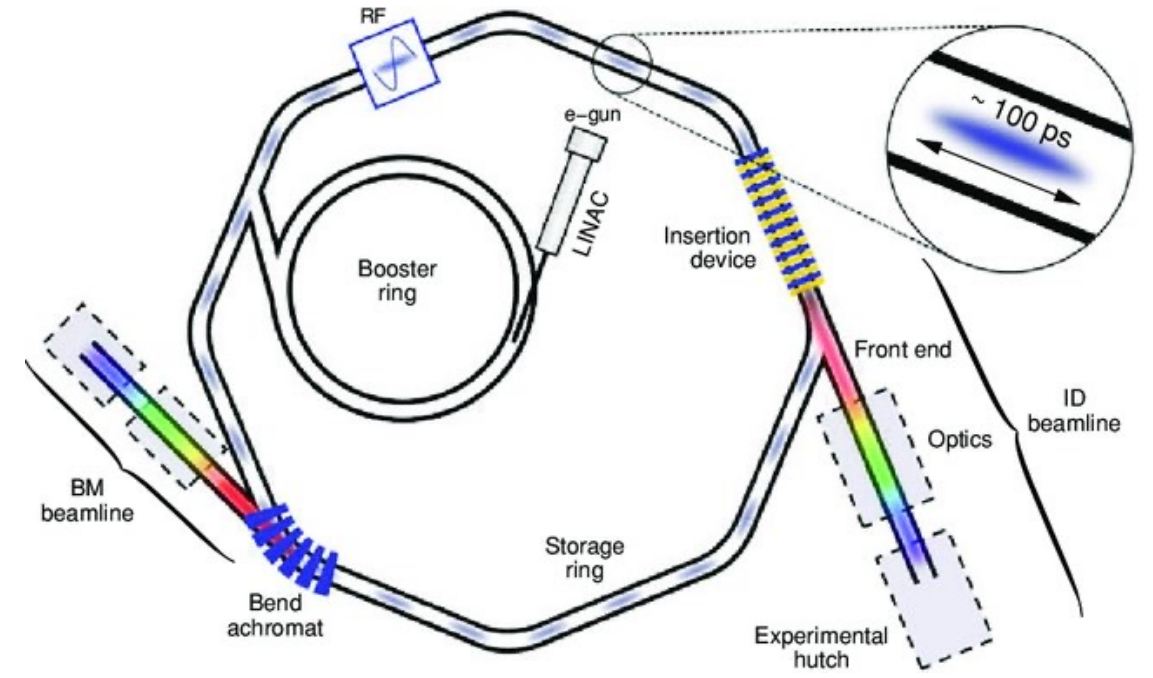
- ❖ Non-destructive 3D imaging technique based on differential X-ray absorption
- ❖ Reconstructs internal structures of materials with micron to sub-micron resolution
- ❖ Contrast arises from density and atomic number differences
- ❖ Applicable to a wide range of materials: metals, polymers, composites, biomaterials
- ❖ Enables quantitative analysis: porosity, cracks, inclusions, grain morphology
- ❖ The Beer-Lambert law:

$$N = N_0 \exp(-\mu_{\rho} \rho t)$$

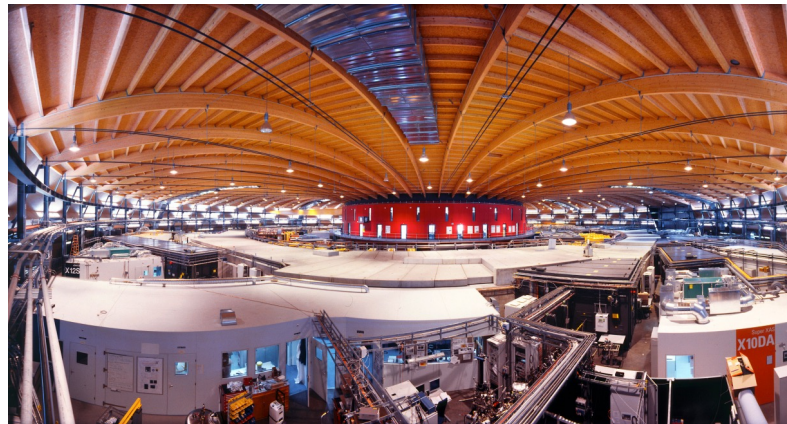
(N : number of photons; N_0 : initial number of photons; ρ : density; t : thickness; μ_{ρ} : mass attenuation coefficient)

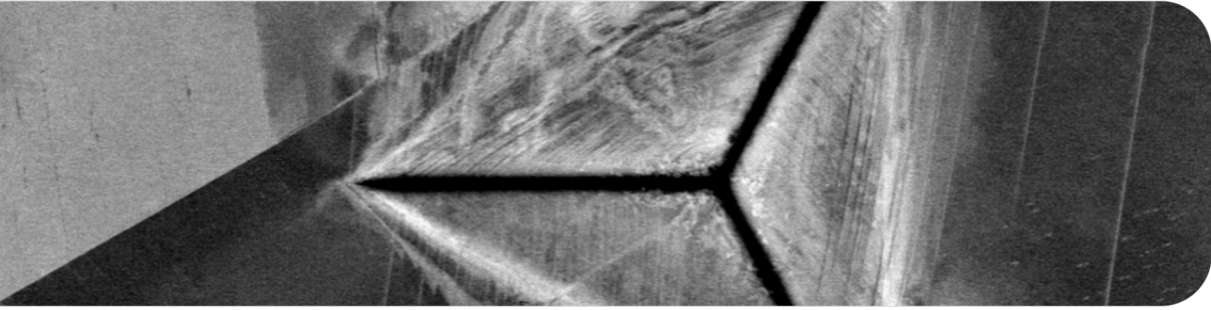


Large x-ray facilities



PSI





Electron microscopy

Micro- Nano-meter scale

The 2 types of electron microscopes

Transmission electron microscope (TEM)



- ❖ Invention: 1931 by Ernst RUSKA et Max KNOLL
- ❖ Specimen : thin foils (~ 100 nm)
- ❖ Price: ~ 1 M€
- ❖ Rare in industry

↳ Nanometric/atomic scale

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Scanning electron microscope (SEM)



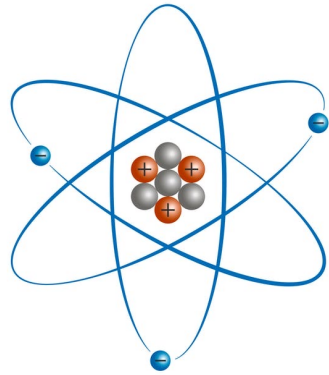
- ❖ Invention: 1937 by Manfred VON ARDENNE but developed in the 1960s
- ❖ Specimen : bulk
- ❖ Price: ~ 500 k€
- ❖ Comon in industry

↳ Meso-/micro-/nano-metric scale

↳ **The two microscopes are complementary.**

Focus on the electron


Particle



Atom structure

- ⊕ Proton
- Neutron
- ⊖ Electron

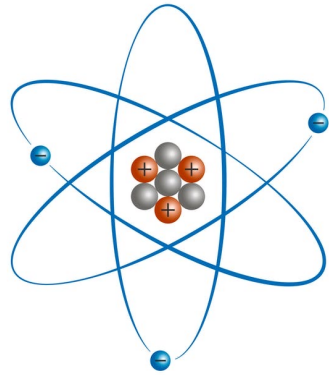


Joseph John THOMSON
(1856–1940) 

- ❖ $m_e = 9.11 \times 10^{-31} \text{ kg}$
- ❖ $e = -1.60 \times 10^{-19} \text{ C}$

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1 Nobel Prize (1929)

Wave

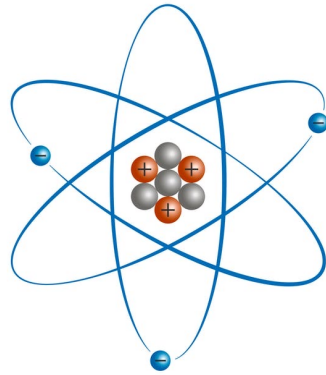
- ❖ All moving particle has wave properties with the wavelength λ being related to the momentum p by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

↳ Electrons act not only as particles but as waves too.

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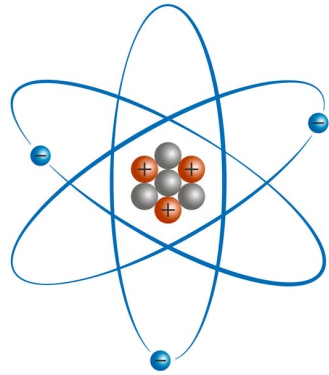
- ❖ Energy of an electron in an electric field of a voltage V :

$$E = eV = \frac{mv^2}{2} \Leftrightarrow v = \sqrt{\frac{2eV}{m}} \Rightarrow \lambda = \frac{h}{\sqrt{2meV}}$$

$$\rightarrow \lambda = \frac{h}{\sqrt{2meV} \left(1 + \frac{eV}{2mc^2}\right)}$$

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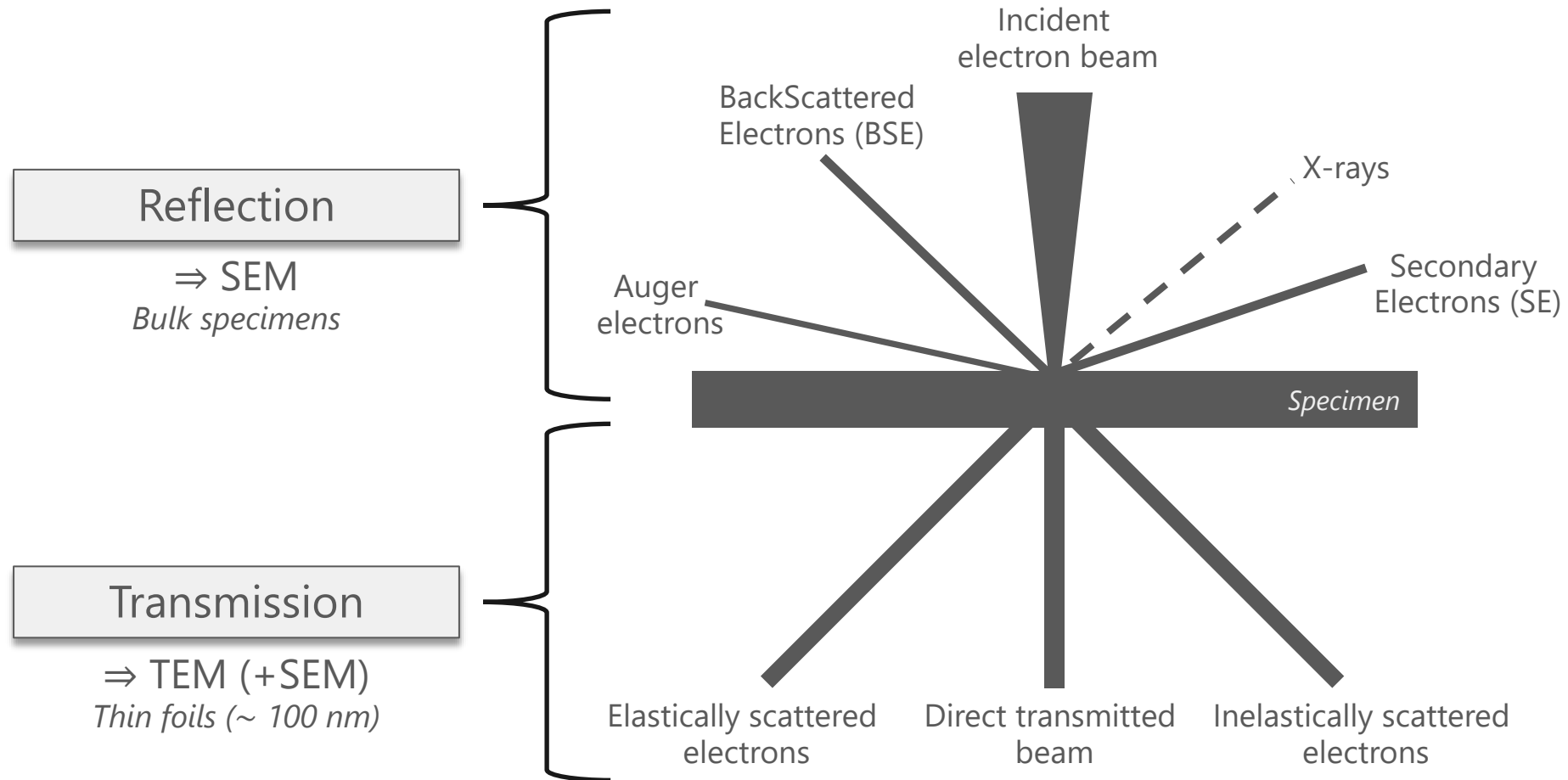
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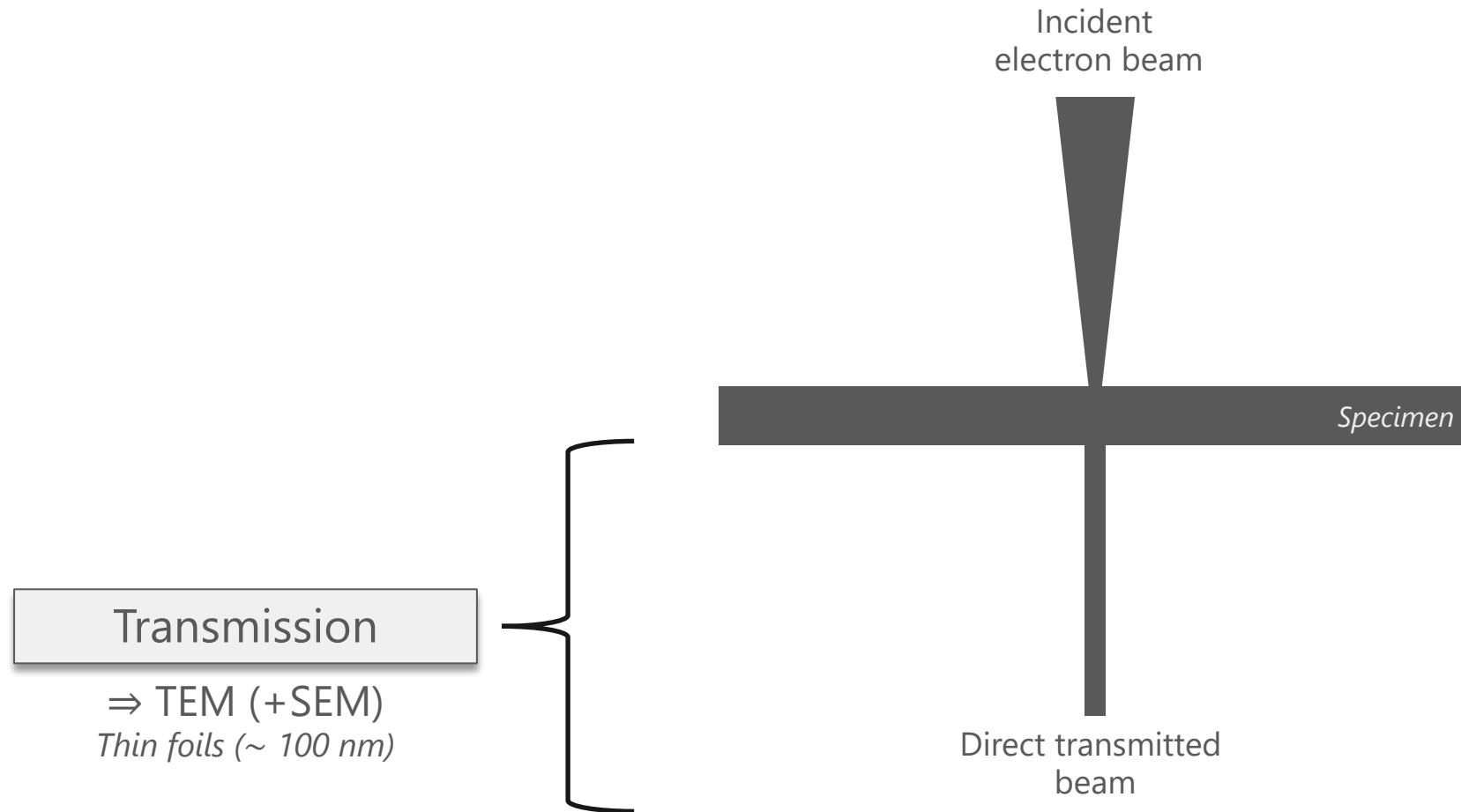
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$\lambda(30 \text{ kV}) = 7 \text{ pm}$
 $\lambda(200 \text{ kV}) = 2.5 \text{ pm}$

Electron-matter interaction

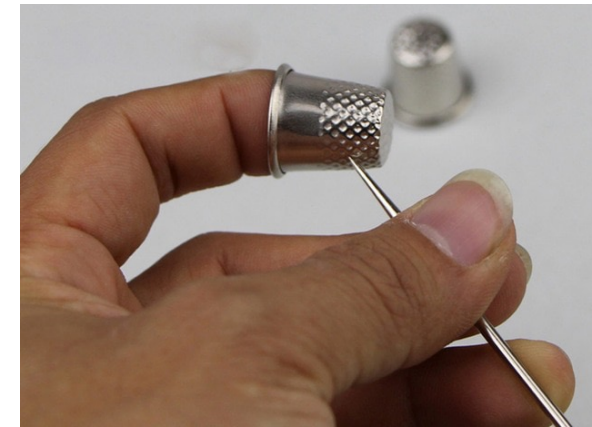
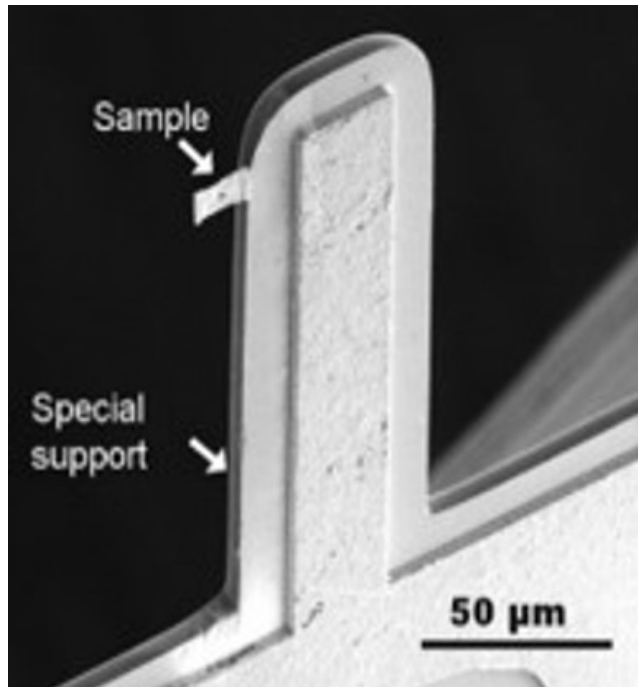


Electron-matter interaction



Specimen thickness

Thickness ≈ 100 nm @ 200 kV (for conventional TEM)

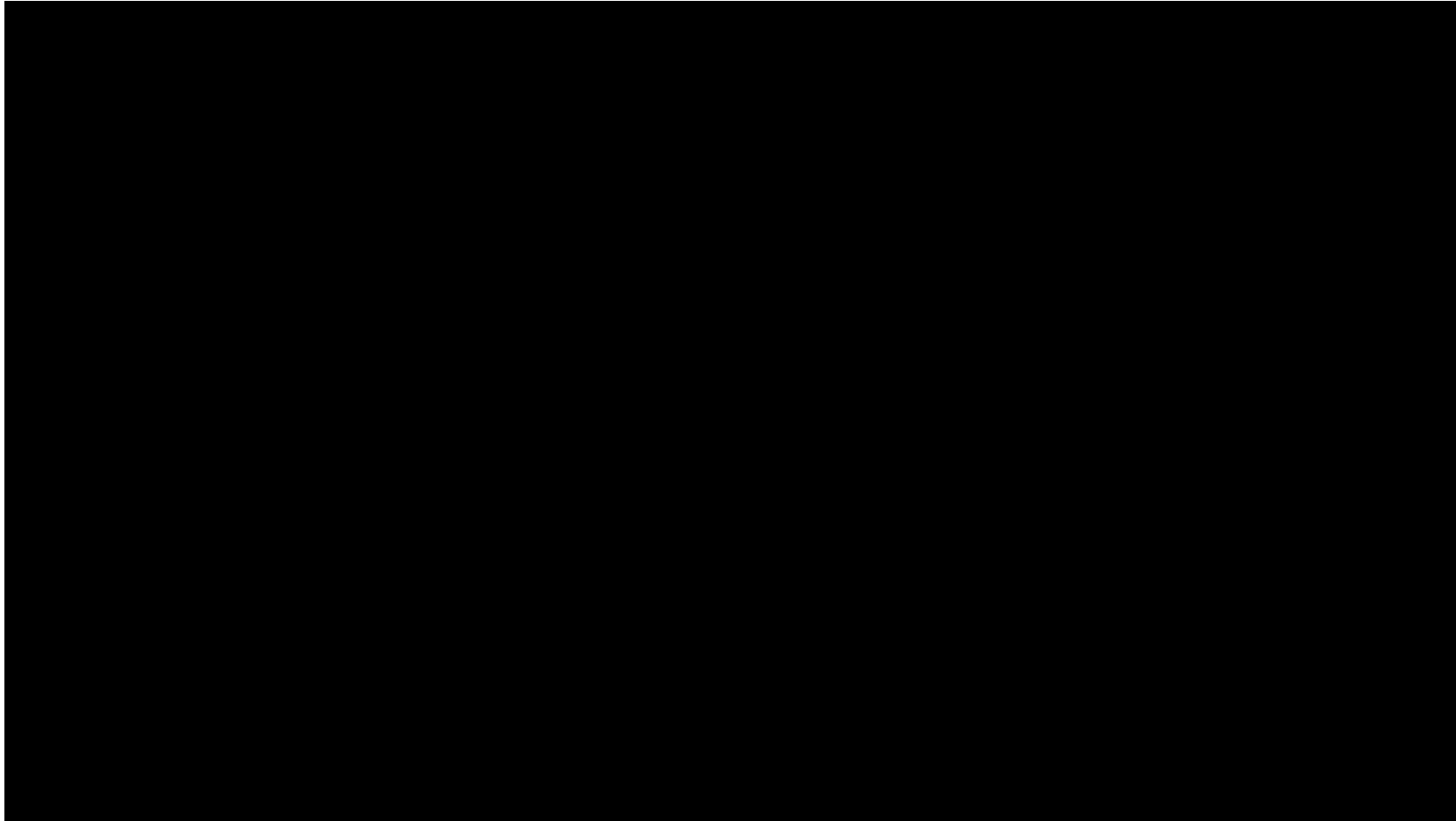


If one is able to assemble all zones studied by TEM since its development in the 40's, they would not even fill a thimble!

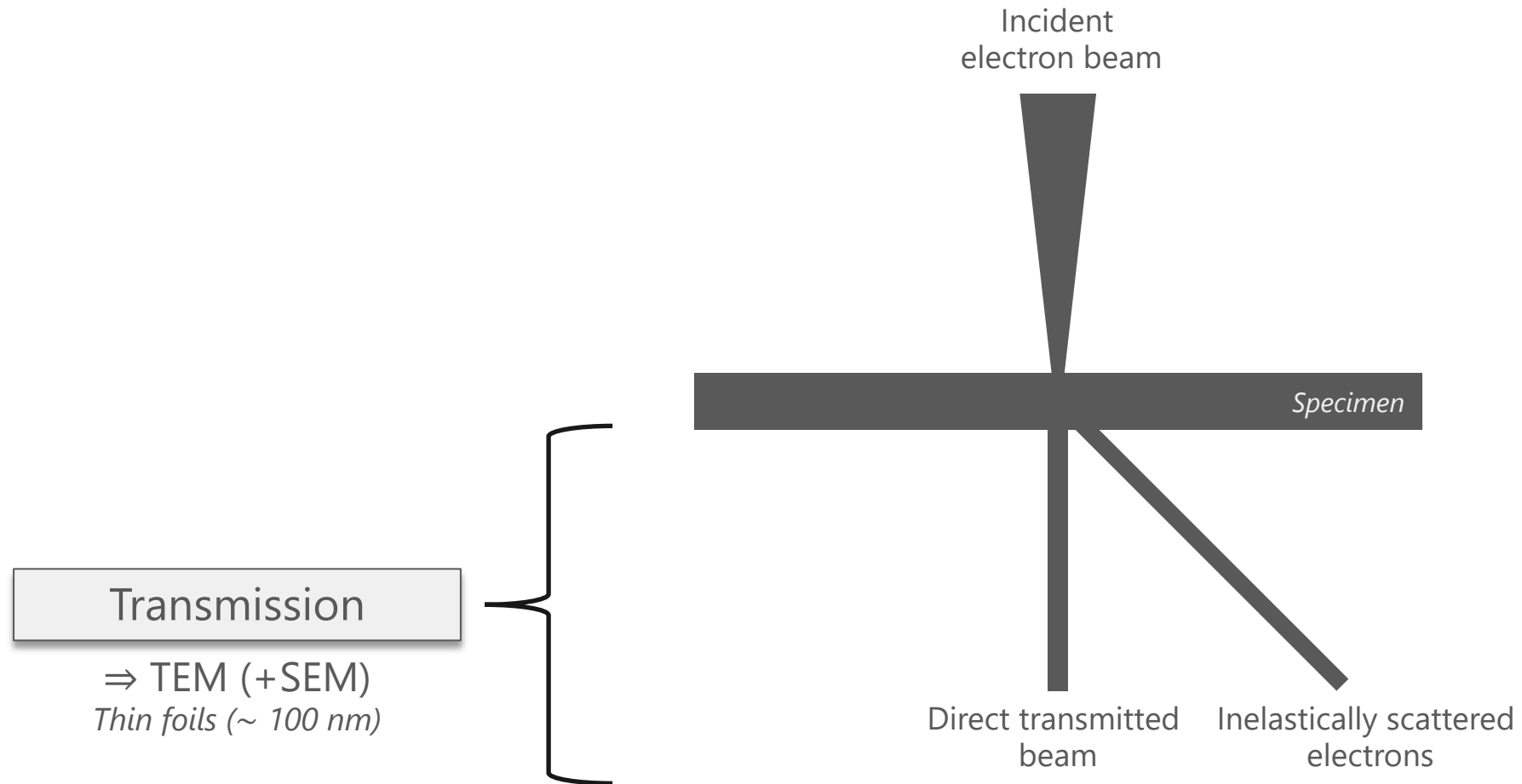
↪ Preparing TEM specimens is both an Art and a Science...

How to machine a TEM specimen?

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Electron-matter interaction



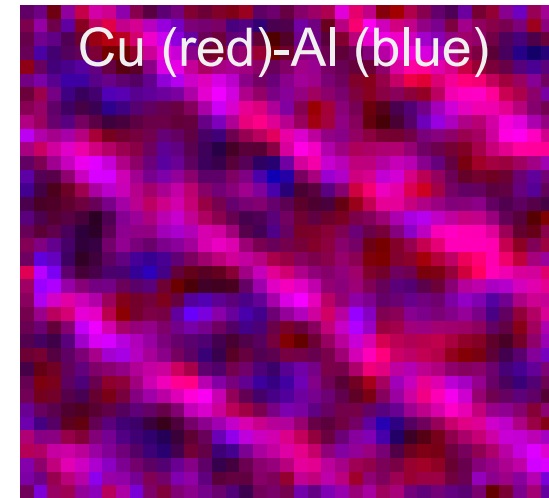
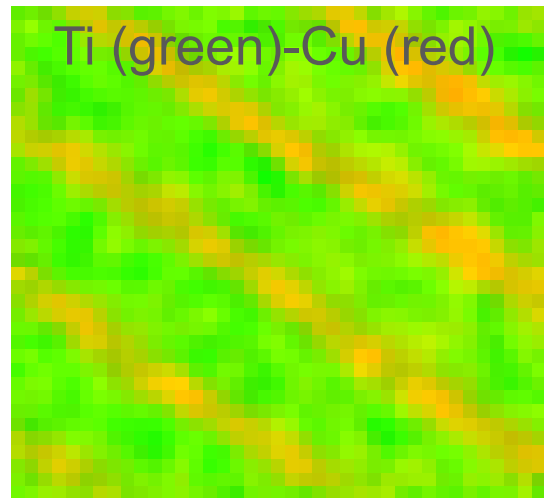
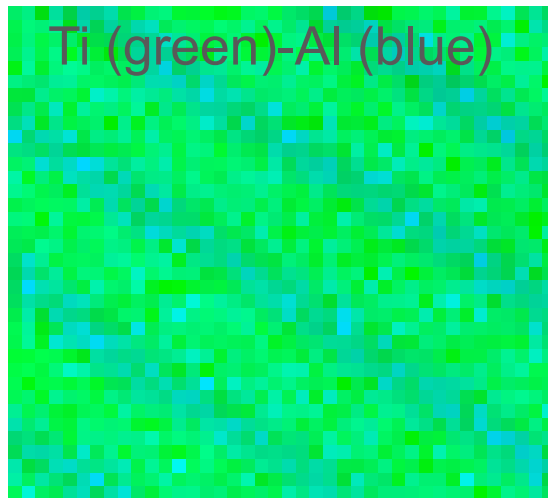
Inelastically scattered electrons

The energy is transferred from the electron to the sample.

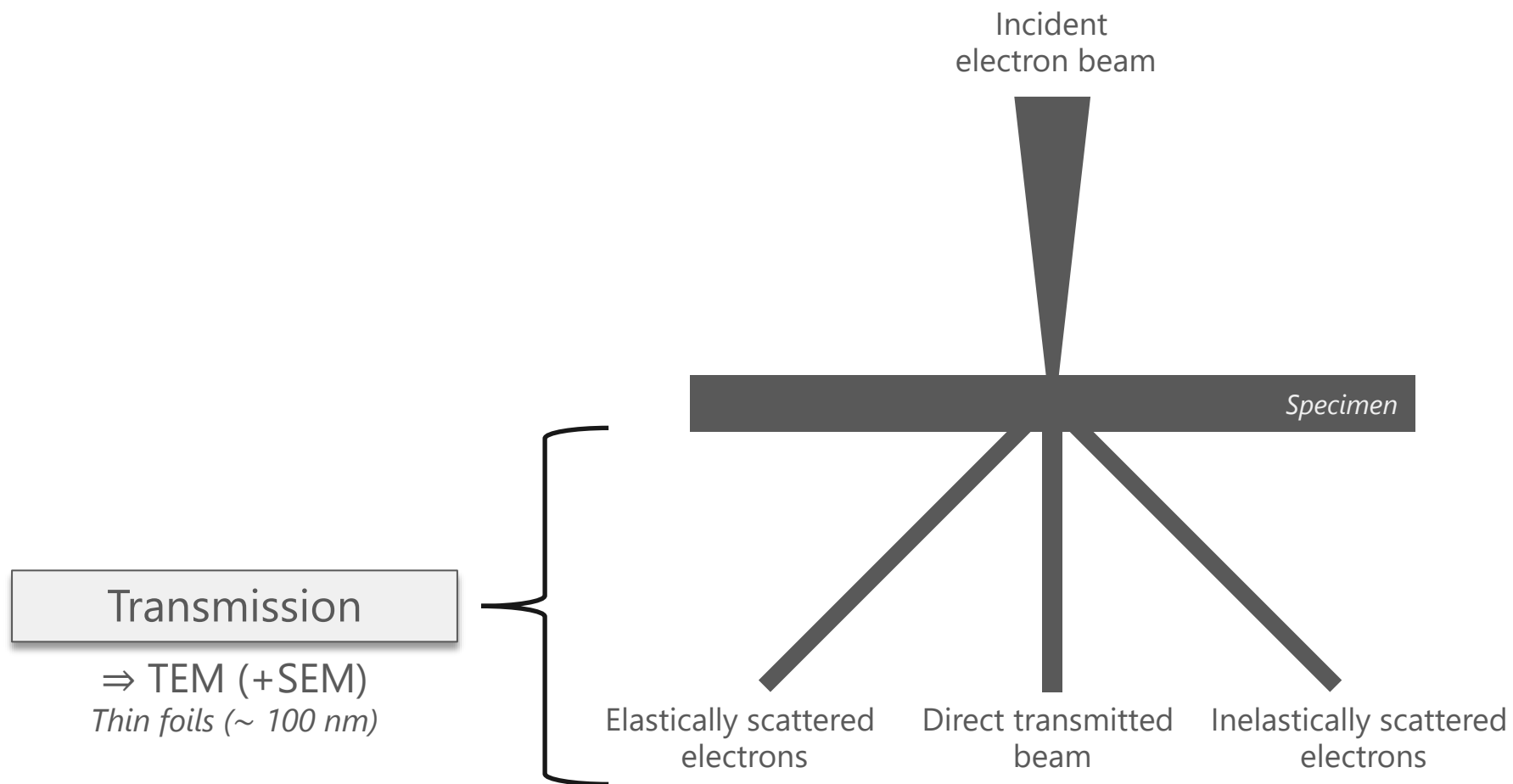
$$\Rightarrow E_0 > E_{e^-}$$

The energy loss is characteristic of the atoms encountered by the electron as it passes through the specimen.

Electron Energy Loss Spectroscopy (EELS)



Electron-matter interaction



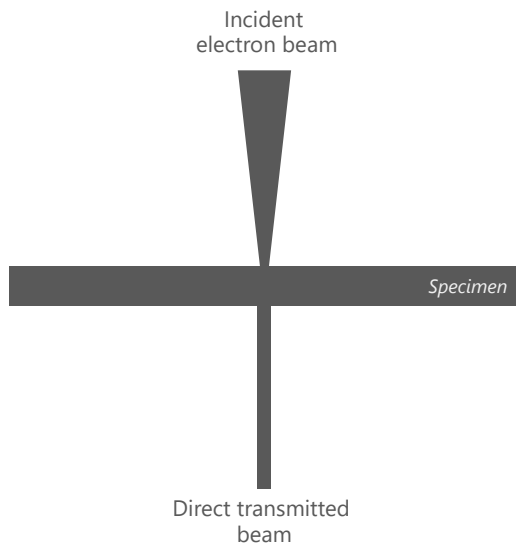
Elastically scattered electrons

No energy is transferred from the electron to the sample.

$$\Rightarrow E_0 \approx E_{e^-}$$

In the direction of the transmitted beam

- ❖ The electron passes through the specimen without any interaction.



↪ Such electrons contribute to the direct transmitted beam

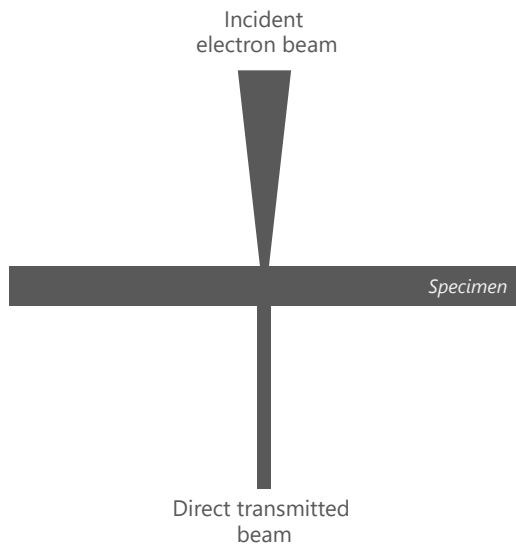
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In other directions

- ❖ The electrons are deflected from their path by Coulomb interactions with the positive potential of the atomic nuclei, screened by the surrounding electron cloud.



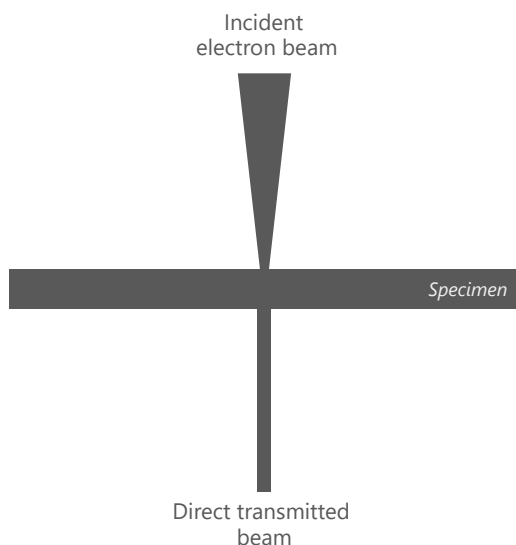
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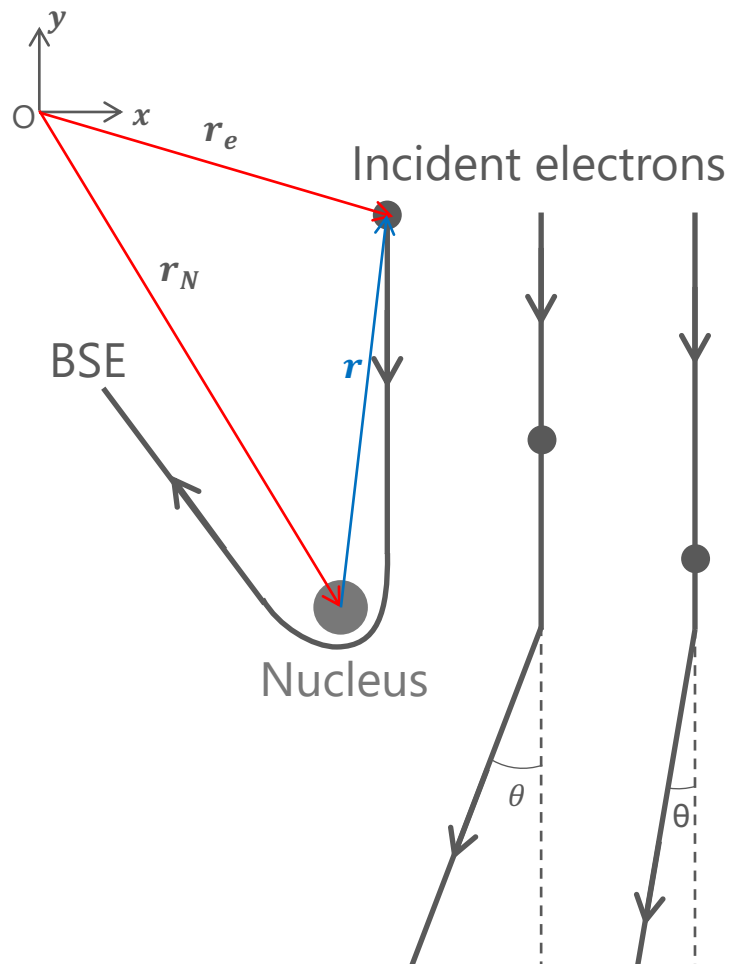
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$$F_{1/2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2 (\mathbf{r}_2 - \mathbf{r}_1)}{\|\mathbf{r}_2 - \mathbf{r}_1\|^3}$$

↳ Elastically scattered electrons are mainly exploited in TEM to generate contrast and diffraction patterns

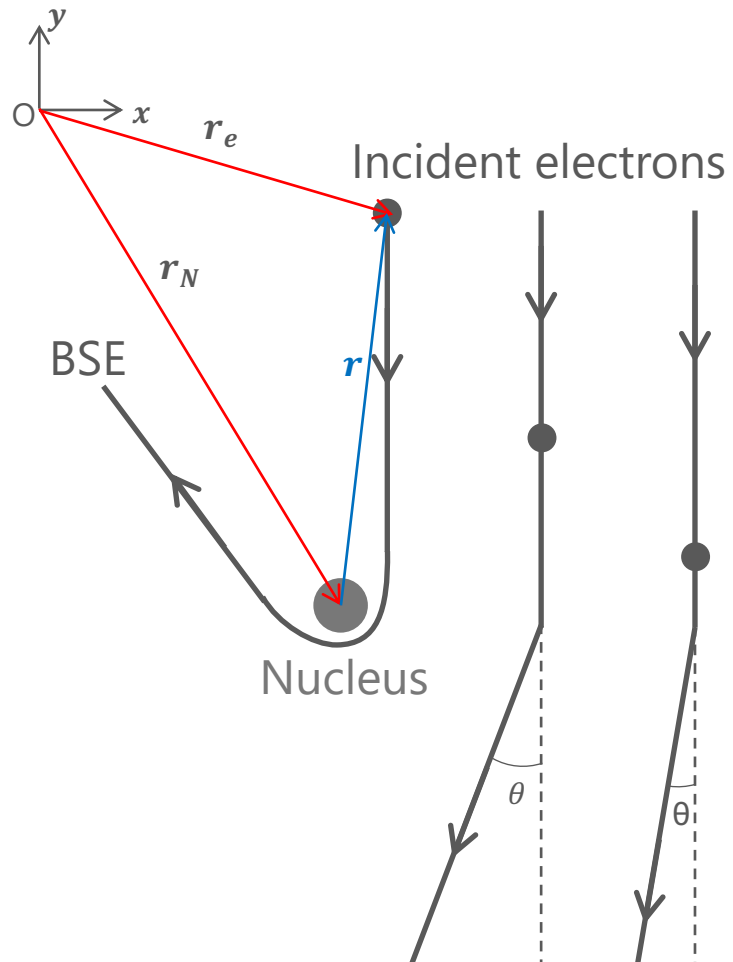
Elastically scattered electrons



- ❖ Electrons are described as particles
- ❖ The electron path is deflected because of the Coulomb force:

$$\mathbf{F}_{N/e} = \frac{1}{4\pi\epsilon_0} \frac{q_N q_e (\mathbf{r}_e - \mathbf{r}_N)}{\|\mathbf{r}_e - \mathbf{r}_N\|^3} = \frac{1}{4\pi\epsilon_0} \frac{q_N q_e}{r^2}$$

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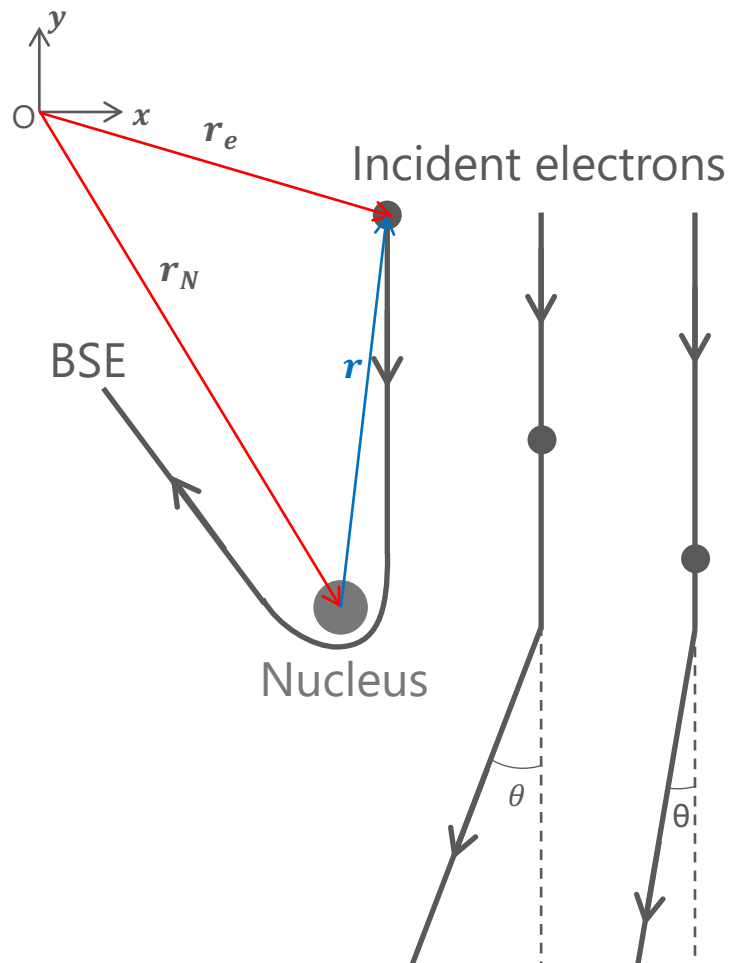
- Scattered angle θ

$$\Rightarrow r \searrow \Rightarrow F_{N/e} \nearrow \Rightarrow \theta \nearrow$$

↪ If $\theta > 90^\circ$

⇒ BackScattered electrons (BSE)

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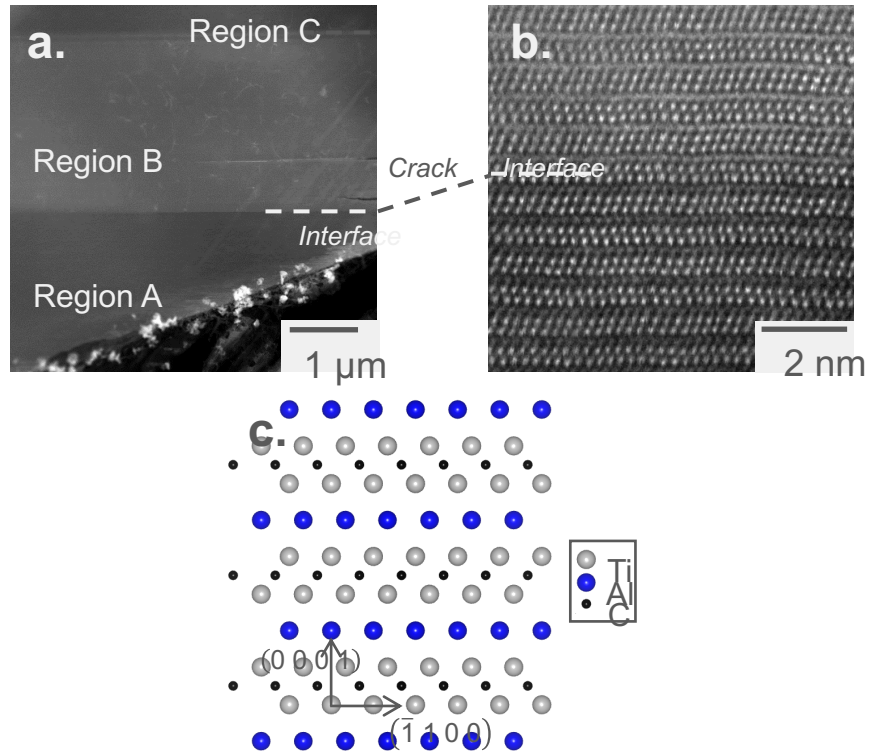
- Influence of Z :

$$q_N \nearrow \Rightarrow F_{N/e} \nearrow$$

$$q_N \nearrow \Rightarrow Z \nearrow$$

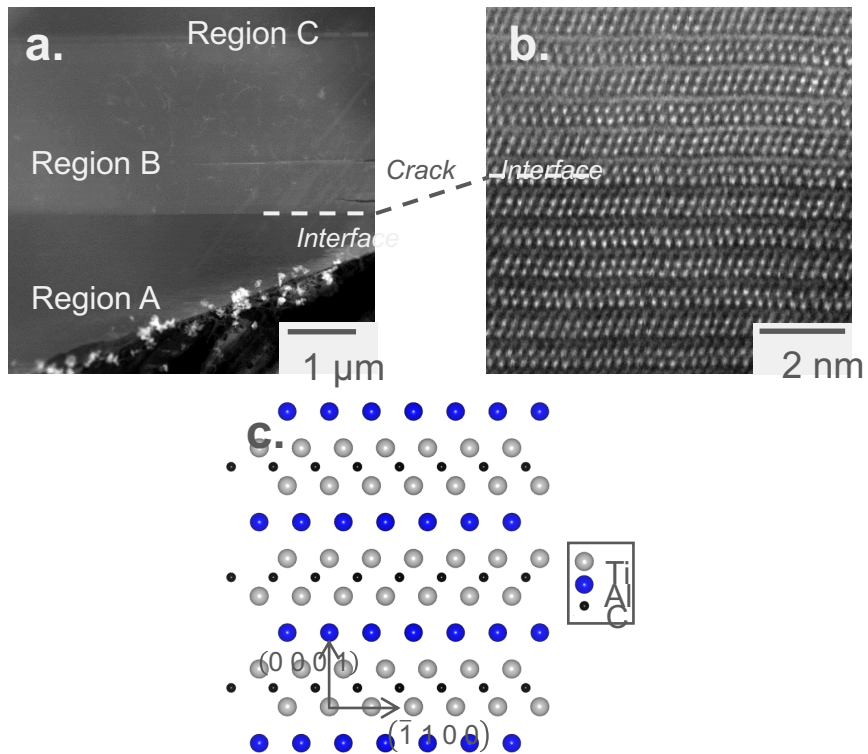
$$Z \nearrow \Rightarrow \theta \nearrow$$

⇒ High Z produces higher intensity at large θ

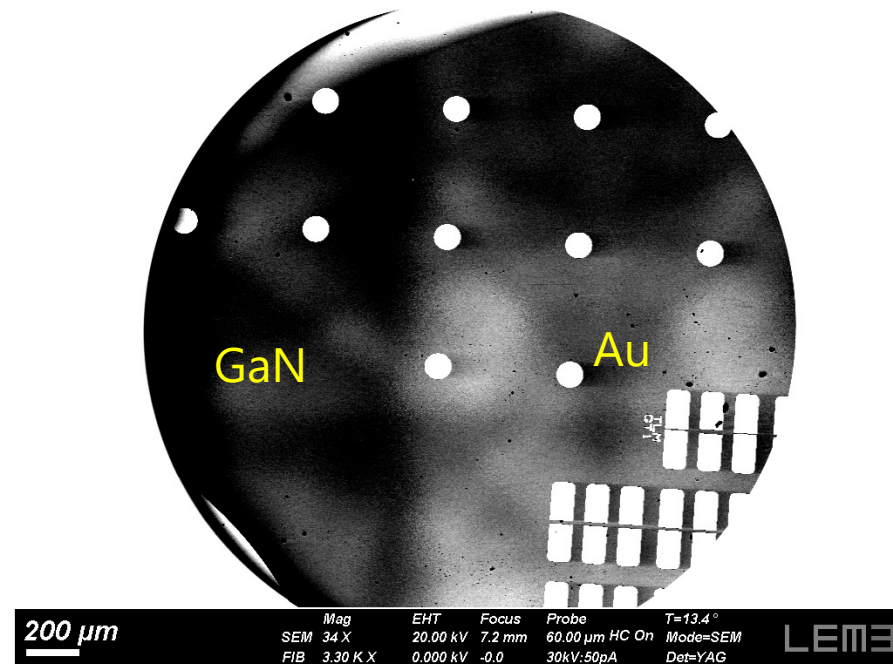
Intensity at large θ *Inside the (S)TEM*

Intensity at large θ

Inside the (S)TEM



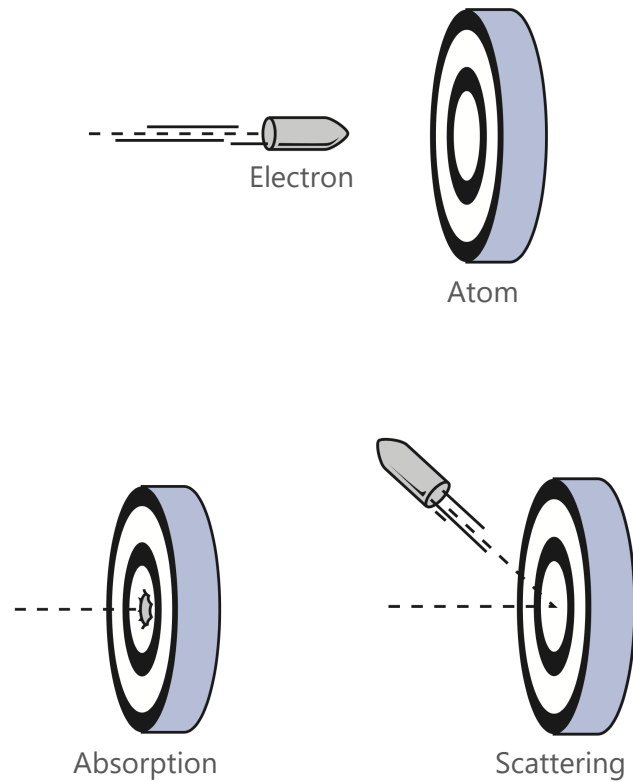
Inside the SEM



Bright contrast \Rightarrow high Z ; Dark contrast \Rightarrow low Z

The cross section of interaction

Cross section the probability that a certain interaction will occur between two particles for a given set of parameters.

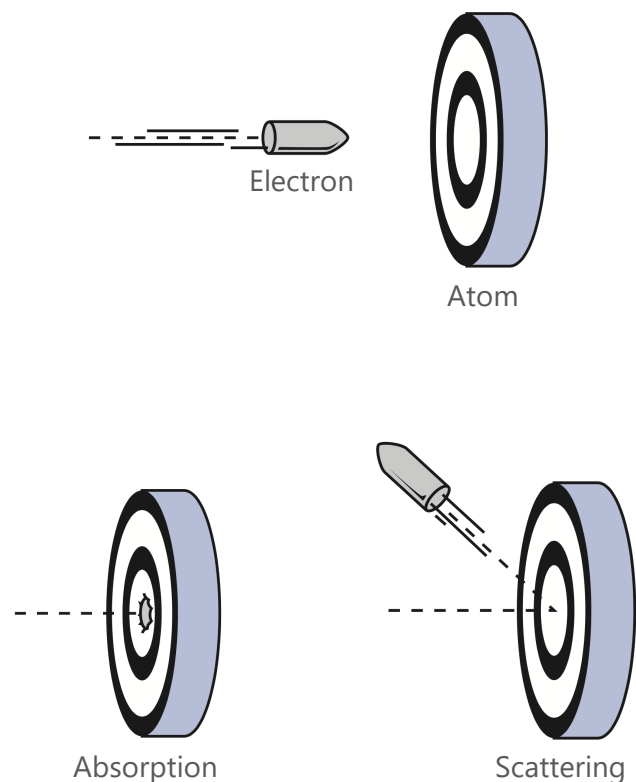


The cross section of interaction

Cross section the probability that a certain interaction will occur between two particles for a given set of parameters.

❖ **There are two kinds of cross-sections:**

- Microscopic (σ): it refers to cross sections of electrons reacting with a specific atom
 $10^{-28}\text{cm}^2 = 1 \text{ b}$

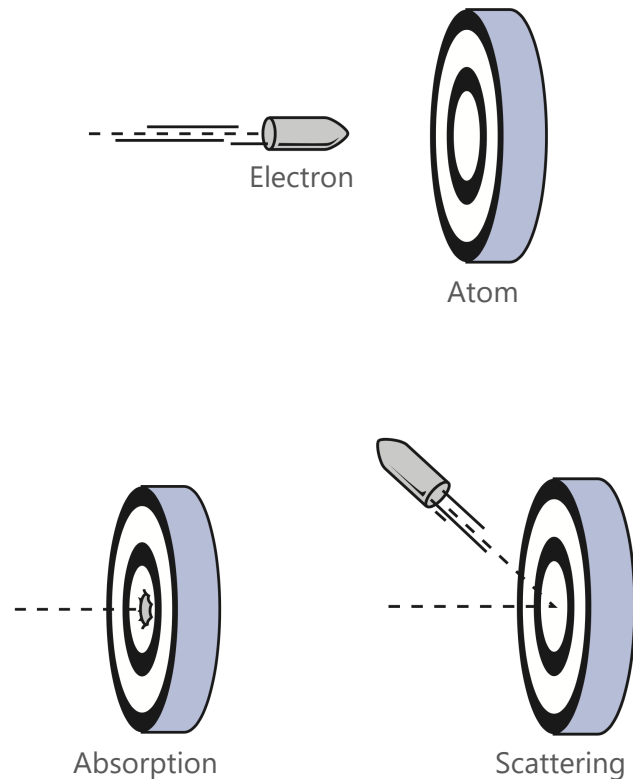


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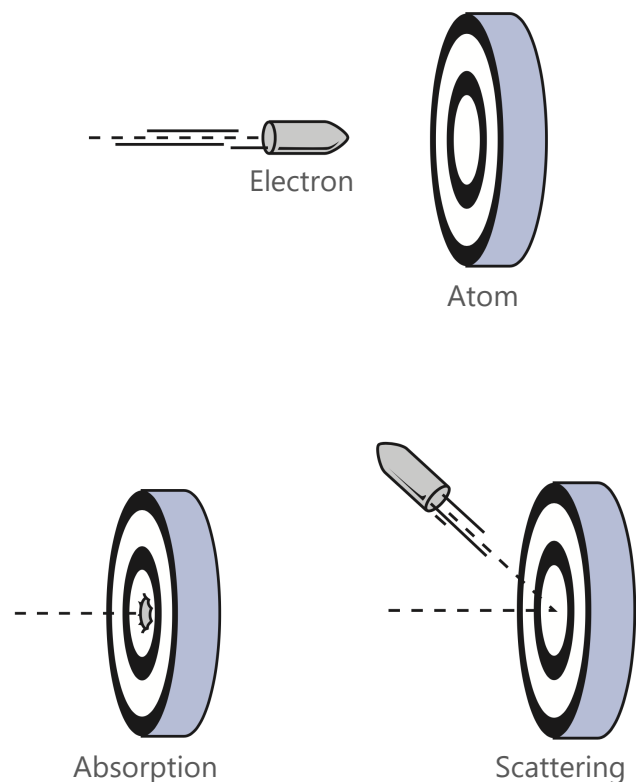
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 $10^{-28}\text{cm}^2 = 1 \text{ b}$
- Macroscopic (Σ): it gives the probability for an electron to interact with a target per unit length.



The cross section of interaction

Cross section the probability that a certain interaction will occur between two particles for a given set of parameters.



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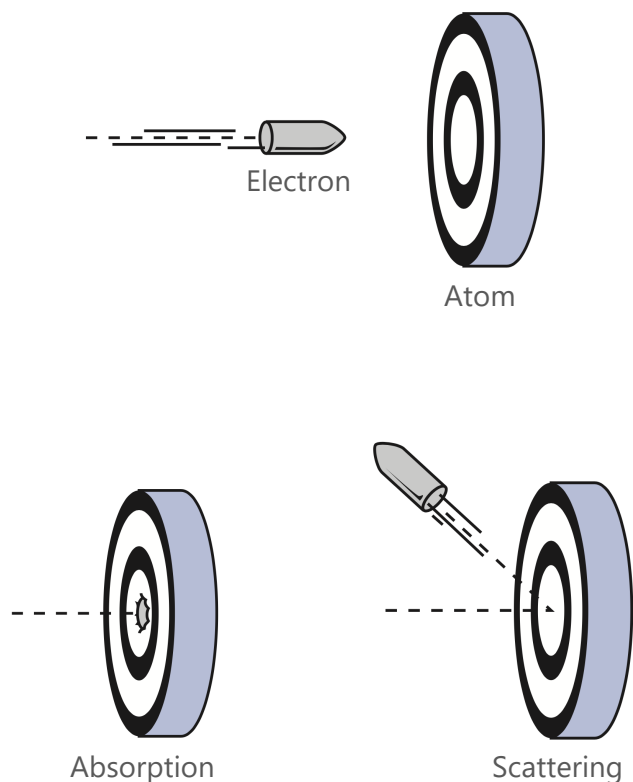
✓ Pure material

$$\Sigma[\text{m}^{-1}] = n\sigma = \frac{N_a\rho}{M}\sigma$$

(n : number of nuclei per volume; M : molar mass; ρ : density; N_a : Avogadro's number)

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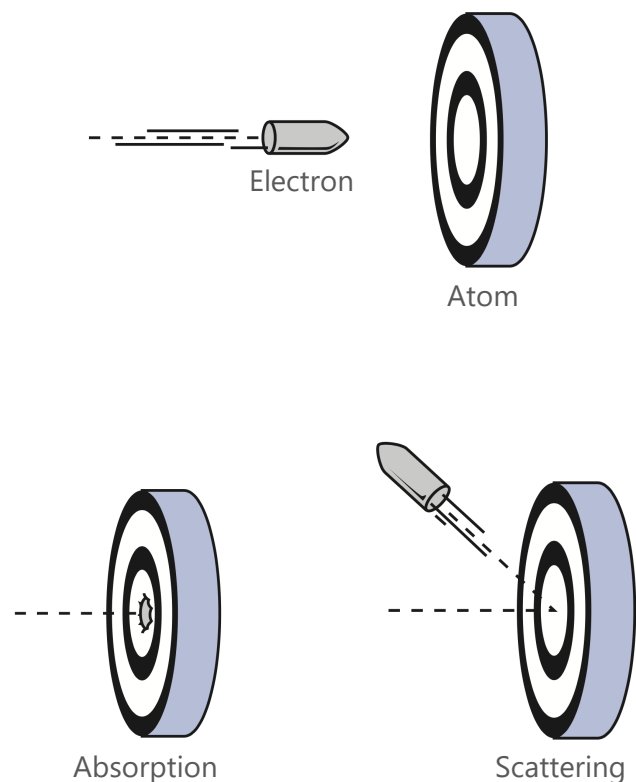
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❖ Mean free path:

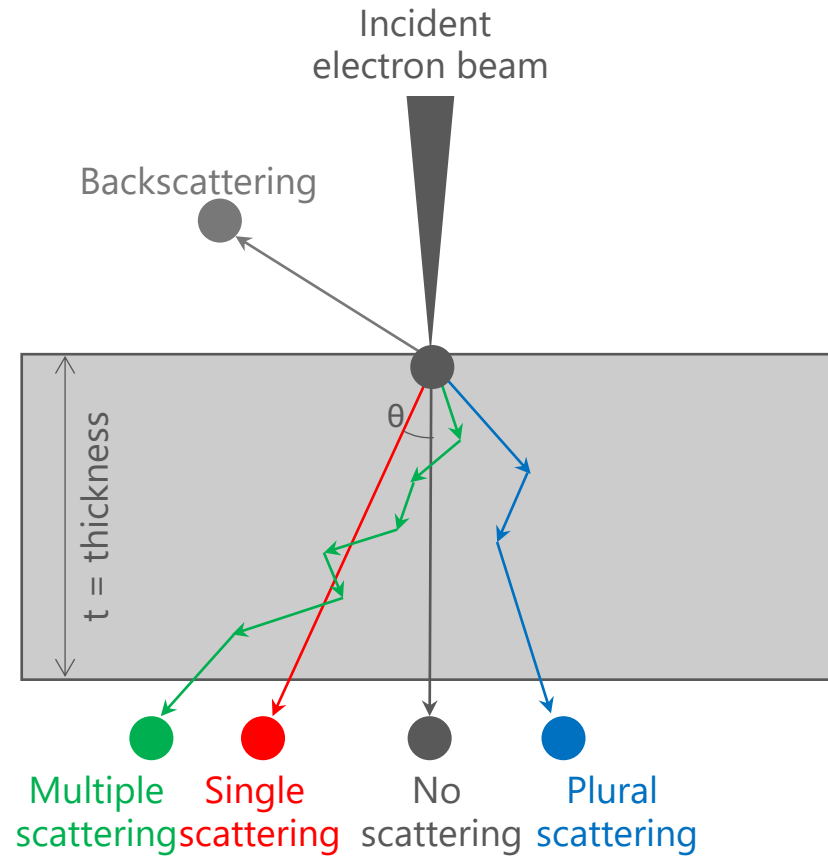
- How far on average an electron will travel before it interacts with an atom

$$\bar{l}[\text{m}] = \frac{1}{\Sigma}$$

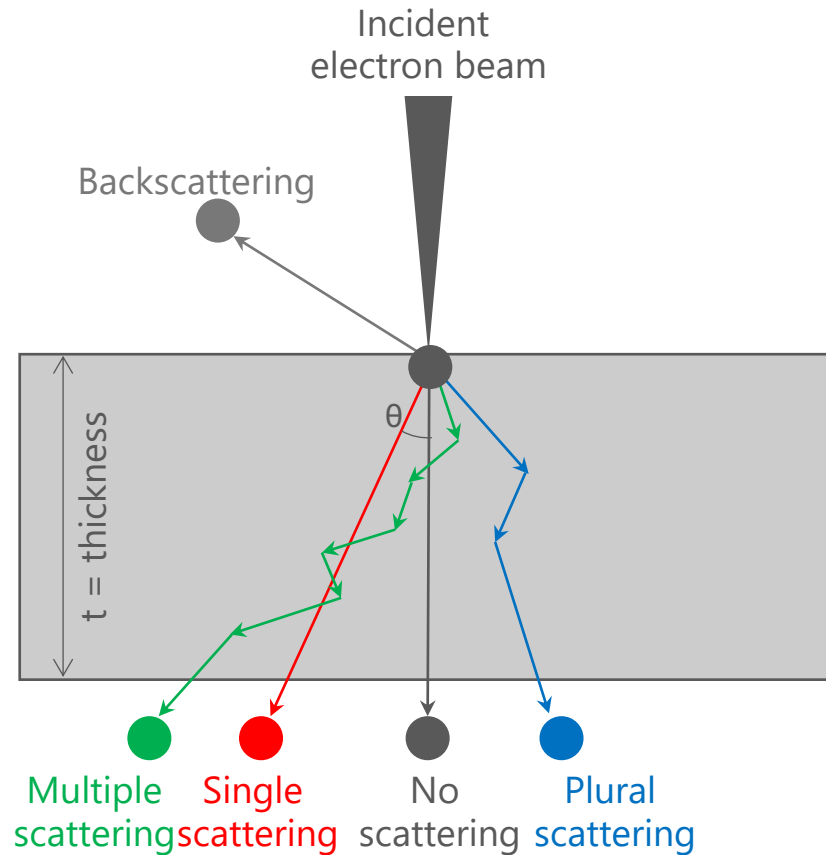
The cross section of interaction

❖ Single scattering:

- Only one interaction (either elastic or inelastic)
- It is the basis for many theoretical models.
- Ideal for quantitative analysis, since the signal is directly related to a single interaction event.



The cross section of interaction



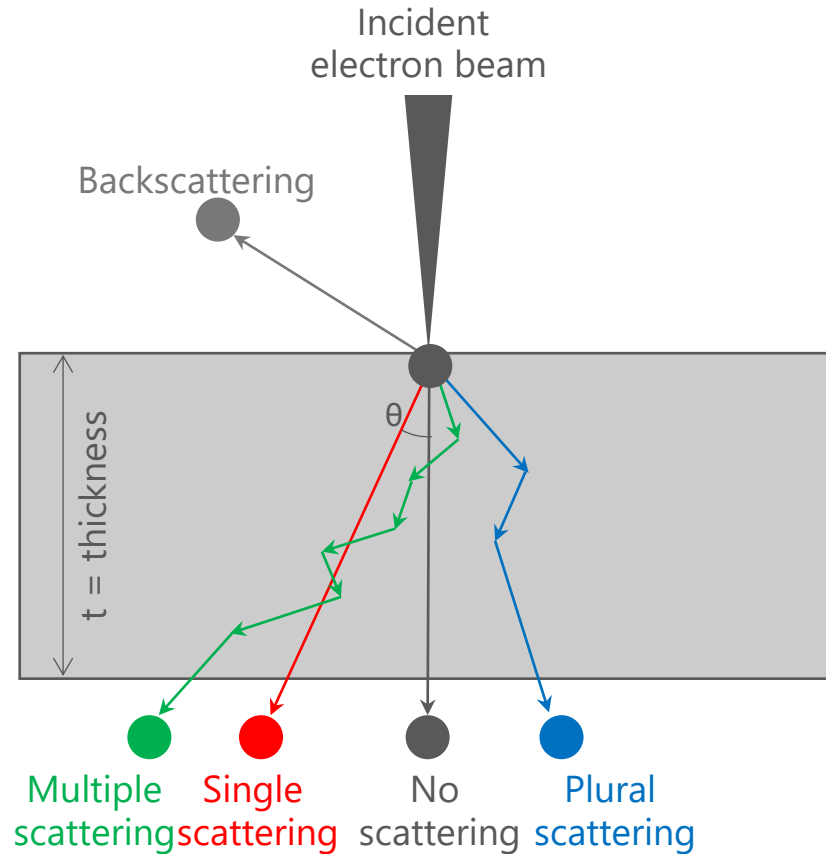
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❖ Multiple scattering:

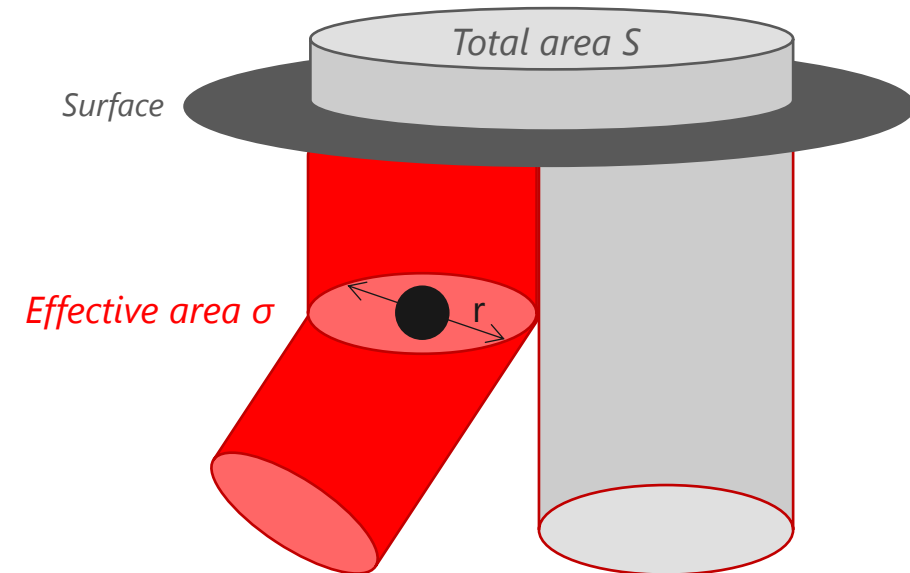
- Many scattering events.
- The path becomes complex, and the original direction and energy of the electron are heavily modified.
- Common in thick or dense samples, especially in SEM where BSE and SE often result from multiple scattering.
- Makes quantitative analysis more difficult, and signal interpretation more complex.

The cross section of interaction

❖ Probability of scattering:

$$P_{Scat} = \frac{\sigma_{tot}}{S} = \frac{\pi r^2}{S}$$

$P_{Scat} \nearrow$ when $\sigma_{tot} \nearrow$



The cross section of interaction

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$$P_{Scat} \nearrow \text{ when } \sigma_{tot} \nearrow$$

❖ Cross section:

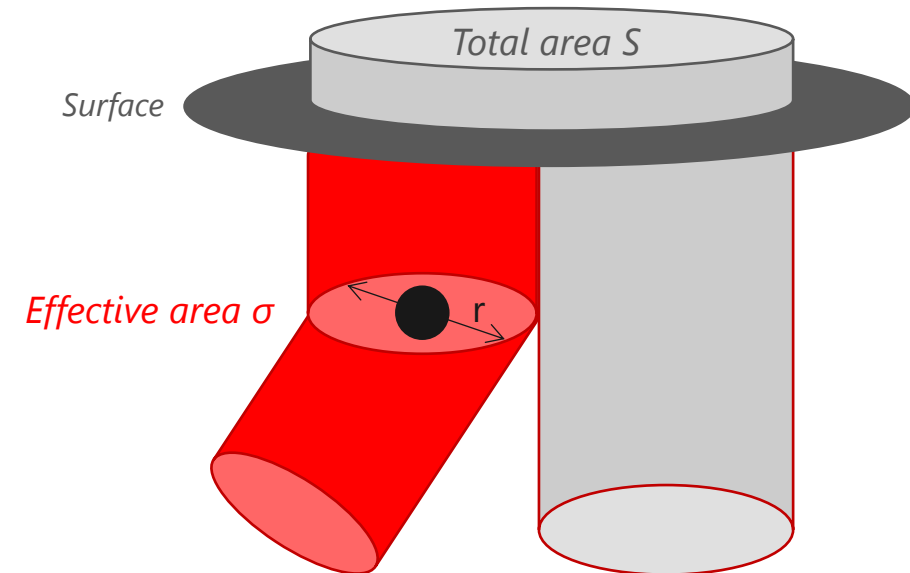
$$\sigma_{tot} = \sigma_{elastic} + \sigma_{inelastic}$$

$$\sigma_{elastic} = \pi r_{elastic}^2 Z e$$

$$r_{elastic} = \frac{1}{V\theta}$$

$$\Rightarrow P_{Scat} \nearrow \text{ when } Z \nearrow$$

$$\theta \nearrow \Rightarrow P_{Scat} \searrow$$



The cross section of interaction

❖ Cross section for a specimen:

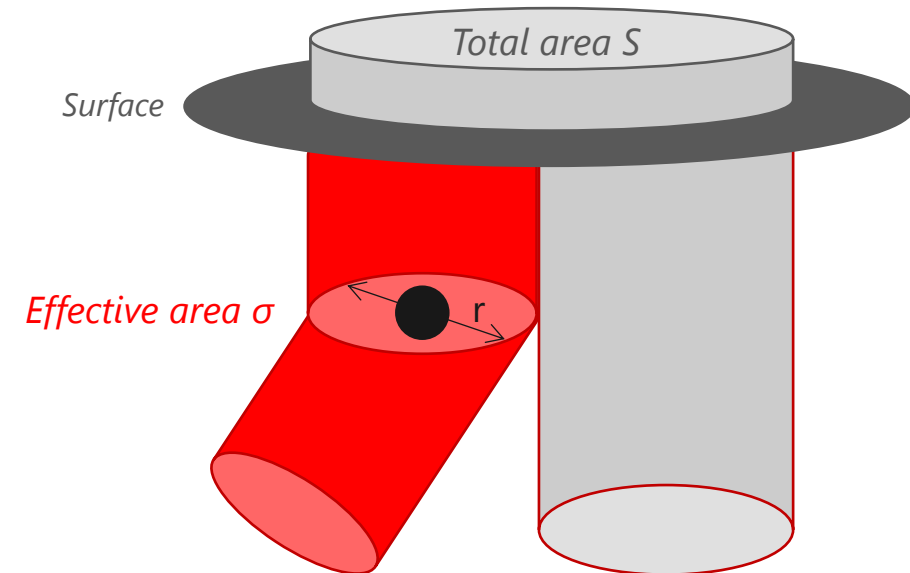
$$\Sigma = n\sigma = \frac{N_a \rho}{M} \sigma_{tot}$$

$$Q_T = t\Sigma = \frac{N_a \rho t}{M} \sigma_{tot}$$

(t : thickness)

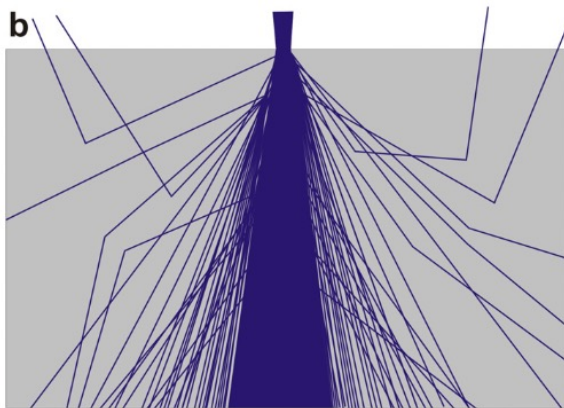
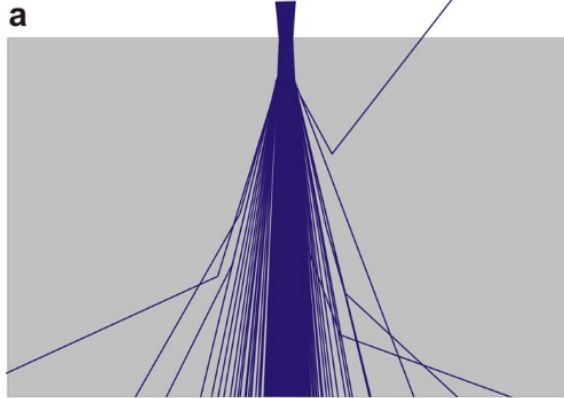
$\Rightarrow P_{scat} \nearrow$ when $t \nearrow$

$\Rightarrow P_{scat} \nearrow$ when $\rho \nearrow$



Interaction volume

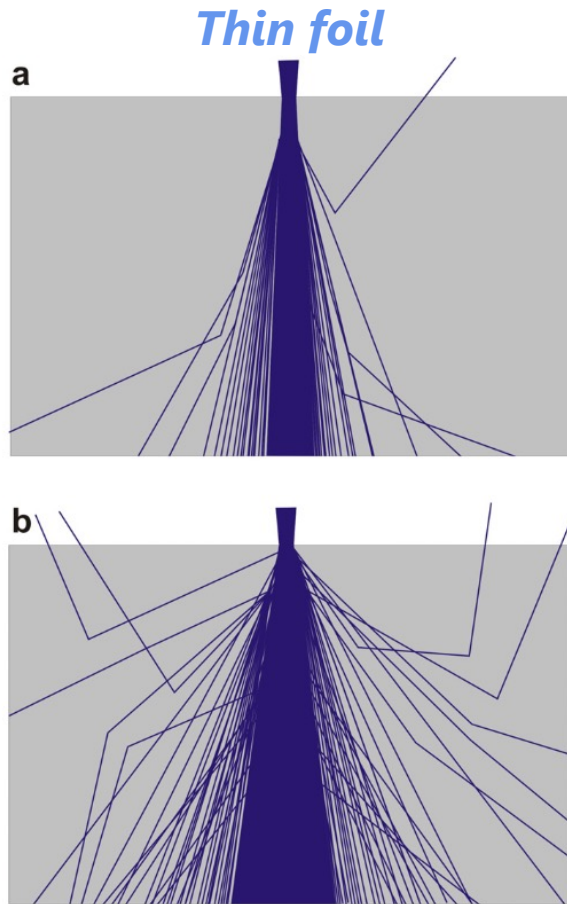
Thin foil



Low Z

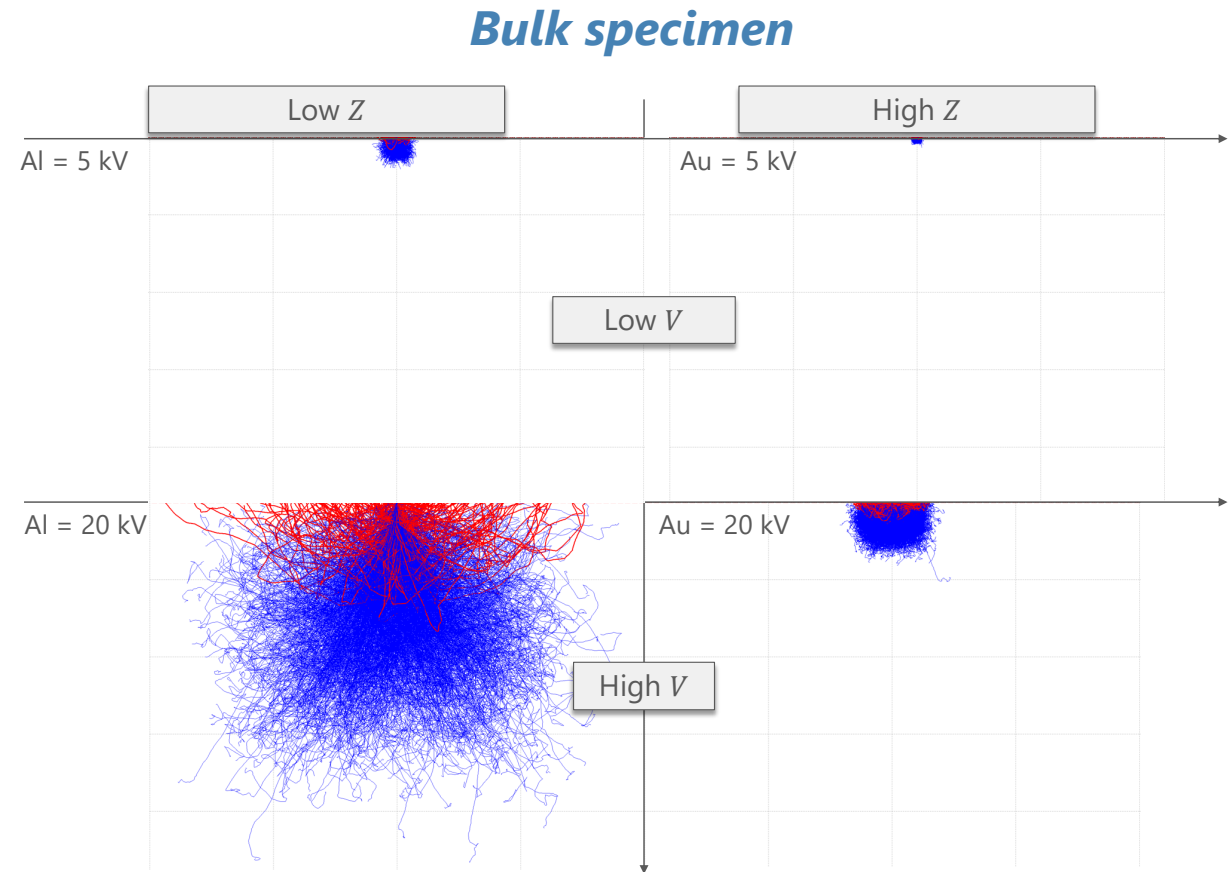
High Z

Interaction volume



Low Z

High Z



The 2 types of electron microscopes

Transmission electron microscope (TEM)



- ❖ Invention: 1931 by Ernst RUSKA et Max KNOL
- ❖ Specimen : thin foils (~ 100 nm)
- ❖ Price: ~ 1 M€
- ❖ Rare in industry

↳ Nanometric/atomic scale

Scanning electron microscope (SEM)



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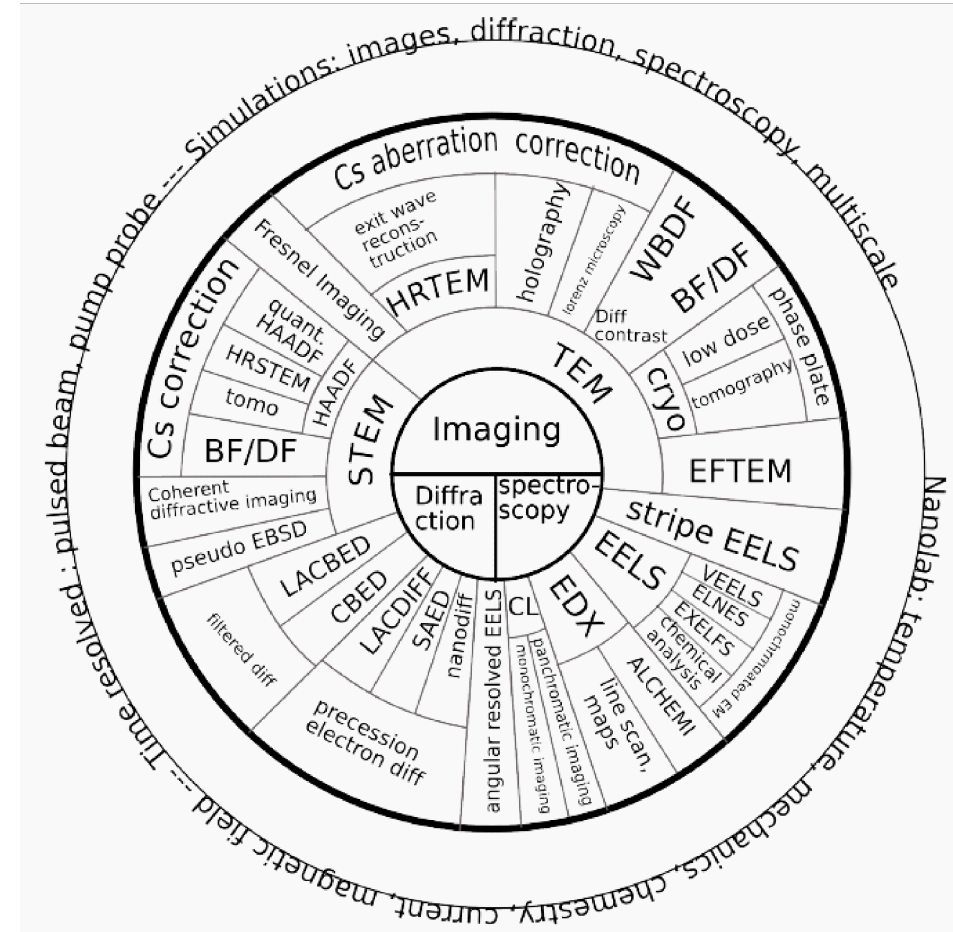
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Basics of TEM contrast

Probability of scattering event:

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Mass contrast:

$Q_T \nearrow$ when $\rho \nearrow$

You shall not study
W by TEM...

High ρ

Low ρ

Basics of TEM contrast

Probability of scattering event:

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Mass contrast:

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High ρ

Low ρ

You shall not study
W by TEM...

Thickness contrast:

$Q_T \nearrow$ when $t \nearrow$

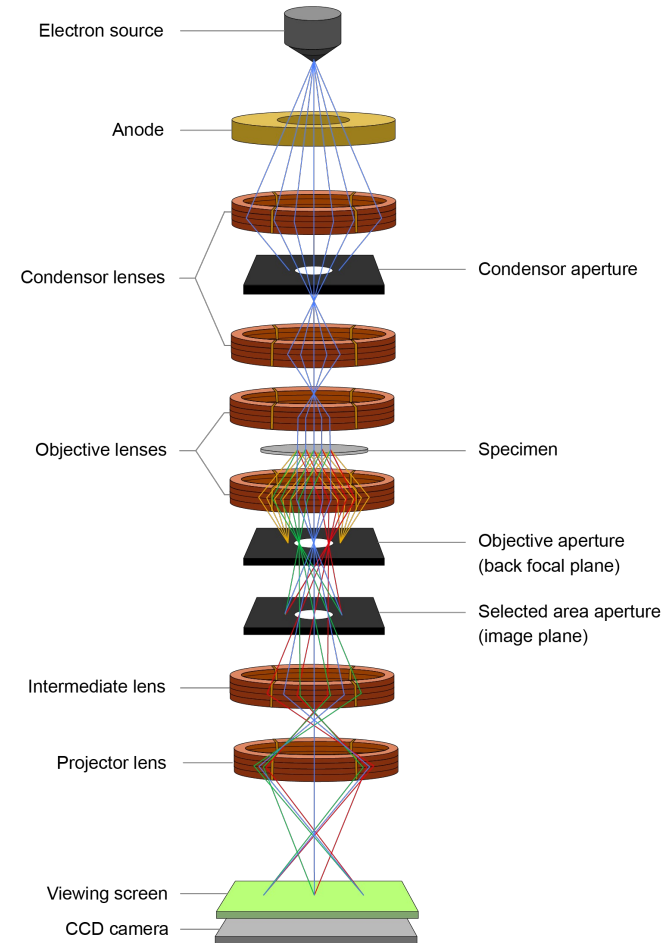
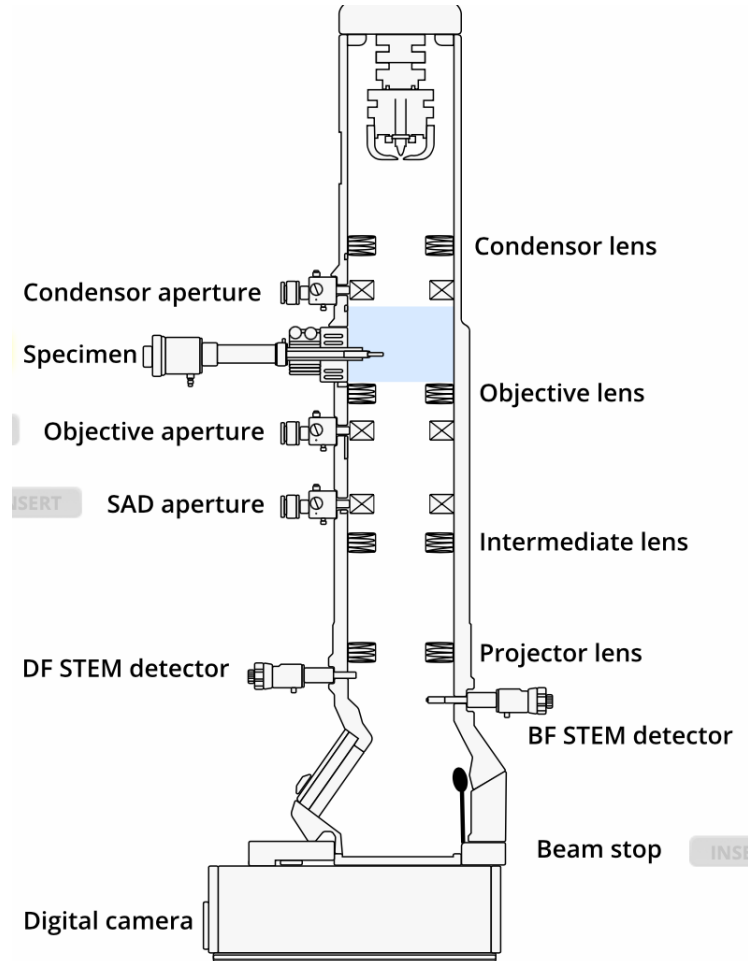
High t

Low t

You shall not study
thick TEM foils...

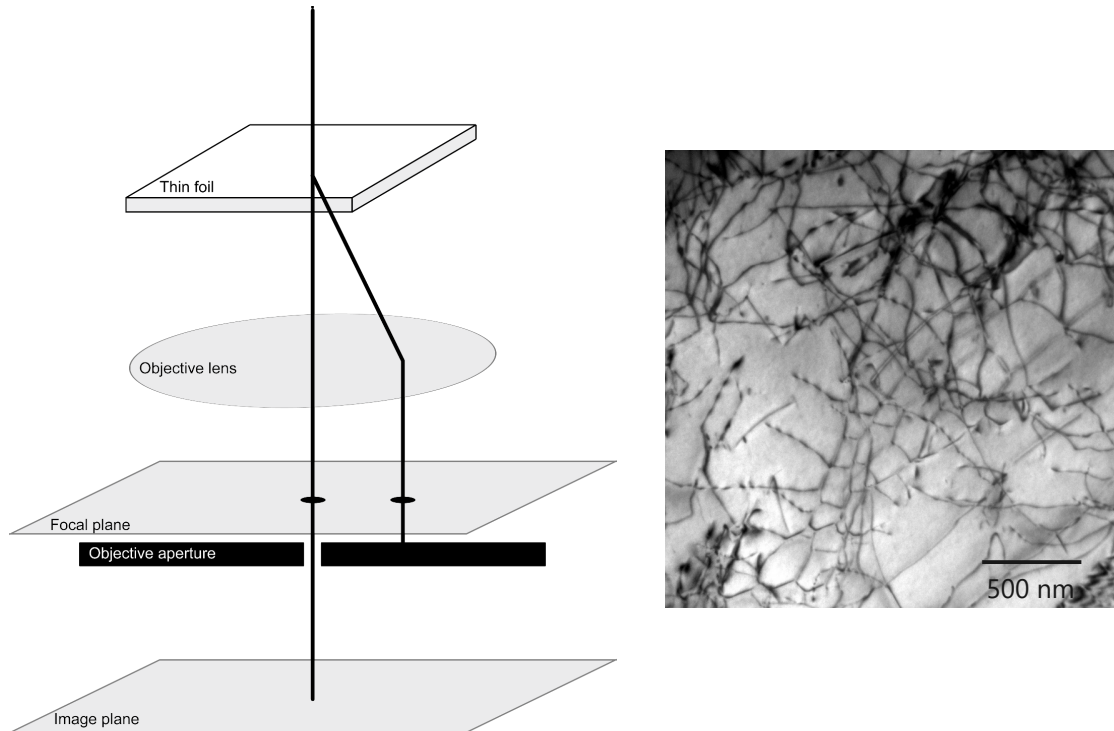
It is the local scattering power that determines the contrast of TEM images

Basics of TEM



Basics of TEM

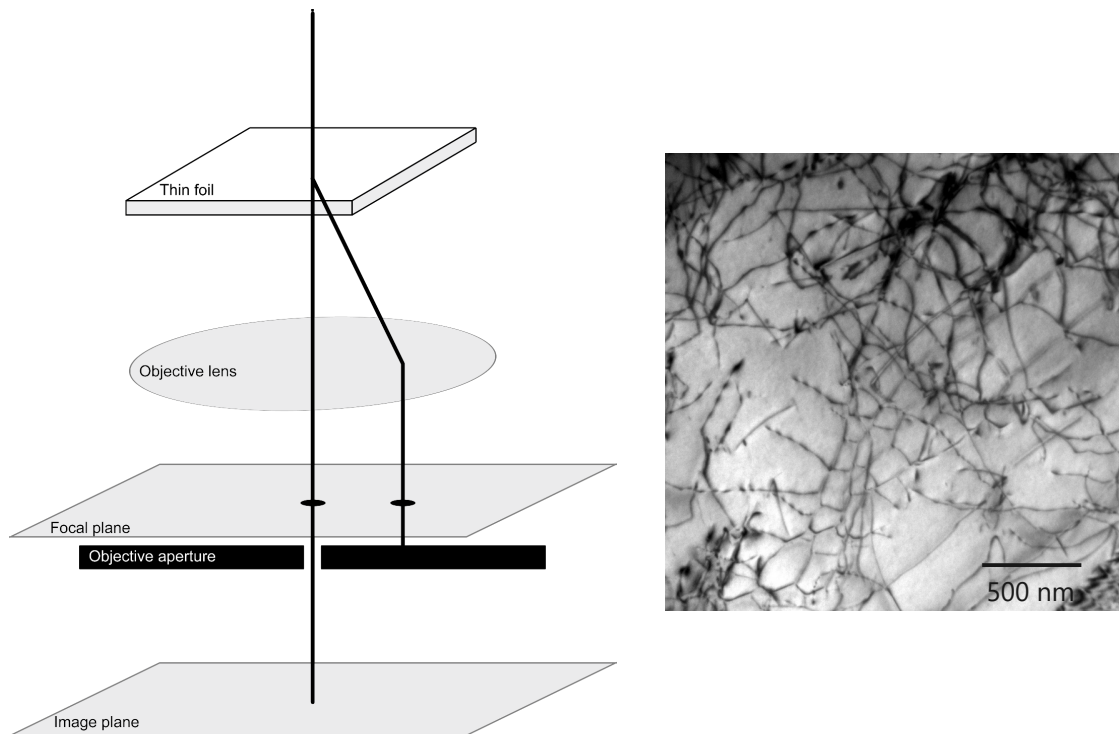
Bright field



Dark contrasts on a bright background

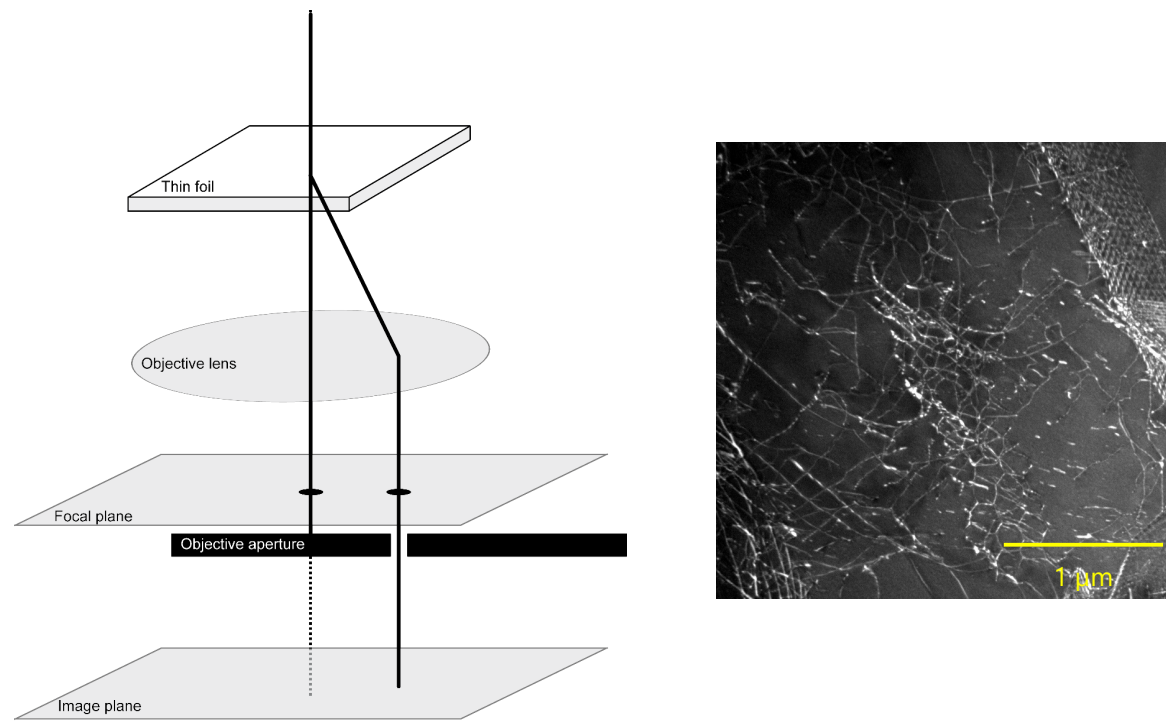
Basics of TEM

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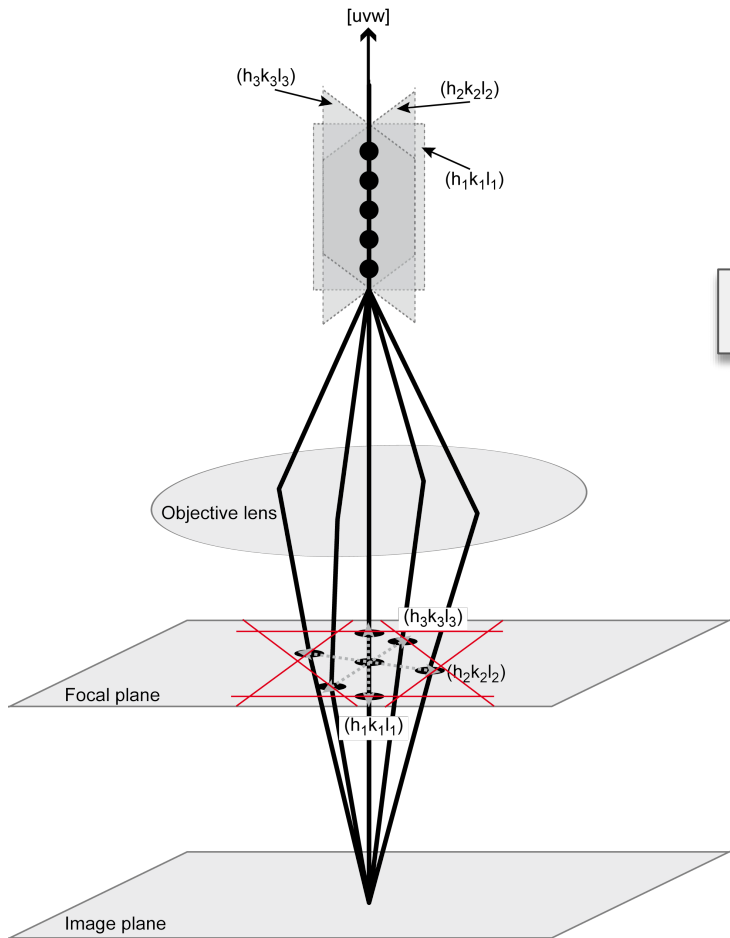
Dark field



↪ Bright contrasts on a dark background

Basics of TEM contrast

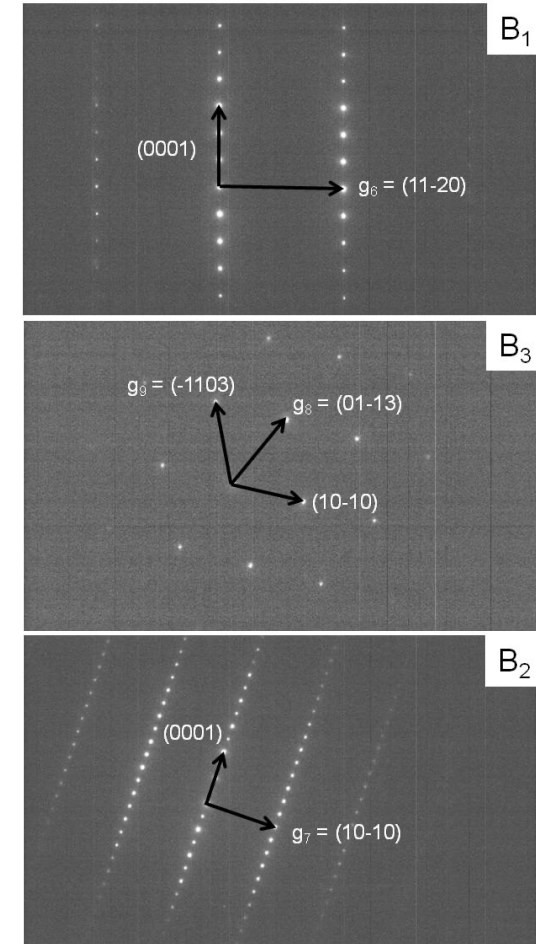
Diffraction mode



Spot pattern



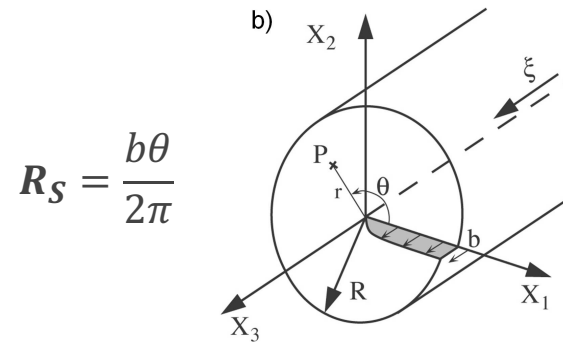
Information on the crystallographic structure



Beams are not convergent in the reality...

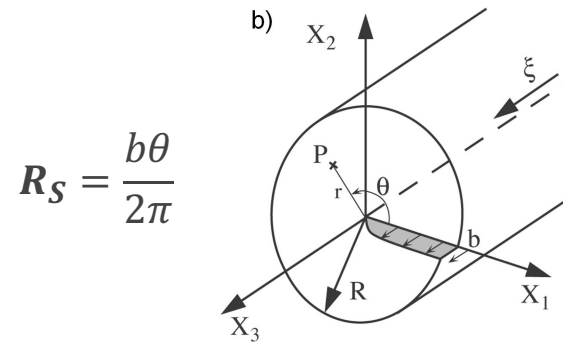
Elastic theory of dislocations

- ❖ Burgers vector: \mathbf{b}
- ❖ Dislocation line direction: ξ
 - Screw dislocation ($\mathbf{b} \parallel \xi$)



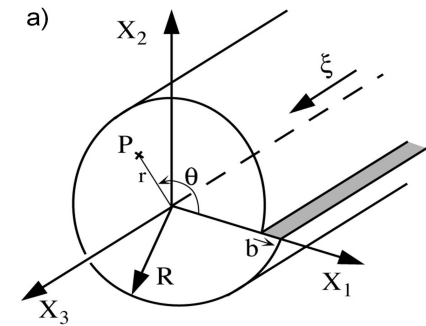
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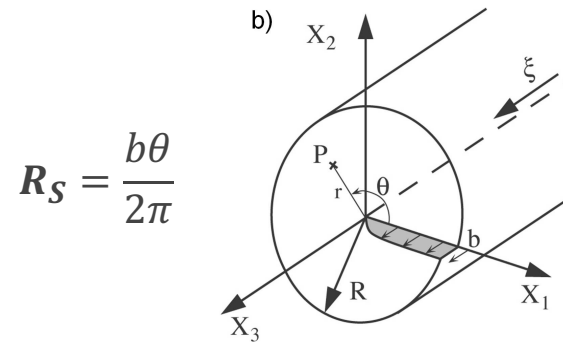
- Edge dislocation ($\mathbf{b} \perp \xi$)

$$R_e = \underbrace{\frac{b\theta}{2\pi} \left[\theta + \frac{\sin 2\theta}{2(1-\nu)} \right]}_{\parallel b} + \underbrace{\frac{\mathbf{b} \times \xi}{2\pi} \left[\frac{1-2\nu}{2(1-\nu)} \ln r + \frac{\cos 2\theta}{4(1-\nu)} \right]}_{\perp b}$$



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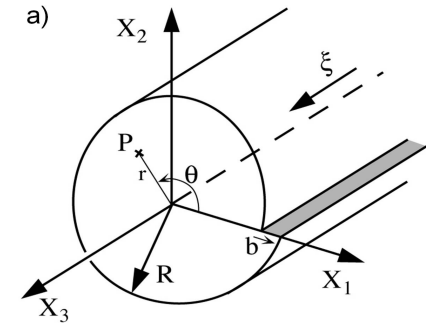


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- Mixed dislocation ($\mathbf{b} \angle \xi$)

Combination of R_S and R_e



Contrast of unperfect crystal

❖ Perfect crystal

- A crystal is composed of several unit cells located at positions \mathbf{R}_m , a Bravais vector

- The atoms inside each unit cell are positioned at \mathbf{r}_j

$$\mathbf{r}_{jm} = \mathbf{R}_m + \mathbf{r}_j$$

$$\mathbf{R}_m = u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3$$

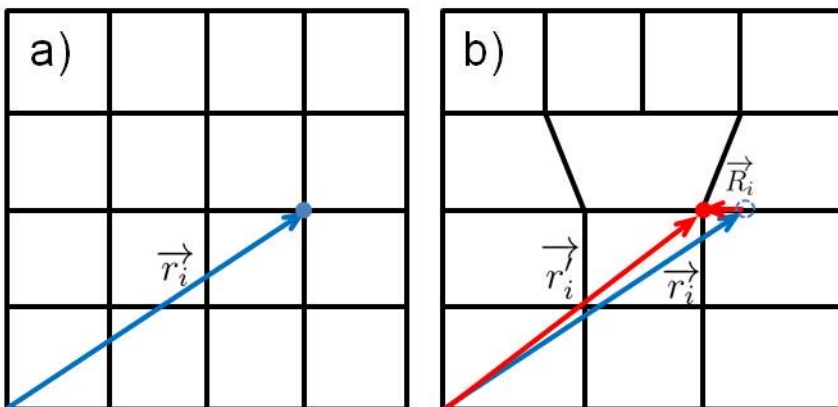
- The total diffracted wave:

$$\psi'(\mathbf{r}, t) \propto \sum_j f_j \exp[2\pi i \mathbf{K} \cdot \mathbf{r}_j] \sum_m \exp[2\pi i \mathbf{K} \cdot \mathbf{R}_m]$$

- With diffraction conditions ($\mathbf{K} = \mathbf{G}$):

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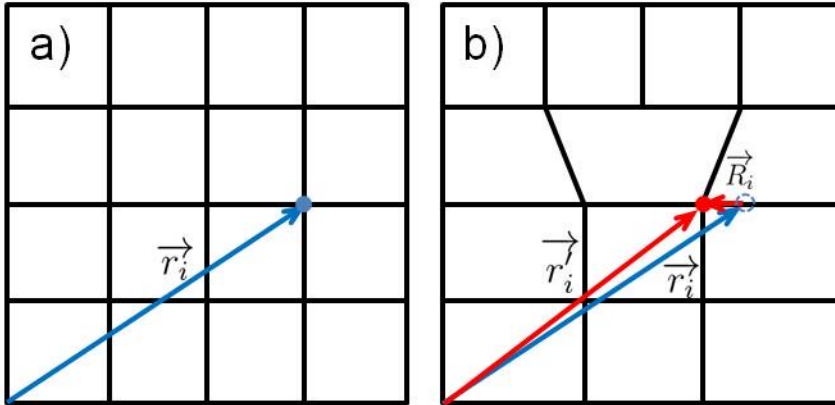
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Contrast of imperfect crystal



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❖ Unperfect crystal

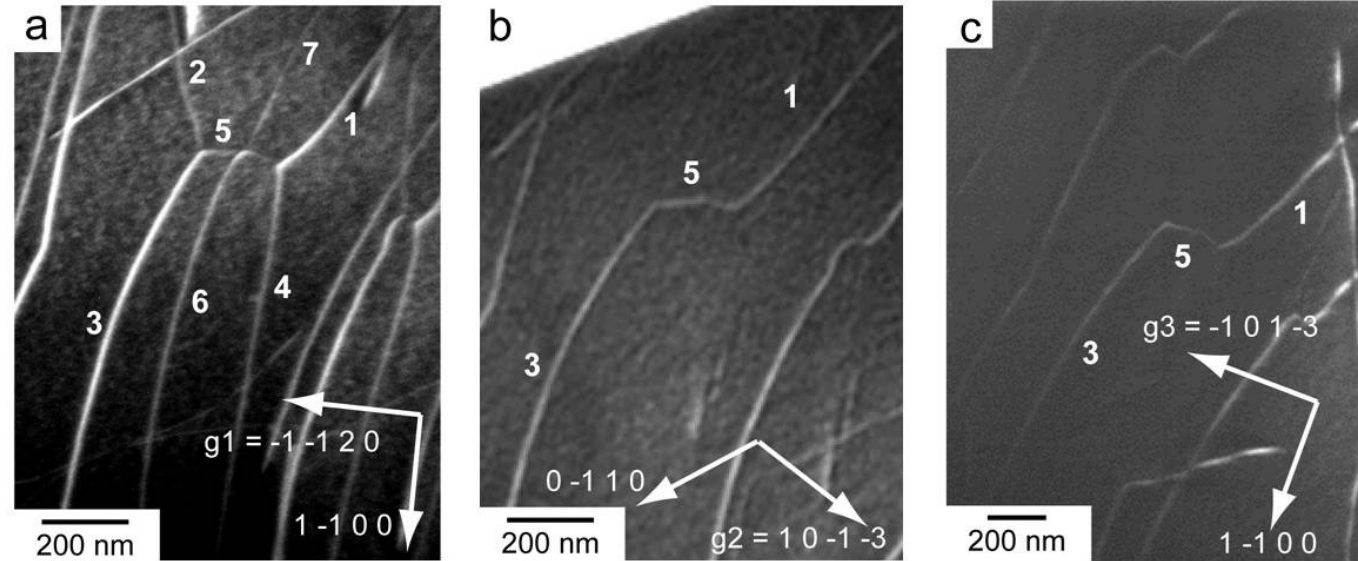
- The atoms inside each unit cell are now positioned at $\mathbf{r}_j + \mathbf{R}_j$

$$\psi'_a(\mathbf{r}, t) \propto \sum_j f_j \exp[2\pi i \mathbf{G} \cdot (\mathbf{r}_j + \mathbf{R}_j)] = \sum_j f_j \exp[2\pi i \mathbf{G} \cdot \mathbf{r}_j] \times \sum_j f_j \exp[2\pi i \mathbf{G} \cdot \mathbf{R}_j] = \sum_j f_j \exp[2\pi i \mathbf{G} \cdot \mathbf{R}_j]$$

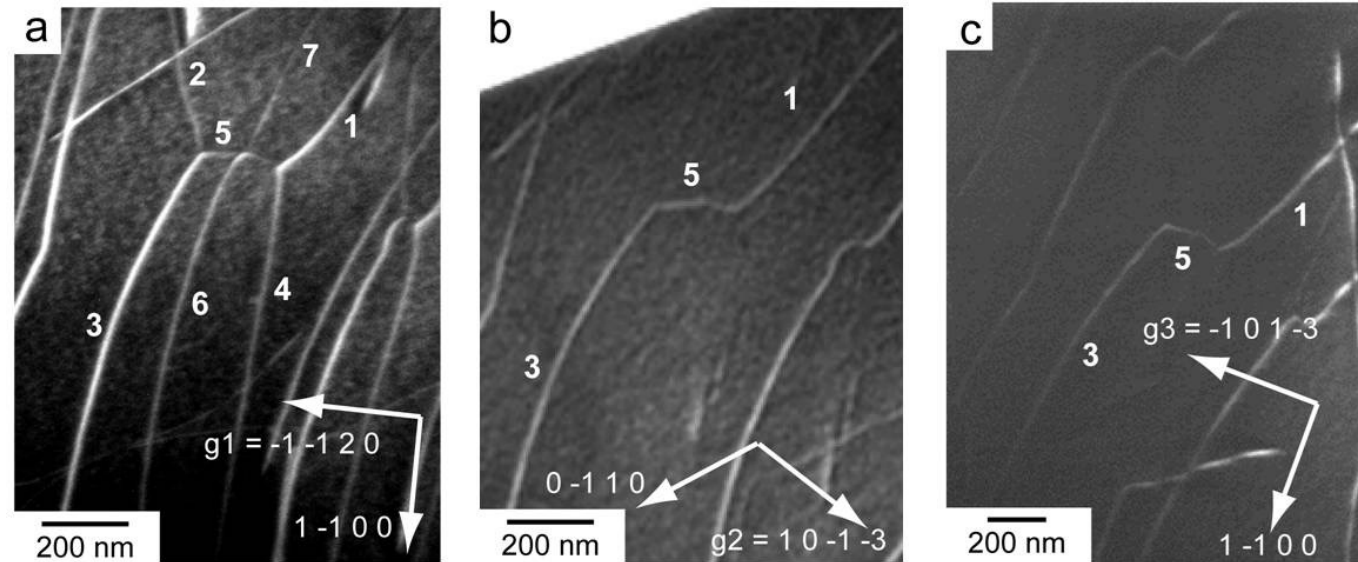
($\mathbf{G} \cdot \mathbf{r}_j = 2\pi$ by definition of \mathbf{G})

↪ $\sum_j f_j \exp[2\pi i \mathbf{G} \cdot \mathbf{R}_j]$ characterizes the defects
 ↪ If $\mathbf{G} \cdot \mathbf{R}_j = 0$ the defect will be out of contrast

Example of extinction condition



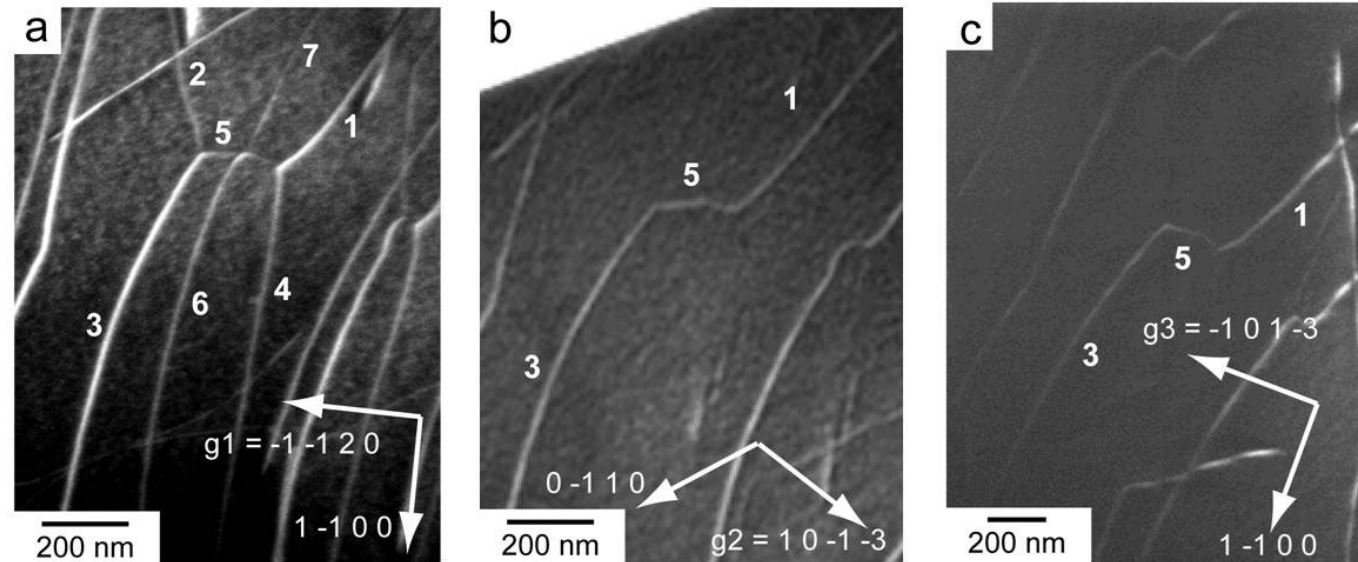
Example of extinction condition



❖ The dislocation 2 is obviously out of contrast for both $g_2 = (1\ 0\ \bar{1}\ \bar{3})$ and $g_3 = (\bar{1}\ 0\ 1\ \bar{3})$

$$\begin{cases} g_2 \cdot b_2 = 0 \\ g_3 \cdot b_2 = 0 \end{cases} \Rightarrow \begin{cases} (1\ 0\ \bar{1}\ \bar{3}) \cdot [h\ k\ i\ l] = 0 \\ (\bar{1}\ 0\ 1\ \bar{3}) \cdot [h\ k\ i\ l] = 0 \end{cases} \Rightarrow \begin{cases} h - i - 3l = 0 \\ -h + i - 3l = 0 \\ -(h + k) = i \end{cases}$$

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$$\Rightarrow b_2 = \pm a/3[\bar{1}\ 2\ \bar{1}\ 0]$$

The 2 types of electron microscopes

Transmission electron microscope (TEM)



- ❖ Invention: 1931 by Ernst RUSKA et Max KNOL
- ❖ Specimen : thin foils (~ 100 nm)
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Scanning electron microscope (SEM)

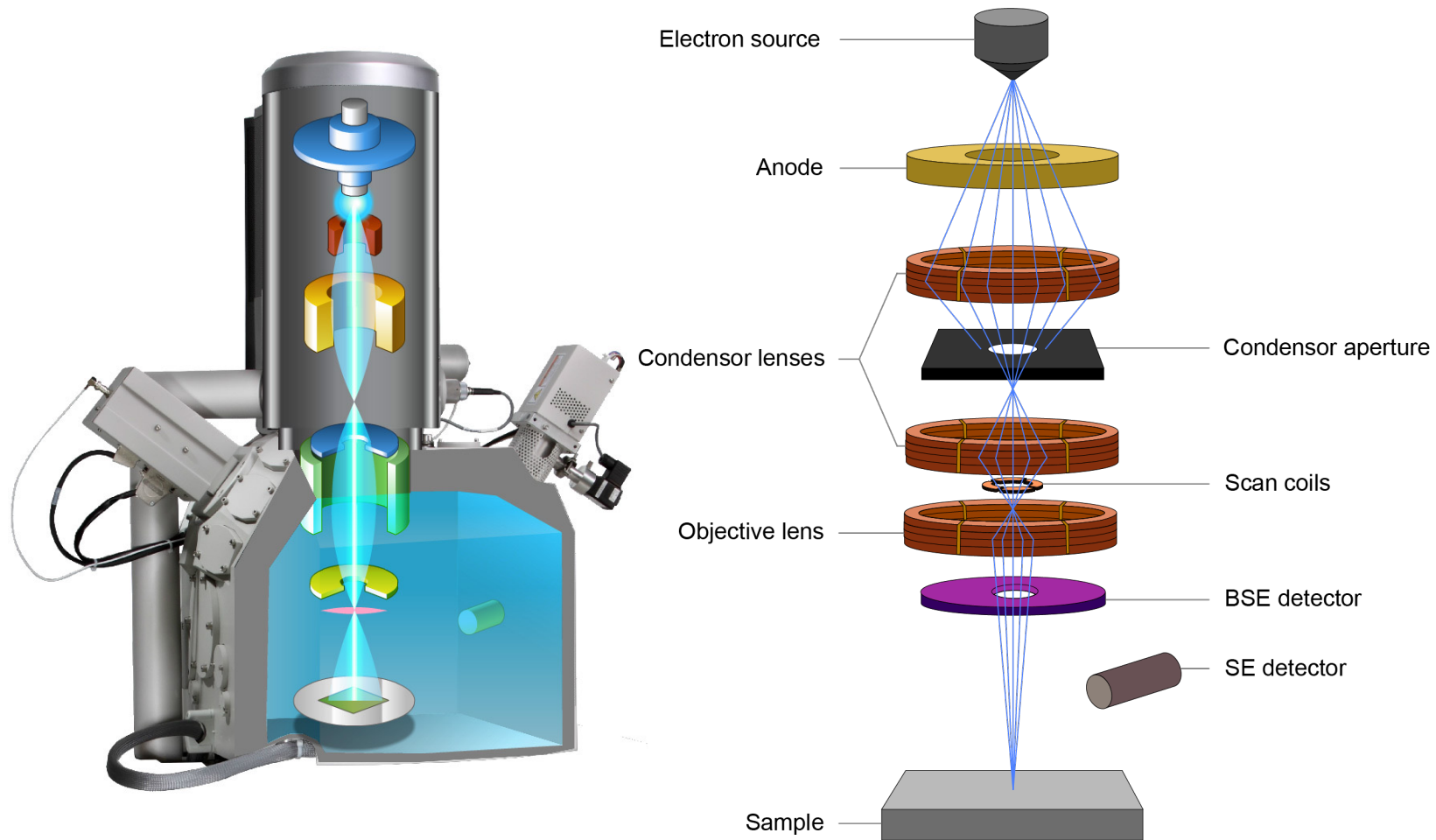


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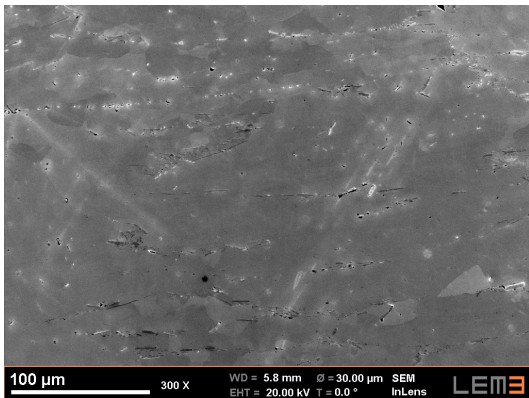
↳ **The two microscopes are complementary.**

Basics of SEM



SEM imaging

Secondary electrons (SE)



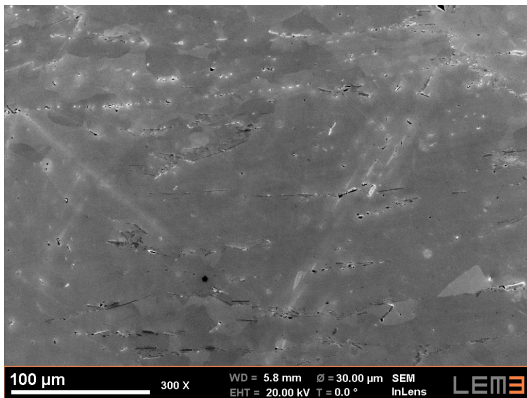
- ❖ Inelastic scattering between incident electrons and outer-shell electrons of sample atoms ⇒ emission of SE
- ❖ Inelastic scattering involving inner-shell electrons, followed by electronic transitions → emission of characteristic X-rays

⇒ EDS

↪ Topographic contrast

SEM imaging

Secondary electrons (SE)

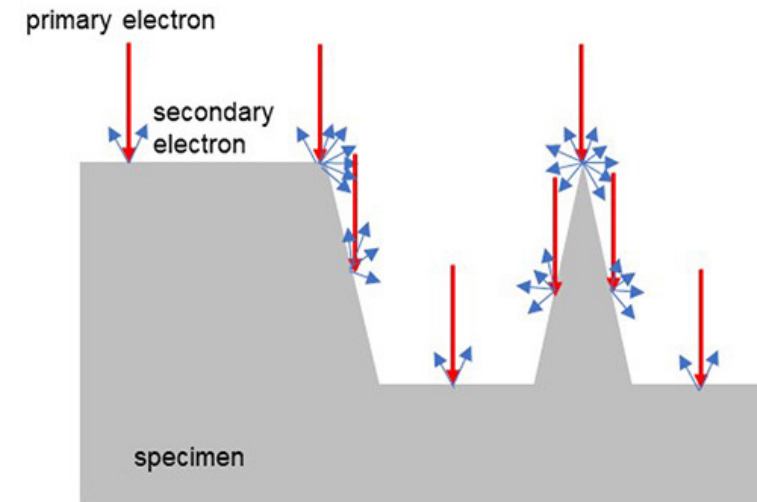


- ❖ Inelastic scattering between incident electrons and outer-shell electrons of sample atoms \Rightarrow emission of SE
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↪ Topographic contrast

The edge effect

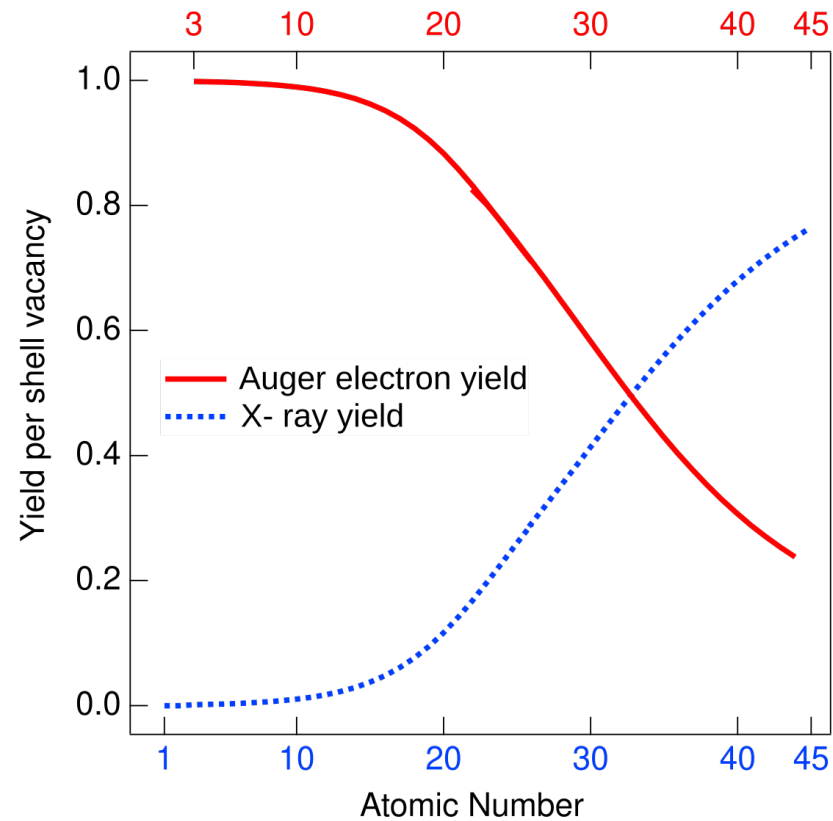
- ❖ Areas at edges and corners appear very bright in SE images due to increased SE emission.



↪ SEM is not 3D, but it gives a strong 3D impression.

Energy Dispersive X-ray Spectroscopy (EDS)

186

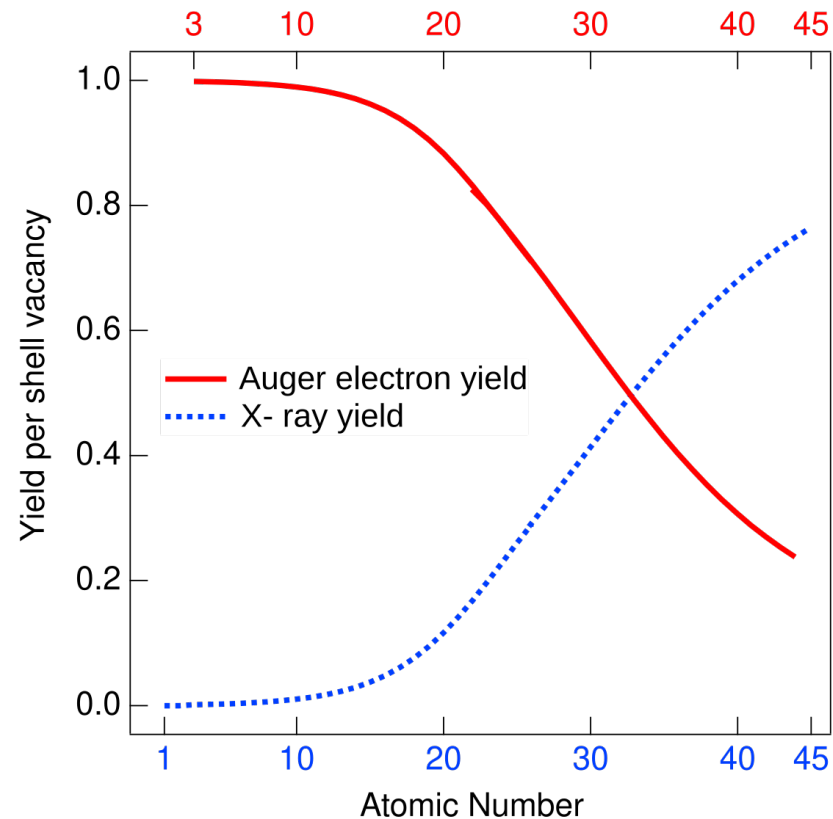


❖ Inner-shell ionization:

- When the high-energy electron beam hits the sample, it can eject inner-shell electrons (usually from K or L shells) in the atoms.

Energy Dispersive X-ray Spectroscopy (EDS)

187



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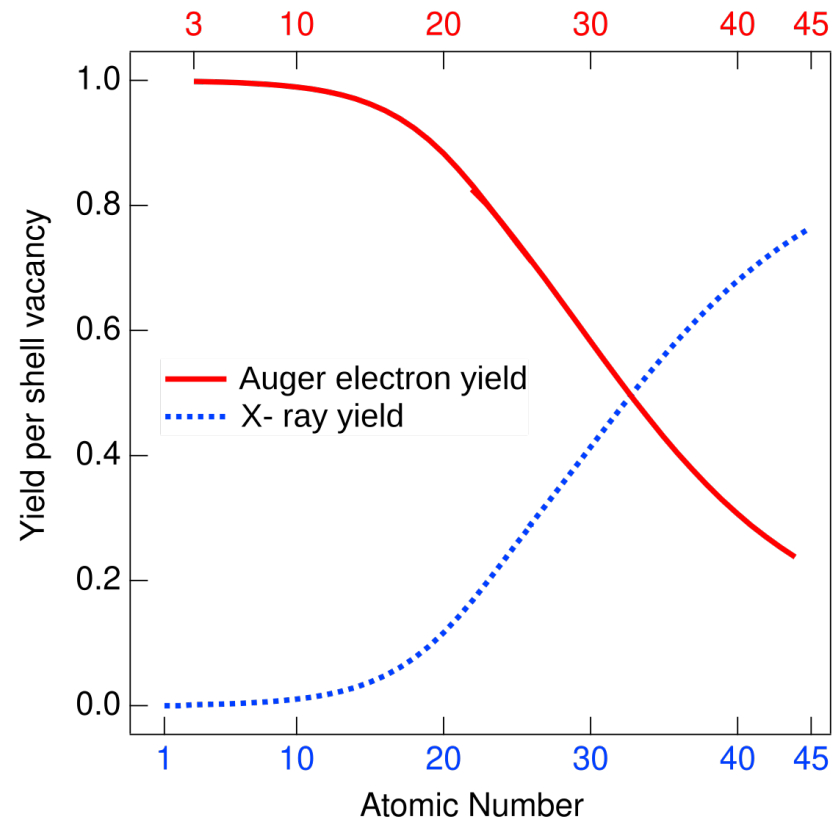
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- An electron from a higher-energy shell falls down to fill the vacancy. This transition releases energy in the form of an X-ray photon.

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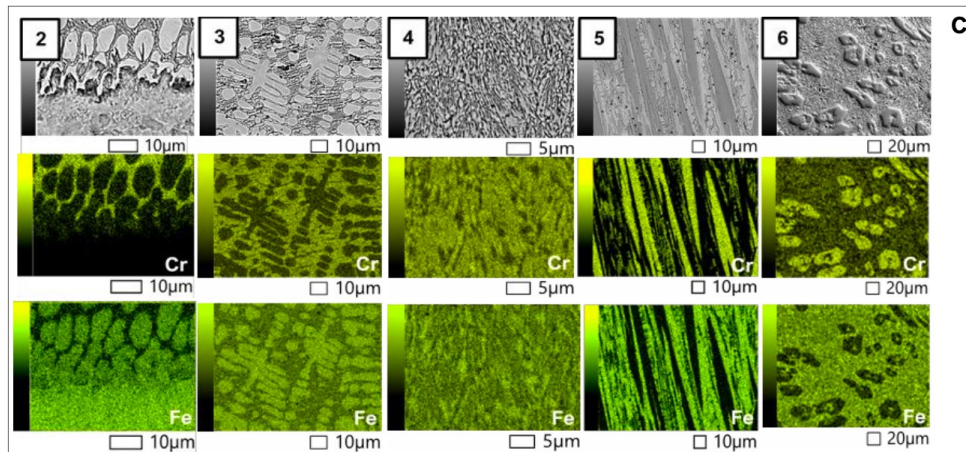
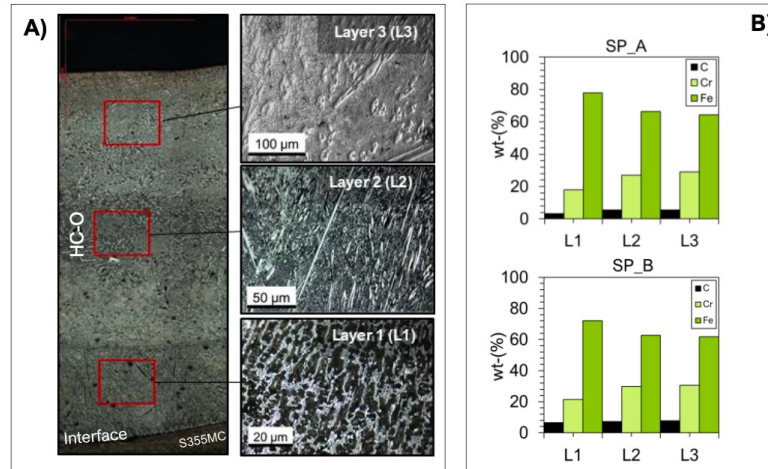
- An electron from a higher-energy shell falls down to fill the vacancy. This transition releases energy in the form of an X-ray photon.

❖ X-ray emission:

- The emitted characteristic X-ray has an energy equal to the difference between the two shells. This energy is unique to each element.

Energy Dispersive X-ray Spectroscopy (EDS)

189



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- When the high-energy electron beam hits the sample, it can eject inner-shell electrons (usually from K or L shells) in the atoms.

❖ Electronic transition:

- An electron from a higher-energy shell falls down to fill the vacancy. This transition releases energy in the form of an X-ray photon.

❖ X-ray emission:

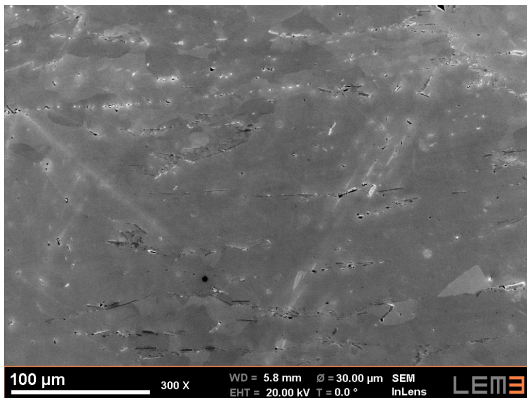
- The emitted characteristic X-ray has an energy equal to the difference between the two shells. This energy is unique to each element.

❖ Detection:

- An EDS detector measures the energy of these X-rays, allowing us to:
 - ✓ Identify the elements present (qualitative analysis)
 - ✓ Estimate their relative amounts (semi-quantitative analysis but not accurate for low Z elements)

SEM imaging

Secondary electrons (SE)

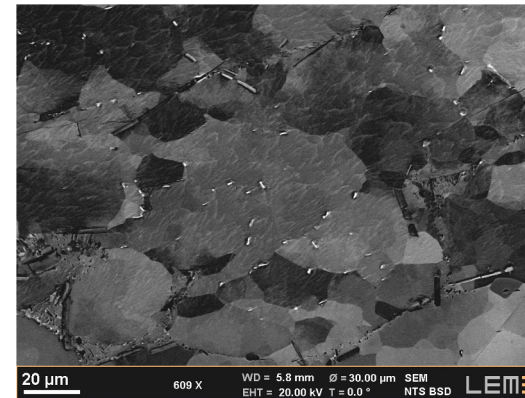


- ❖ Inelastic scattering between incident electrons and outer-shell electrons of sample atoms \Rightarrow emission of SE
- ❖ Inelastic scattering involving inner-shell electrons, followed by electronic transitions \rightarrow emission of characteristic X-rays

\Rightarrow EDS

\hookrightarrow Topographic contrast

BackScattered Electrons (BSE)



- ❖ Elastic scattering: the incident electron is deflected by the electrostatic field of the atomic nucleus, without significant energy loss.

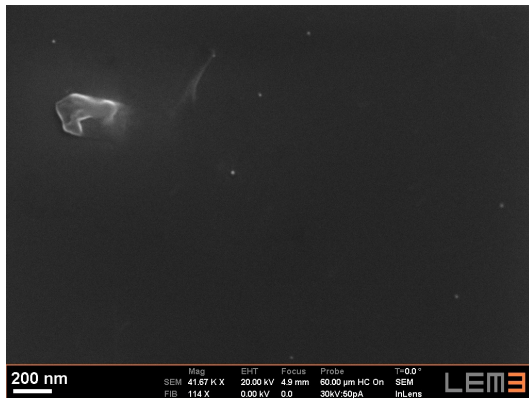
\Rightarrow EBSD, ECCI

\hookrightarrow Topographic contrast
 \hookrightarrow Chemistry contrast
 \hookrightarrow Orientation contrast

\hookrightarrow **The two imaging modes are complementary.**

SEM imaging

Secondary electrons (SE)

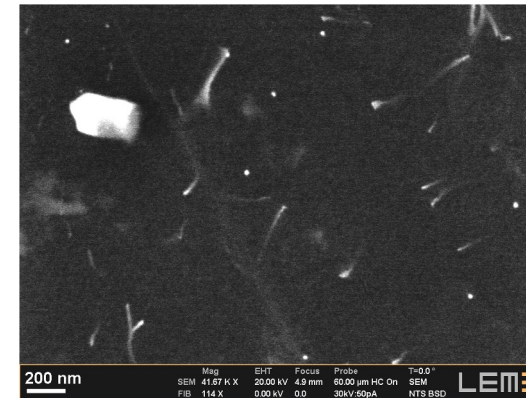


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Fate of incident electrons in SEM

❖ X-rays:

- ~0.1–1% of incident electrons generate X-rays

❖ Secondary electrons (SE):

- ~5–40%

❖ Backscattered electrons (BSE):

- ~10–50%
- The BSE cross section increases with Z

❖ Other energy losses (heat, phonons, electronic excitations)

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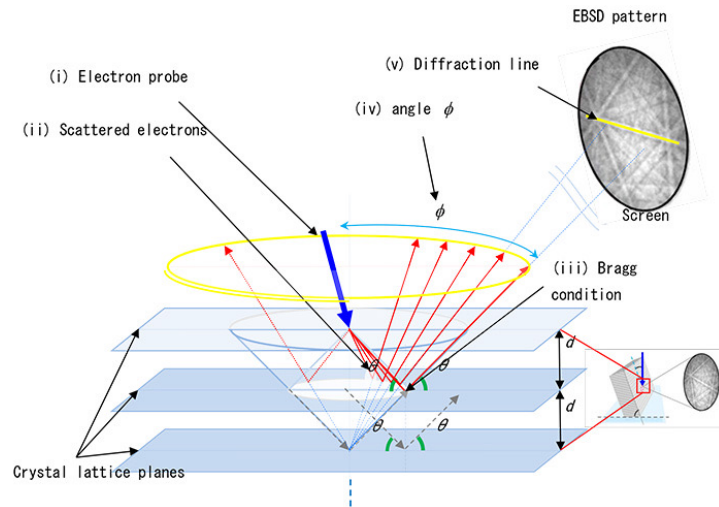
❖ Other energy losses (heat, phonons, electronic excitations)

❖ Relative contributions within the BSE signal

- Orientation contrast (electron channeling): ~2–5% of the BSE contrast
- Atomic number contrast (Z-contrast)
- ~10–30% of the BSE contrast
- Topographic contrast: ~65–90% of the BSE contrast

Basics of the Electron BackScattered Diffraction (EBSD)

194

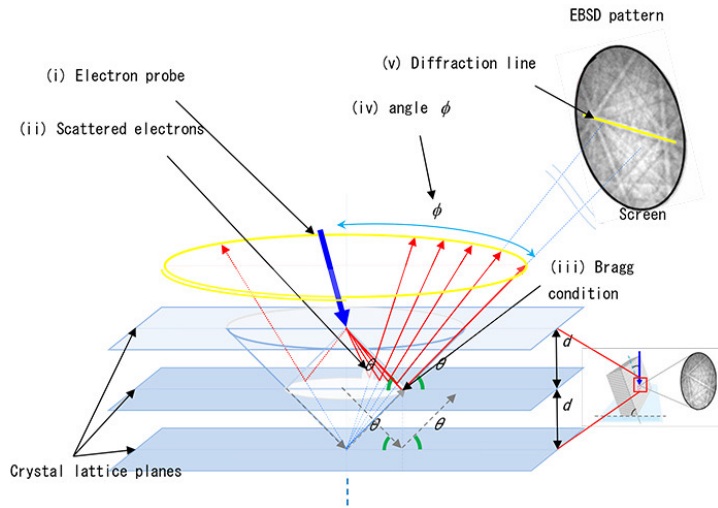


❖ Generation of Kikuchi patterns:

- The incident electron beam interacts with the crystal lattice, generating BSE through inelastic and elastic scattering.
- Some of these BSE undergo coherent elastic diffraction by the crystal planes, fulfilling the Bragg condition.
- This diffraction produces Kikuchi bands, which appear as pairs of parallel lines corresponding to specific crystallographic planes.

Basics of the Electron BackScattered Diffraction (EBSD)

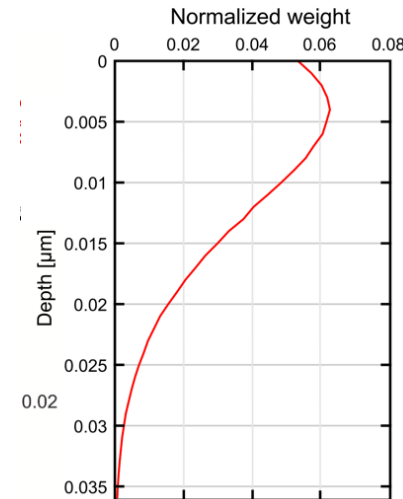
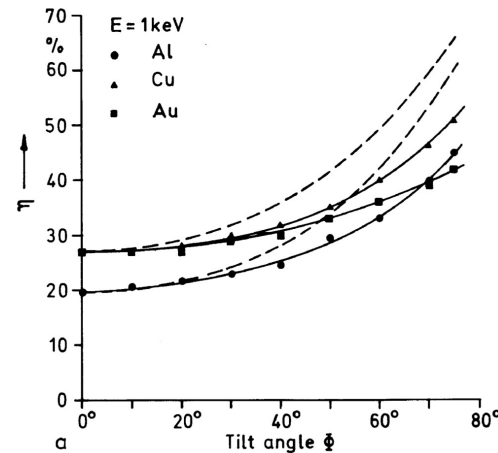
195



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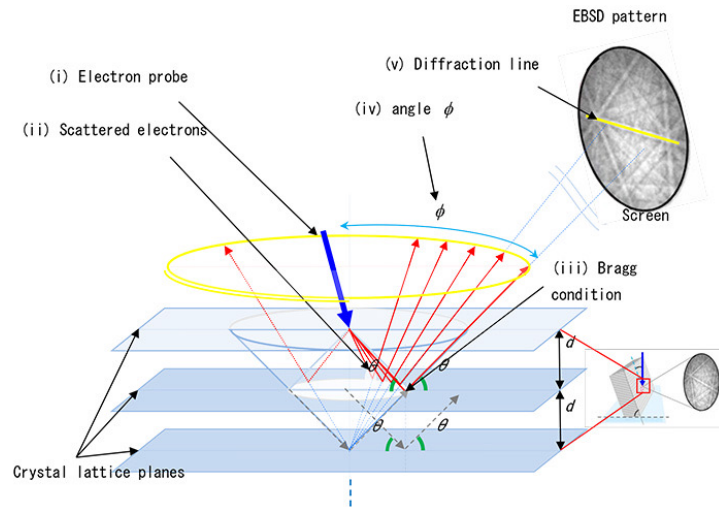
↪ The BSE yield increases with increasing tilt angle



↪ The probing depth in GaN at 15 keV is ~35 nm below the surface.

Basics of the Electron BackScattered Diffraction (EBSD)

196

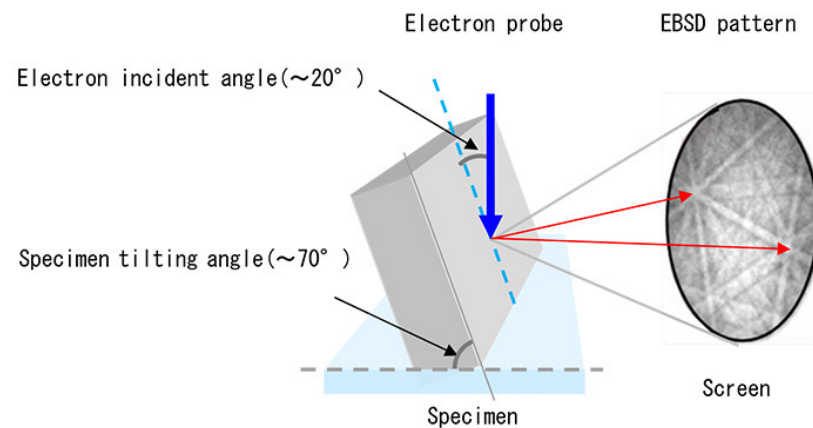


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❖ Basics of the EBSD:

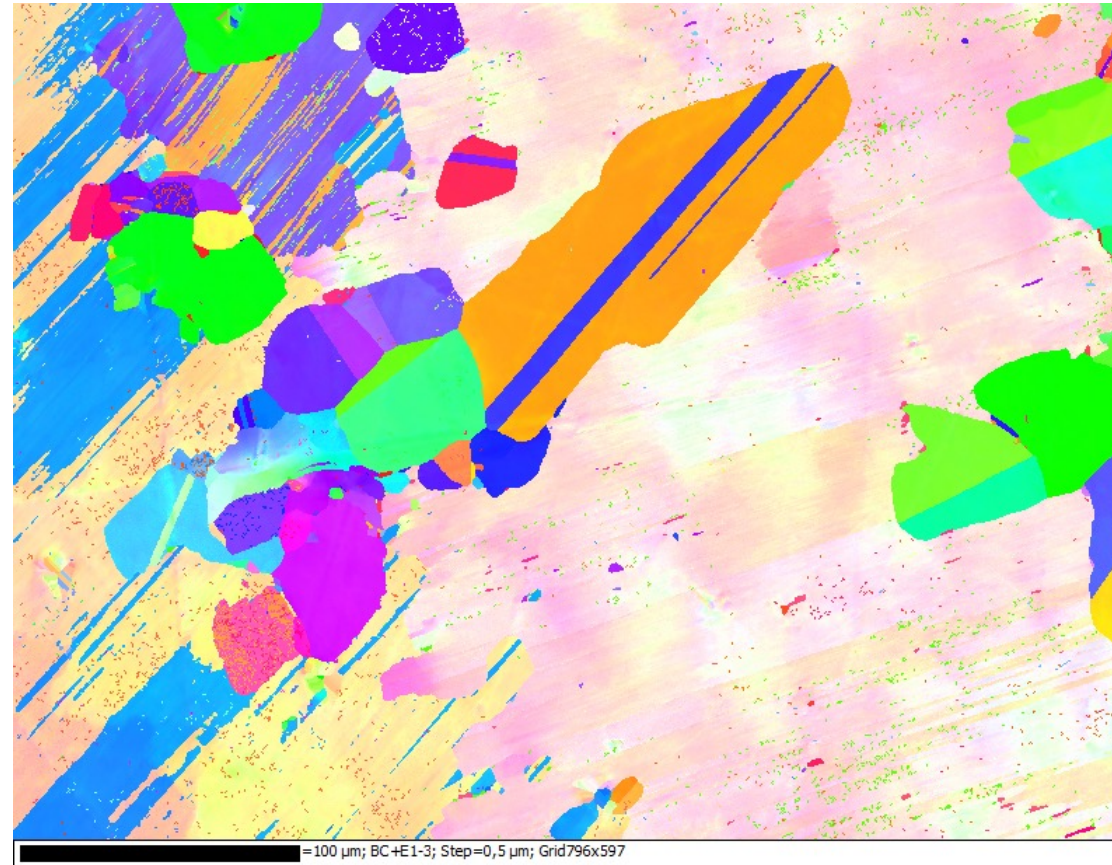
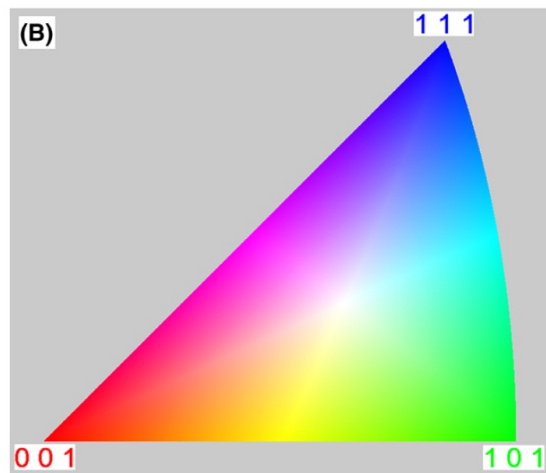
- EBSD involves directing a focused electron beam onto a tilted crystalline specimen
- The BSE form Kikuchi patterns, which are captured by a phosphor screen and recorded using a camera.
- These patterns are analyzed to extract information about the material's grain structure, orientation, and phase.



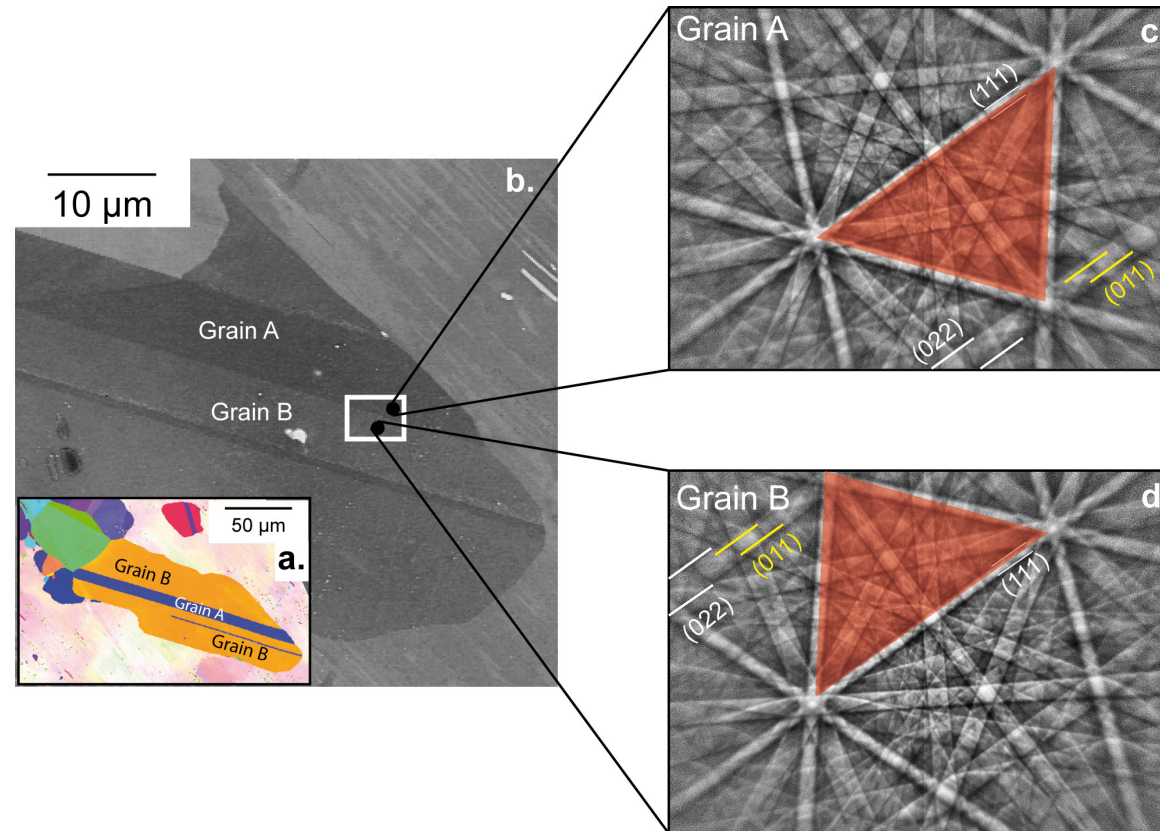
Example of extinction condition

❖ Capabilities of EBSD:

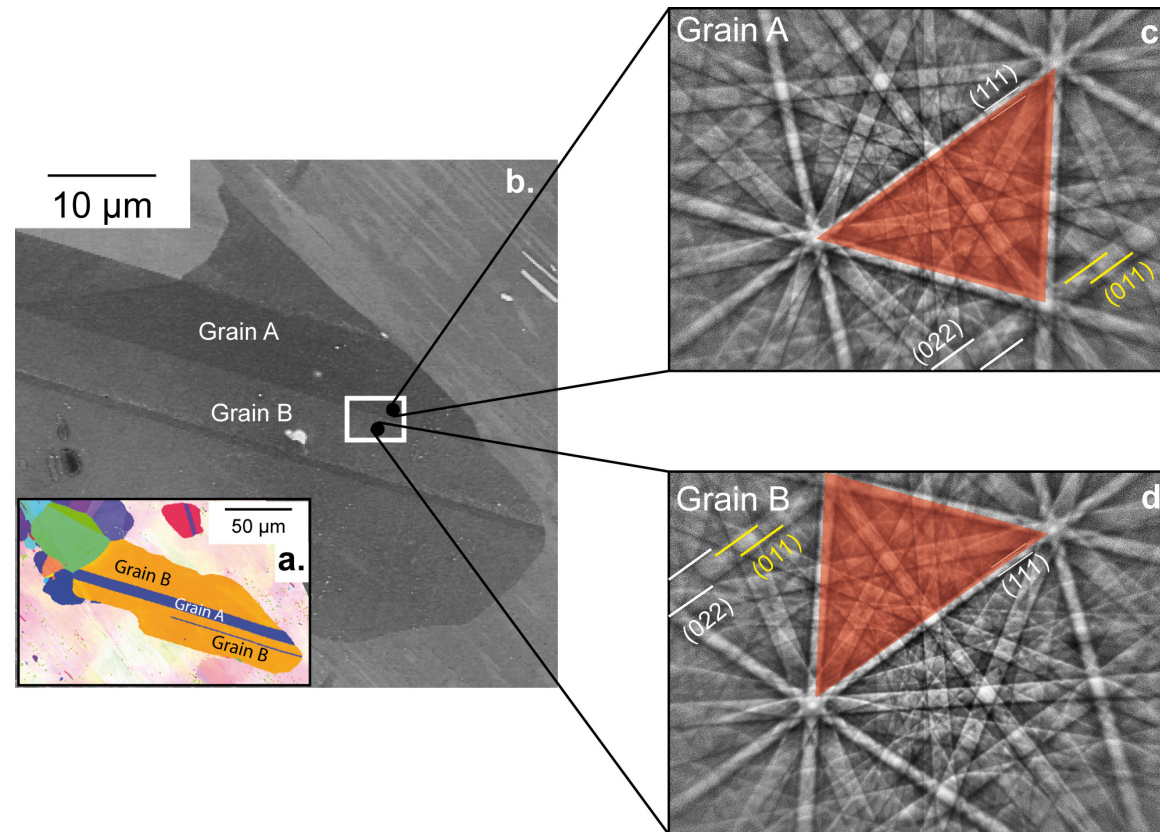
- Orientation measurements
- Local misorientation measurements
- Grain identification
- Phase identification
- Statistics on the microstructure



What is the nature of the boundary?

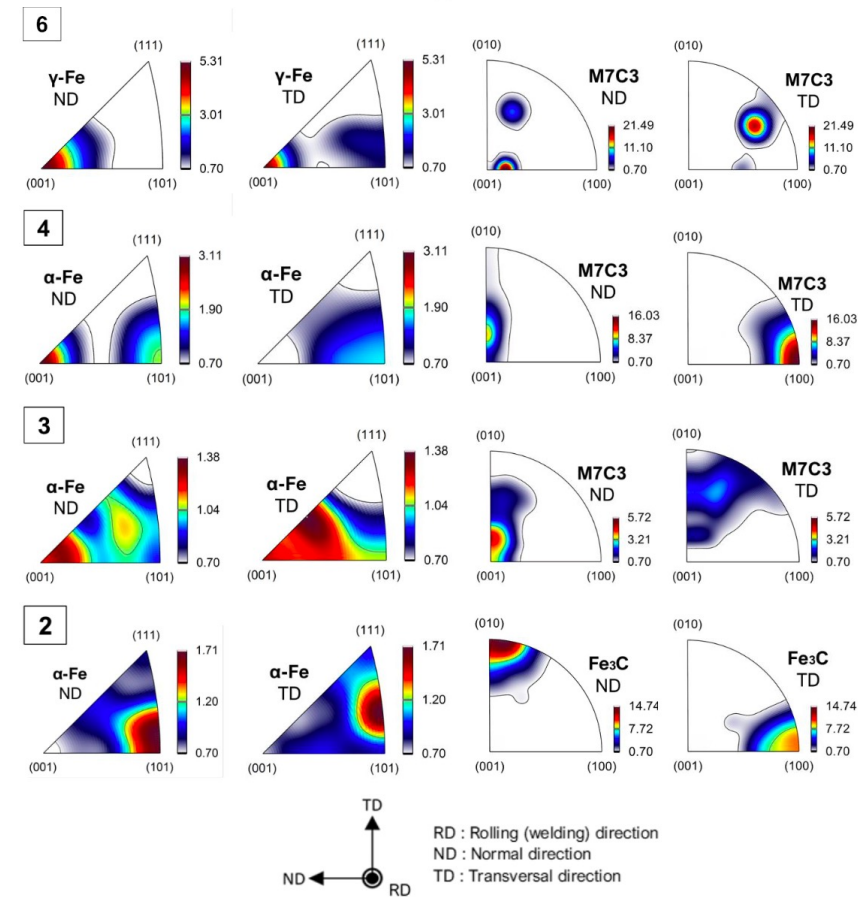
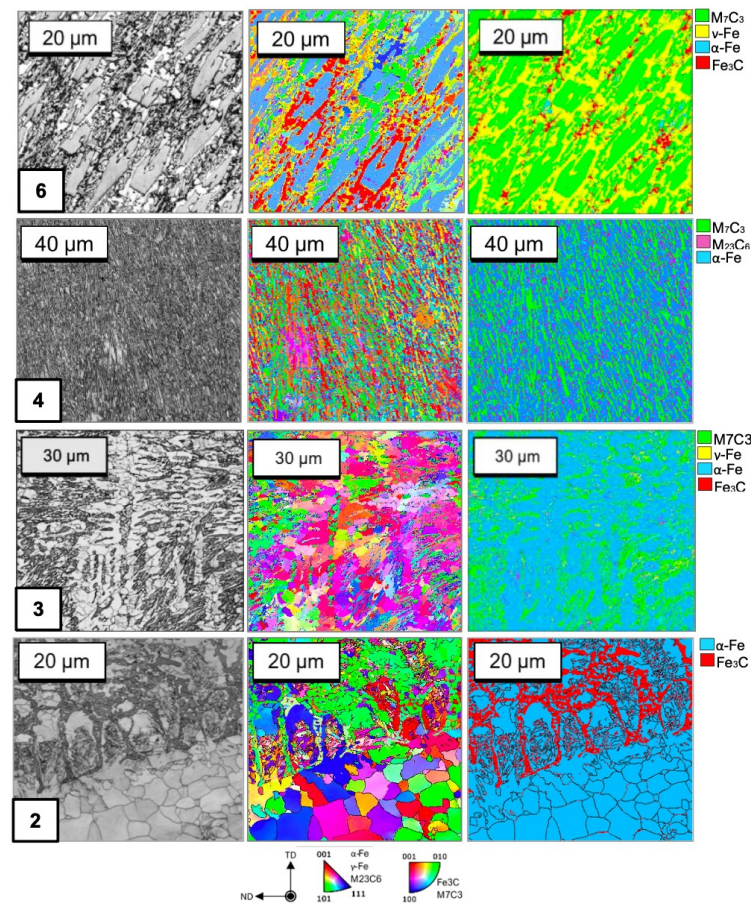
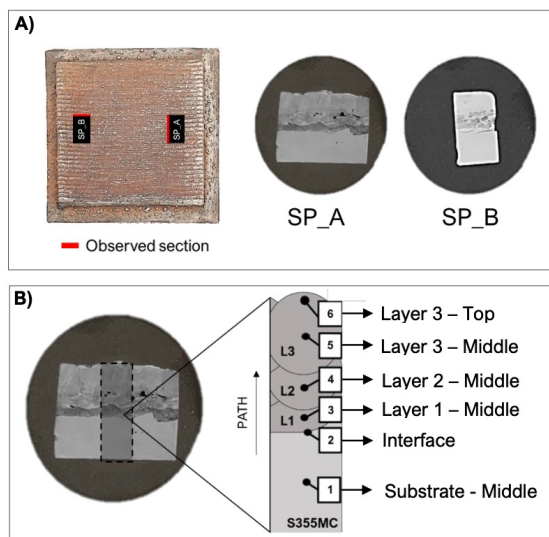


What is the nature of the boundary?



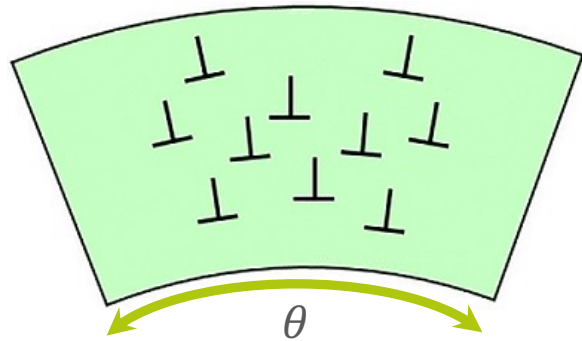
↪ This is a (1 1 1) twin boundary.

Texture measurements by EBSD



Different types of dislocation densities

Geometrically necessary dislocations (GND)



⇒ Lattice curvature at long range

$$\rho_{\text{GND}} \approx \frac{1}{\|\mathbf{b}\|} \|\boldsymbol{\alpha}\|; \|\boldsymbol{\alpha}\| = \sqrt{\alpha_{ij} \cdot \alpha_{ij}}$$

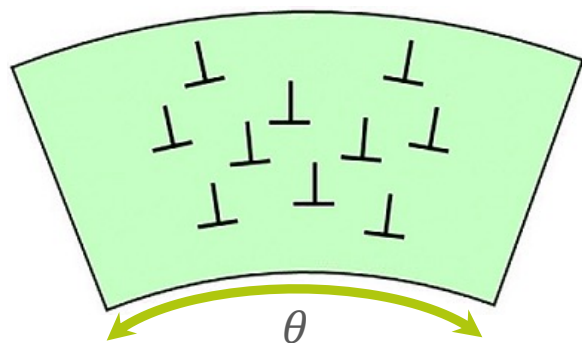
$$\boldsymbol{\alpha} = \text{curl } \boldsymbol{\varepsilon} + \text{tr}(\boldsymbol{\kappa}_e) \cdot \mathbb{I} - \boldsymbol{\kappa}_e^T$$

$$\kappa_{ij} \cong \frac{\Delta \omega_i}{\Delta x_j}; \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$



Different types of dislocation densities

Geometrically necessary dislocations (GND)



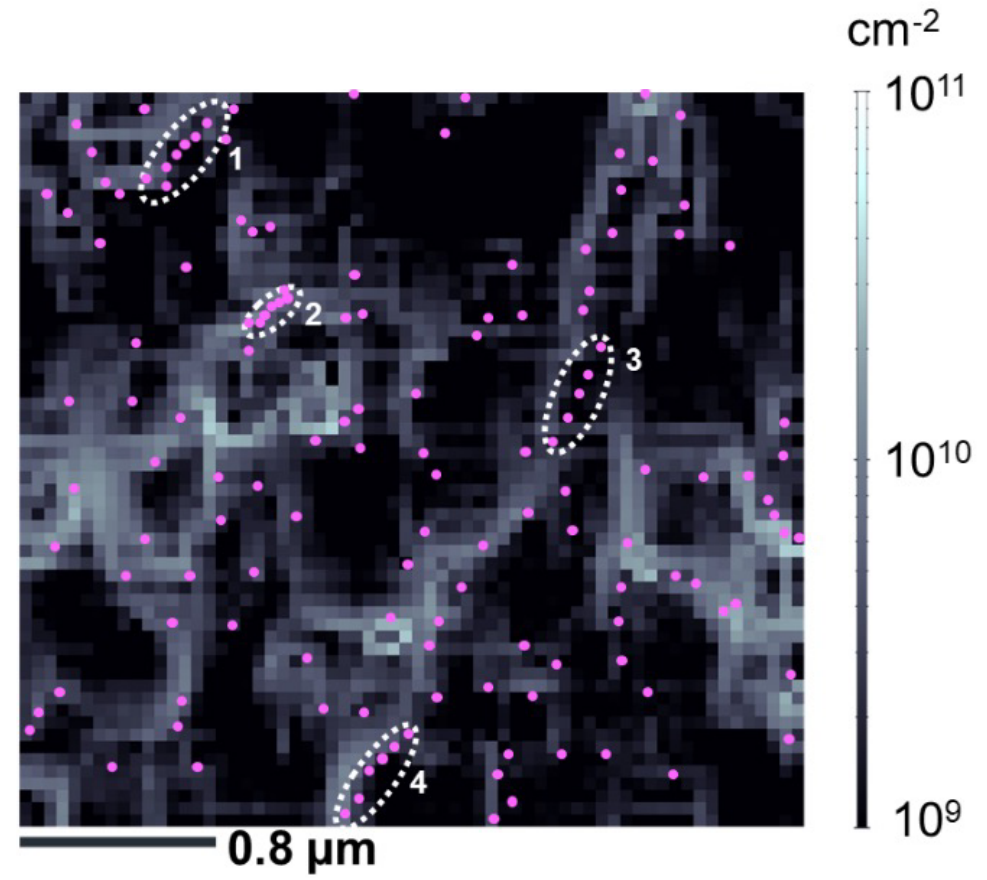
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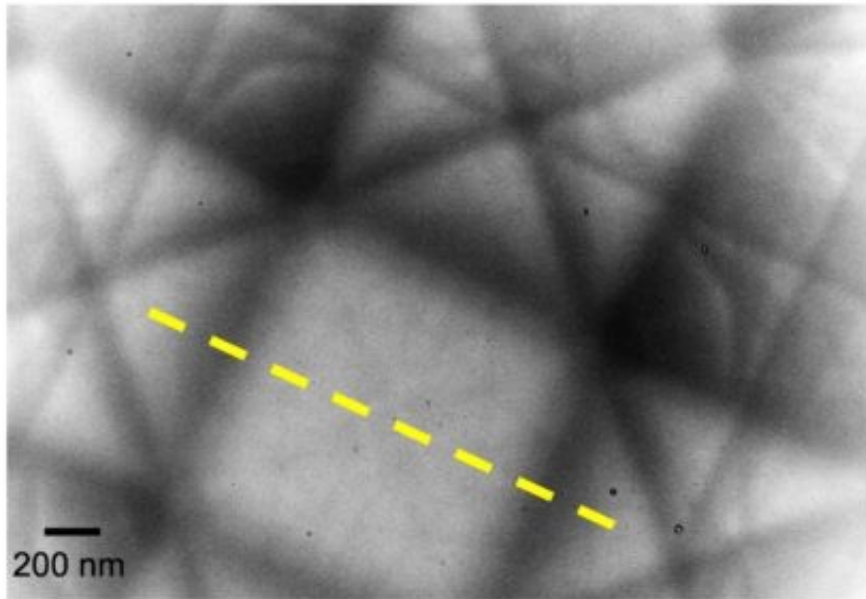
Norm of the Nve tensor ($\|\alpha\|$)



Origin of the Electron Channeling Pattern (ECP)

203

- ❖ Kikuchi-like bands are observed at low magnification on a bulk single crystal.



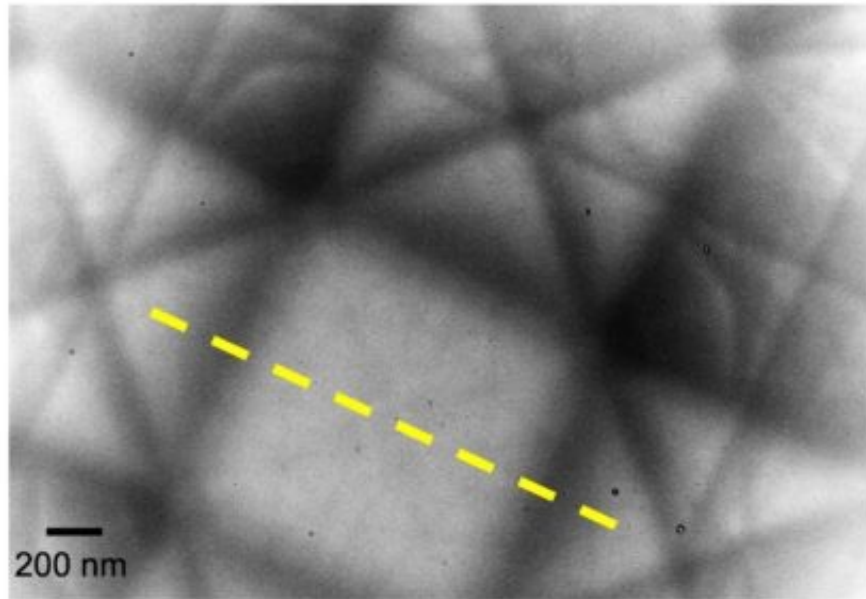
Electron Channeling Pattern "ECP"
obtained from Si single crystal.

Origin of the Electron Channeling Pattern (ECP)

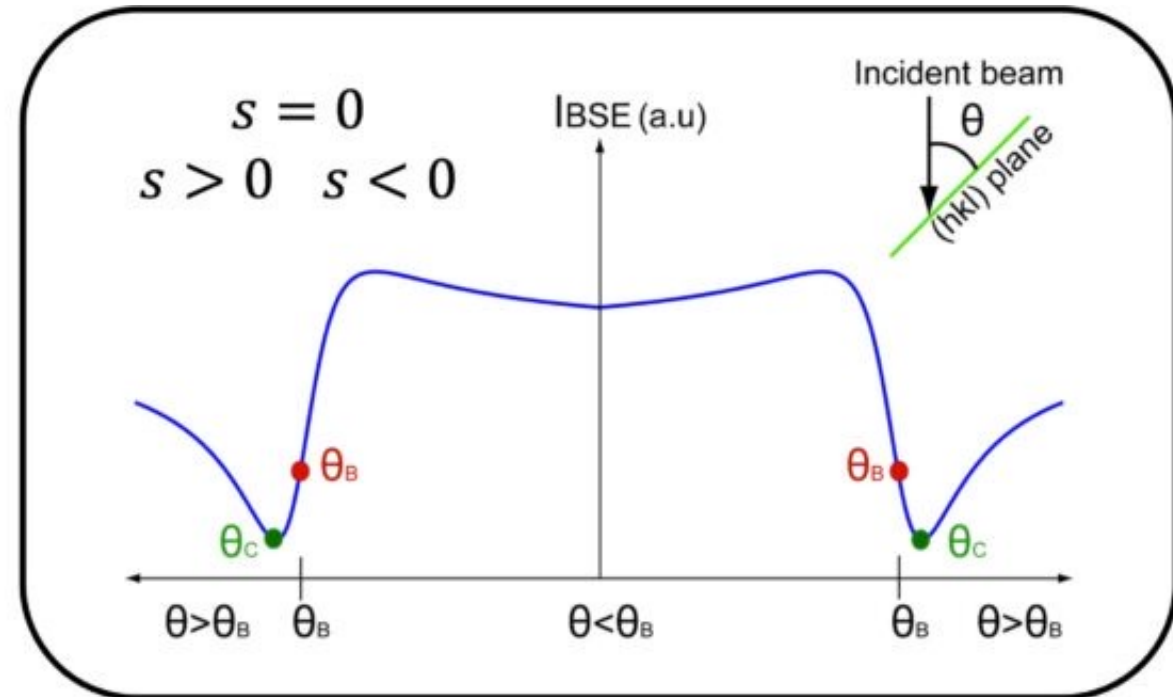
204

❖ Kikuchi-like bands are observed at low magnification on a bulk single crystal.

❖ BSE yield depends on the specimen orientation relative to the incident beam.

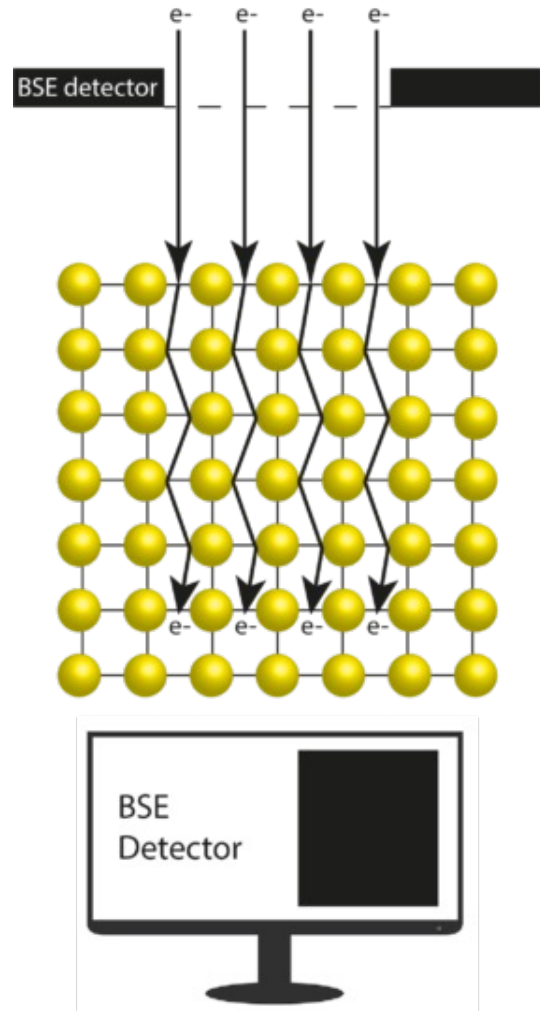


Electron Channeling Pattern "ECP" obtained from Si single crystal.

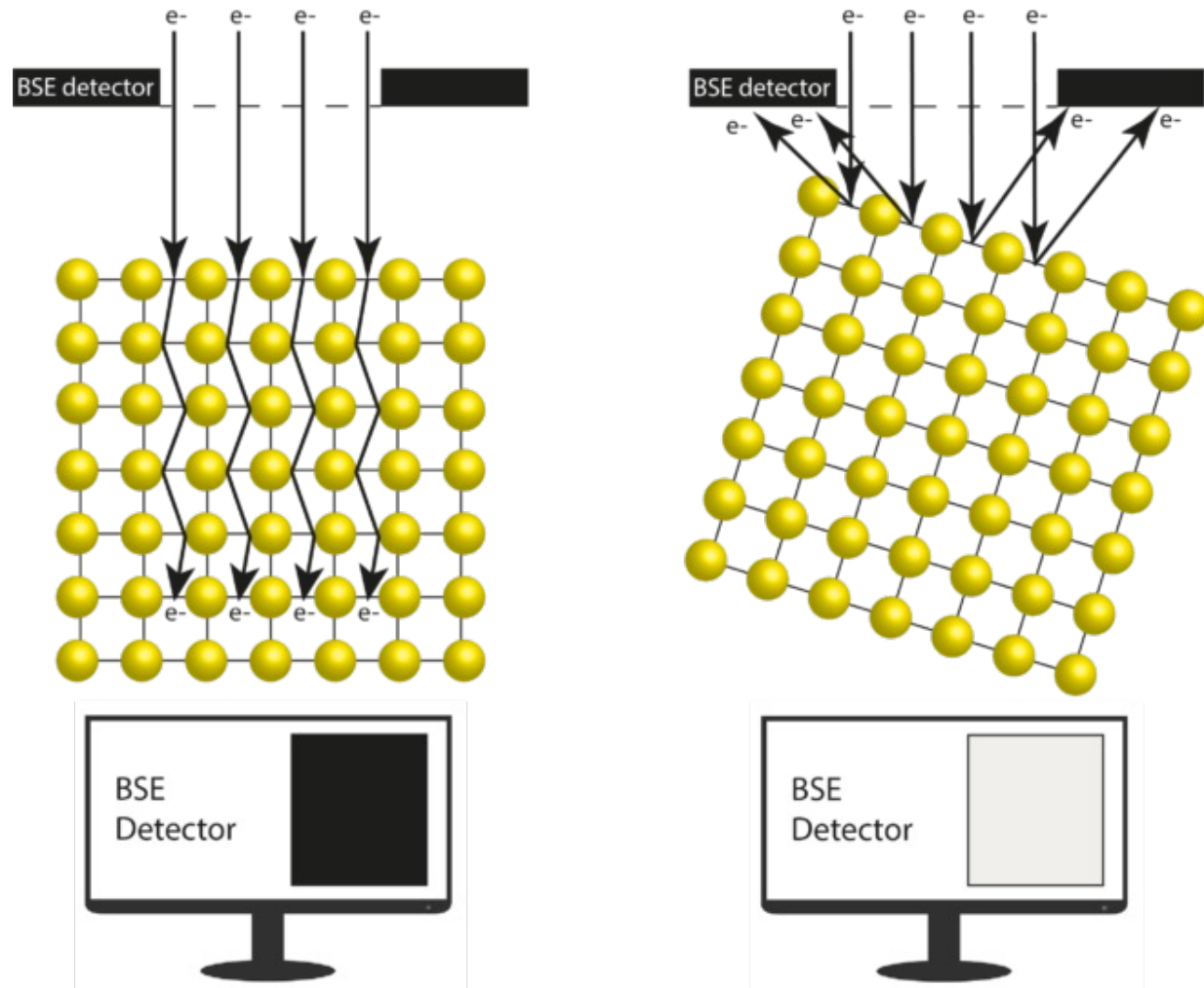


Channeling condition \neq Bragg condition

What is the Electron Channeling Contrast Imaging? 205

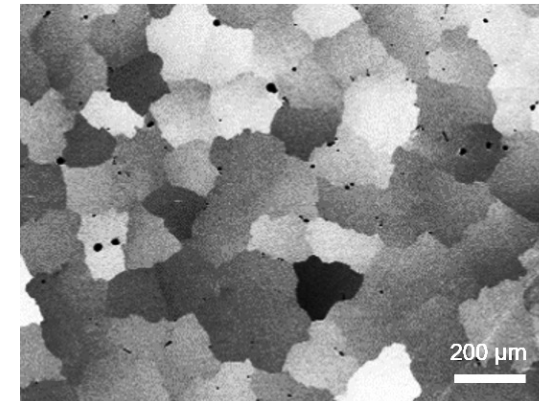


What is the Electron Channeling Contrast Imaging? 206



This is not a polycrystal!

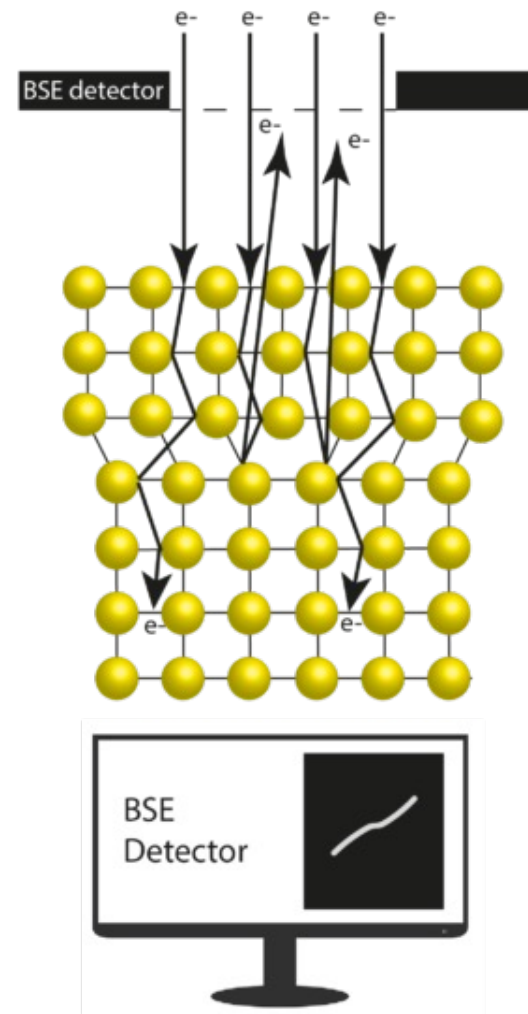
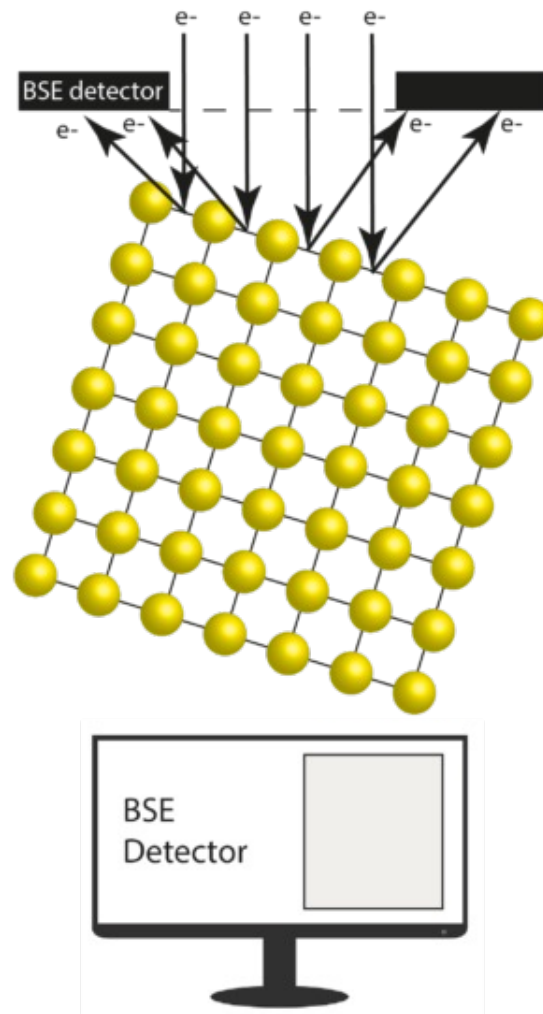
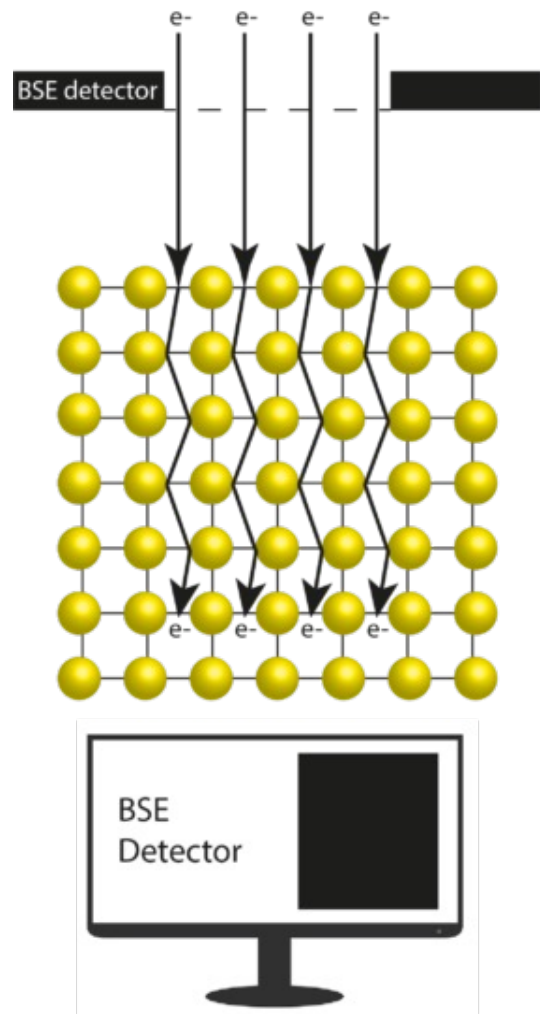
F. Habiyaemye et al., *MSEA* (2021)



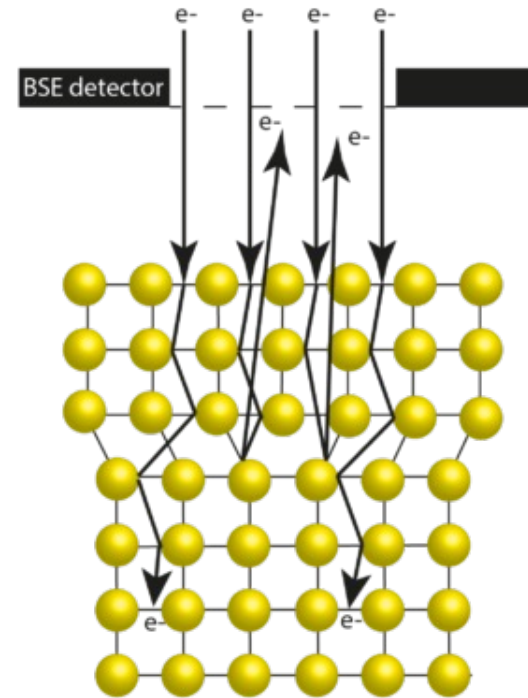
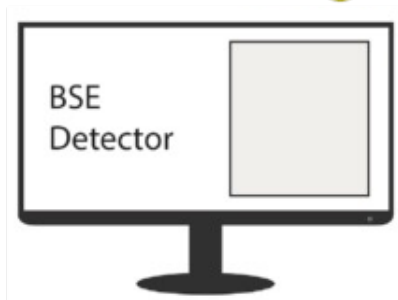
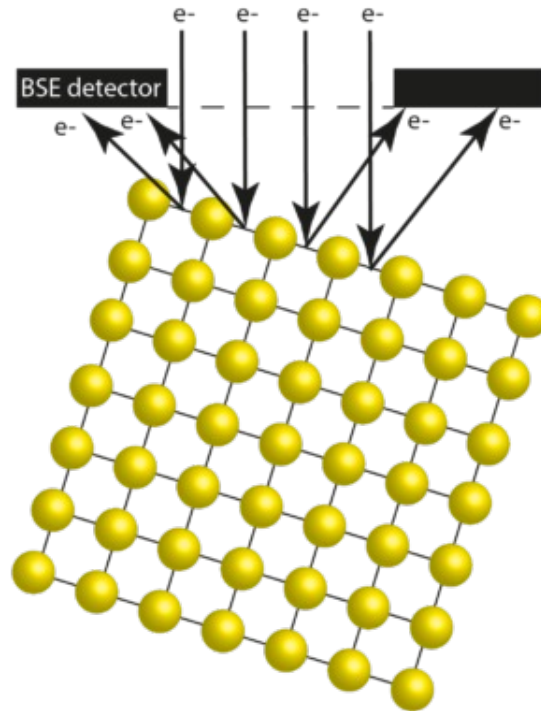
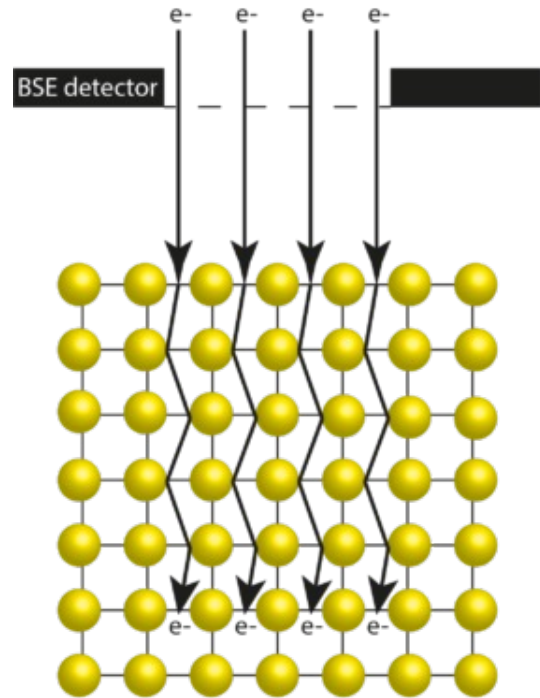
Single crystal of CrCoNi with dendrites
(mean misorientation: $< 1^\circ$)

What is the Electron Channeling Contrast Imaging?

207



What is the Electron Channeling Contrast Imaging? 208



$$\text{Dislocation} \Rightarrow \begin{cases} \mathbf{g} \cdot \mathbf{b} = 0 \\ \mathbf{g} \cdot \mathbf{b} \times \boldsymbol{\xi} = 0 \end{cases}$$

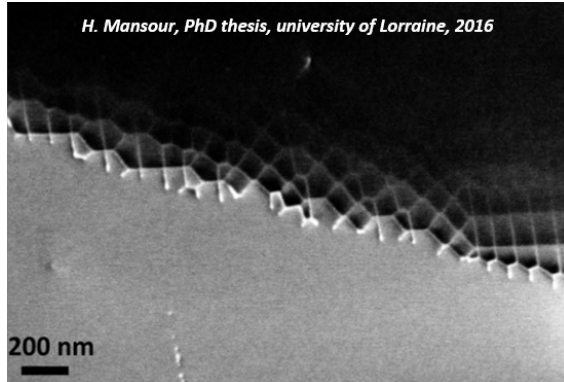
$$\text{Stacking fault} \Rightarrow \mathbf{g} \cdot \mathbf{R} = 0$$



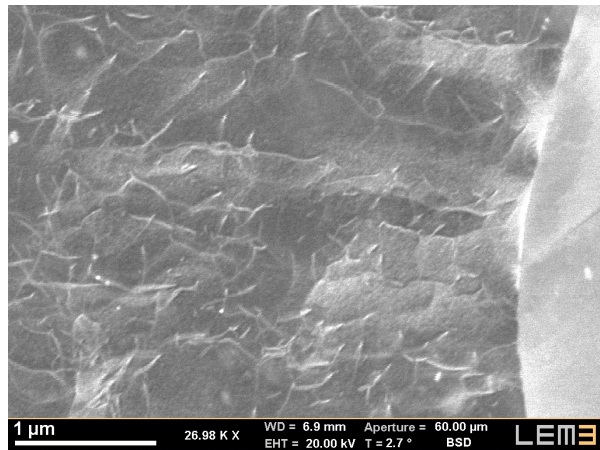
Nature of the crystalline defects

Detailed characterization of crystalline defects

209

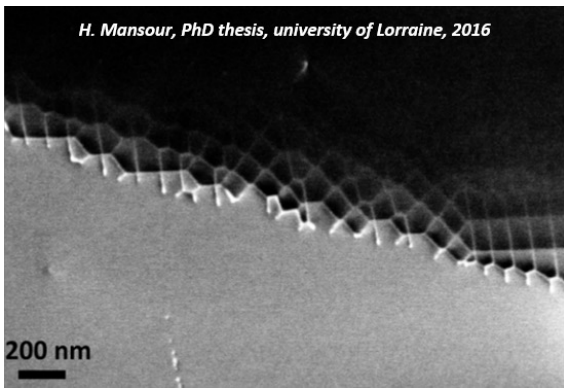


Ceramics (UO_2)

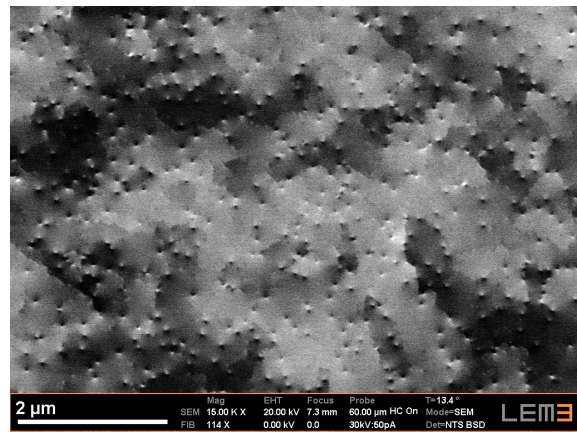


Ceramics (MAX phase)

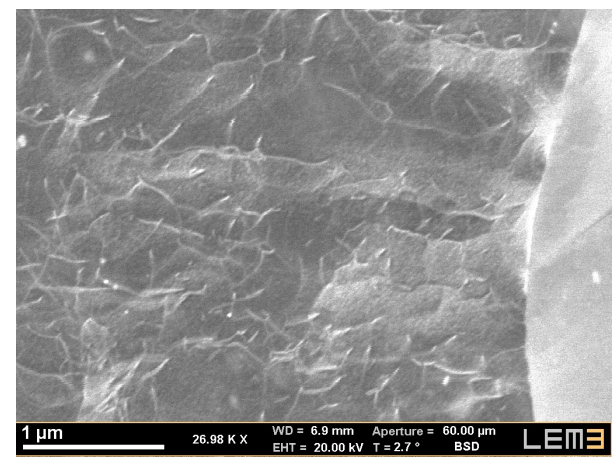
Detailed characterization of crystalline defects



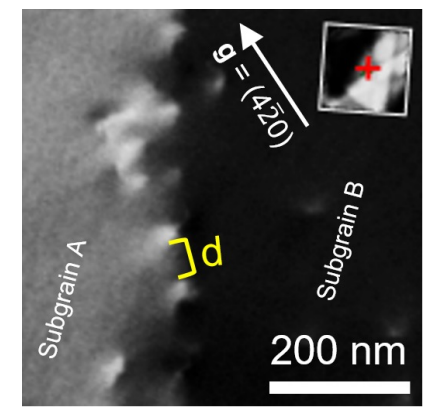
Ceramics (UO_2)



Semi-conductors (GaN)

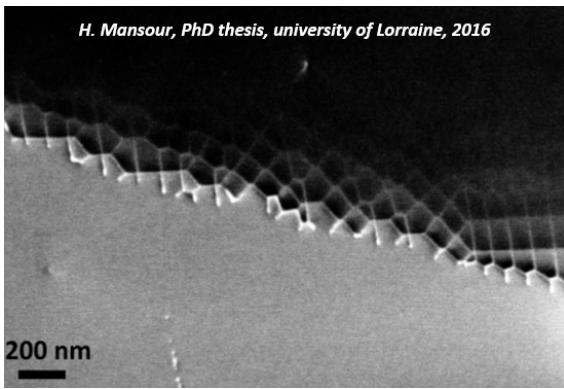


Ceramics (MAX phase)

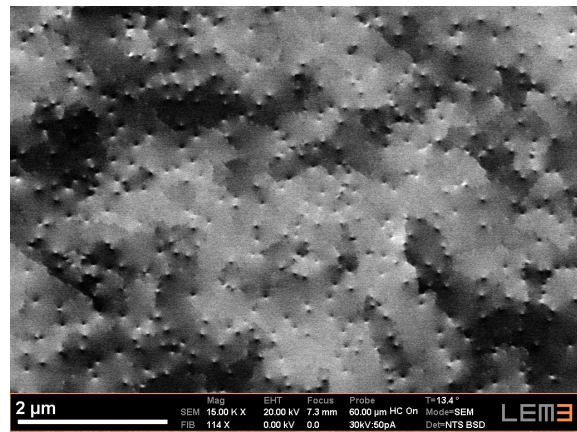


Metals (CrCoNi)

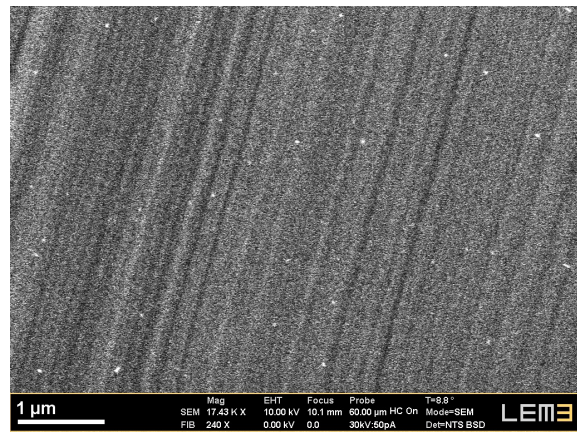
Detailed characterization of crystalline defects



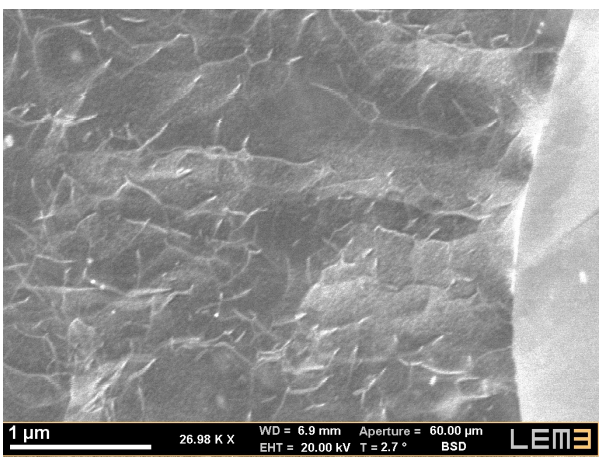
Ceramics (UO_2)



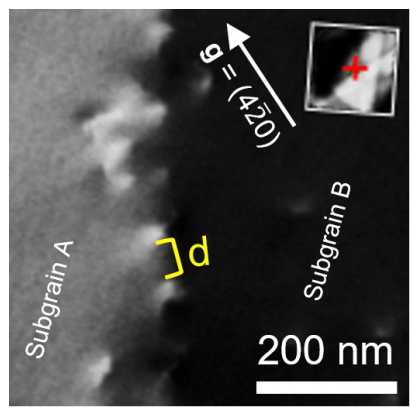
Semi-conductors (GaN)



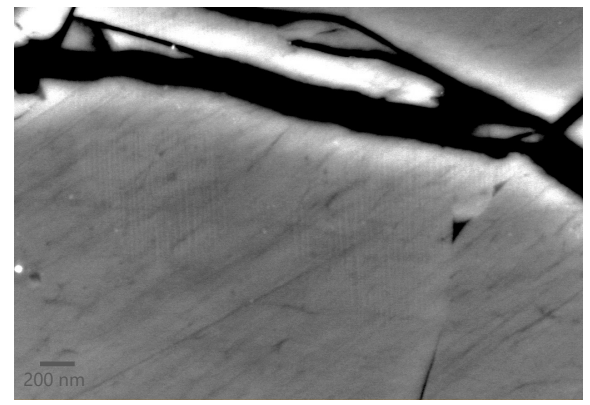
Diamond



Ceramics (MAX phase)

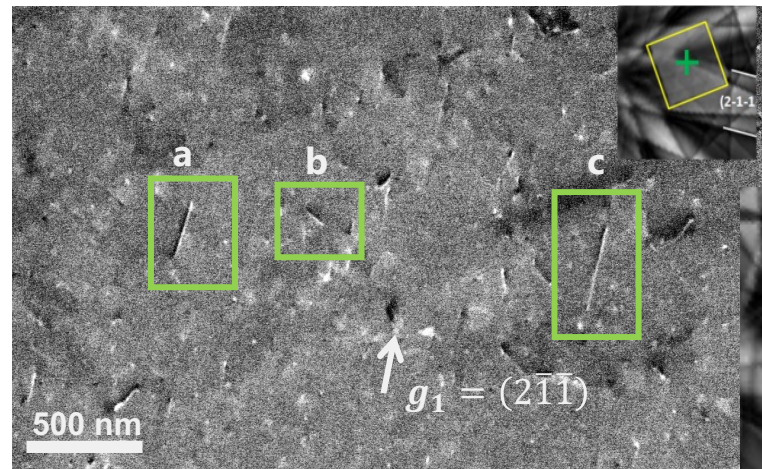


Metals (CrCoNi)



Rock (meteorite)

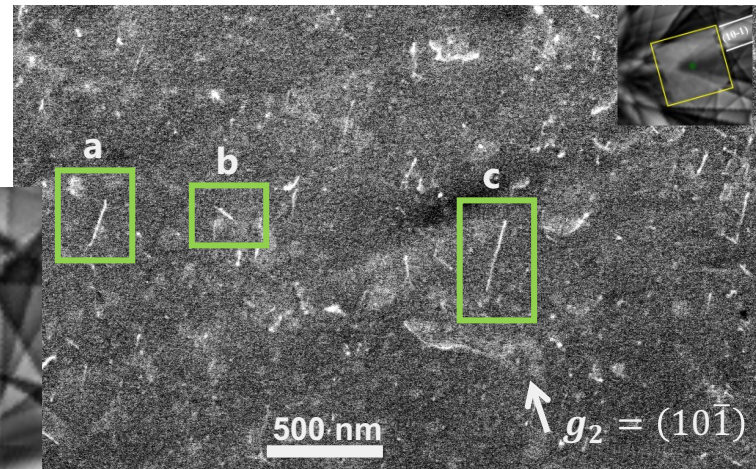
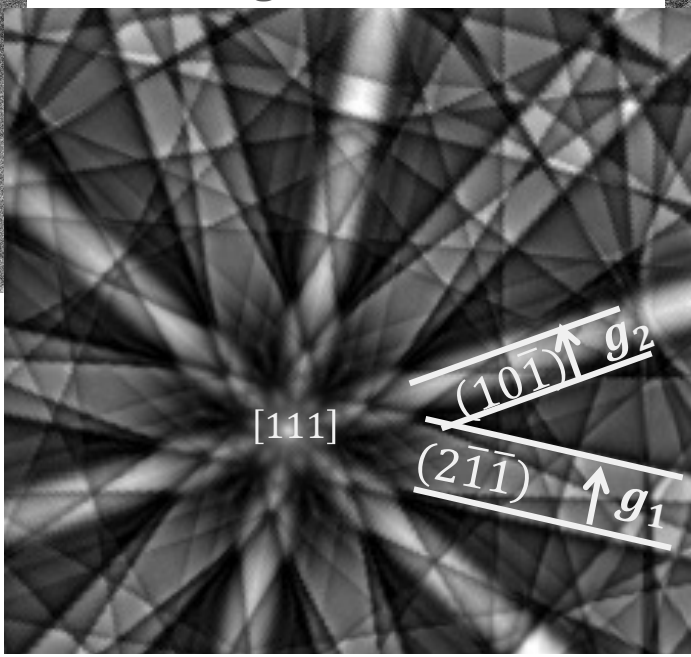
How to characterize dislocations by ECCL?



g : diffraction vector

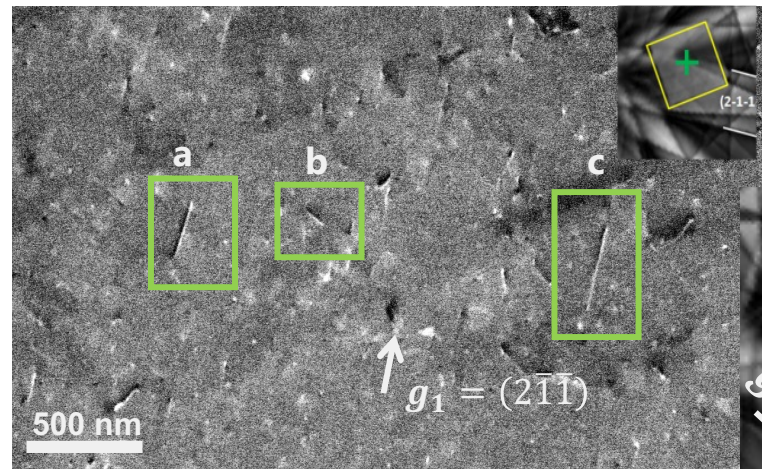
$$b = \frac{1}{2} \langle 111 \rangle$$

$$g \cdot b \neq 0$$



b : Burgers vector

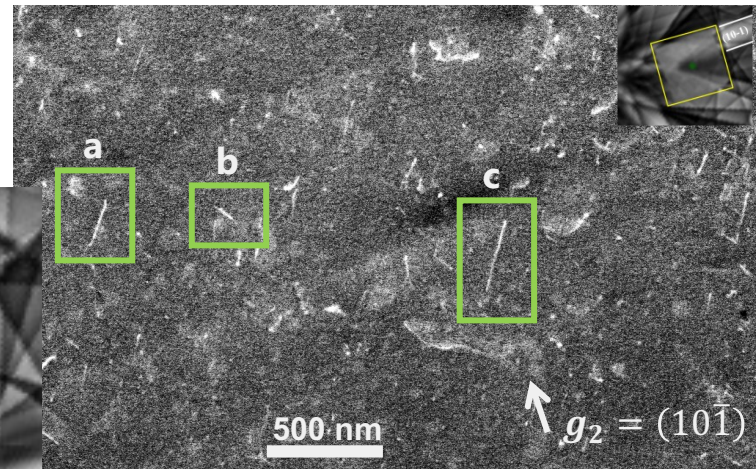
How to characterize dislocations by ECCL?



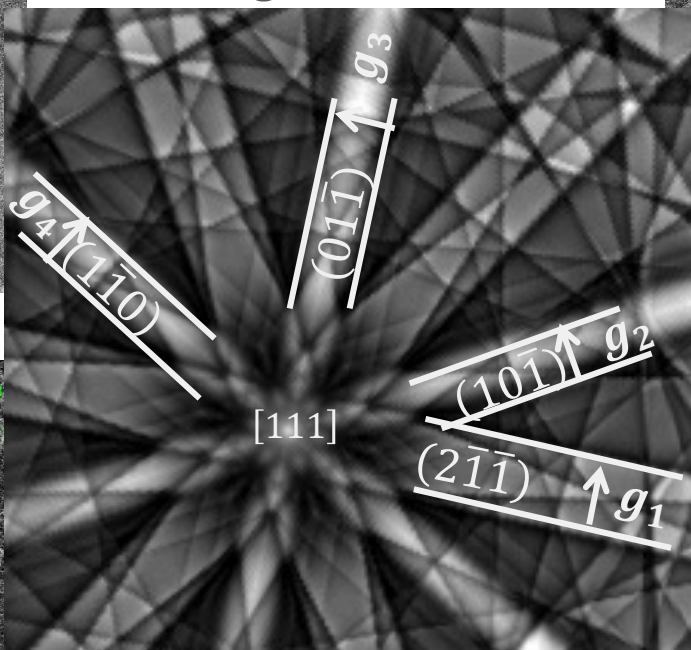
g : diffraction vector

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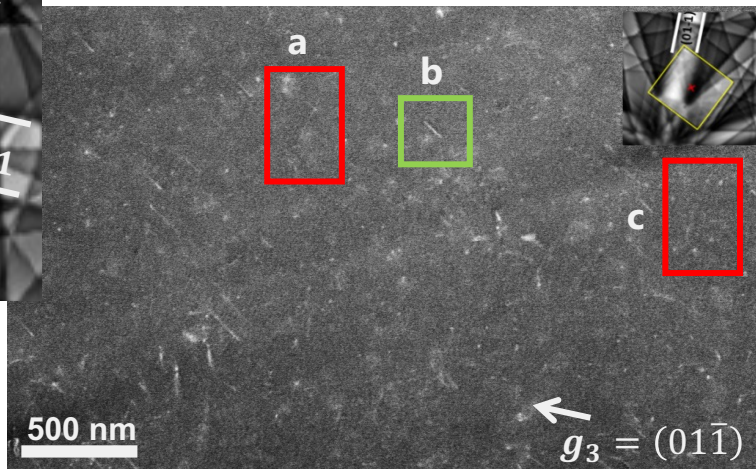
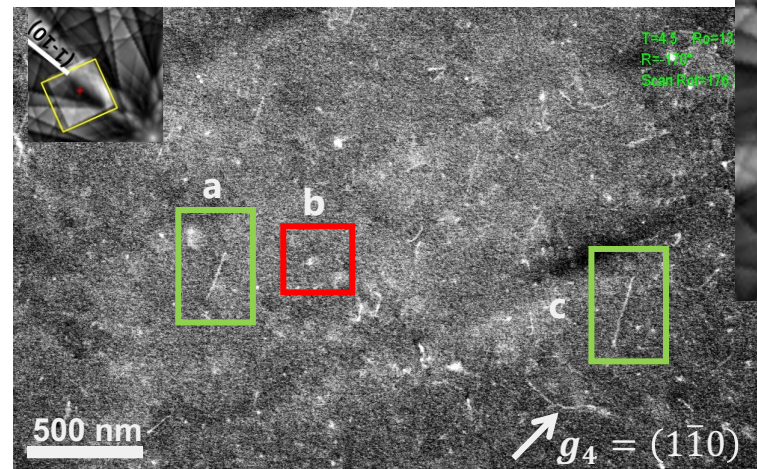
$$g \cdot b \neq 0$$



b : Burgers vector



$$g \cdot b = 0$$



How to characterize dislocations by ECCEI?

Experimental invisibility criteria: $g \cdot b = 0$; $g \cdot (b \times \xi) = 0$

	$g_1 = (2\bar{1}\bar{1})$	$g_2 = (10\bar{1})$	$g_3 = (01\bar{1})$	$g_4 = (1\bar{1}0)$	$g_5 = (11\bar{2})$	Burgers vector b	Line direction ξ	$\alpha = (b, \xi)$
Dislocation a	✓	✓	X	✓	✓	$\pm 1/2[-111]$	[011]	35°
Dislocation b	✓	✓	✓	X	✓	$\pm 1/2[11-1]$	[110]	35°
Dislocation c	✓	✓	X	✓	✓	$\pm 1/2[-111]$	[011]	35°

Note: ✓: visibility; X: invisibility, α : angle between b and ξ

How to characterize dislocations by ECCL?

Experimental invisibility criteria: $g \cdot b = 0$; $g \cdot (b \times \xi) = 0$

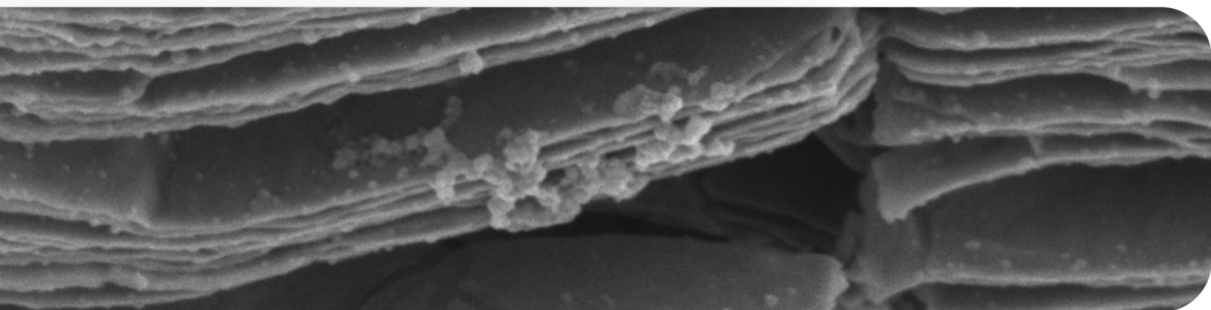
	$g_1 = (2\bar{1}\bar{1})$	$g_2 = (10\bar{1})$	$g_3 = (01\bar{1})$	$g_4 = (1\bar{1}0)$	$g_5 = (11\bar{2})$	Burgers vector b	Line direction ξ	$\alpha = (b, \xi)$
Dislocation a	✓	✓	X	✓	✓	$\pm 1/2[-111]$	[011]	35°
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Dislocation c	✓	✓	X	✓	✓	$\pm 1/2[-111]$	[011]	35°

Note: ✓: visibility; X: invisibility, α : angle between b and ξ

Comprehensive analysis of crystalline defects in SEM on bulk samples

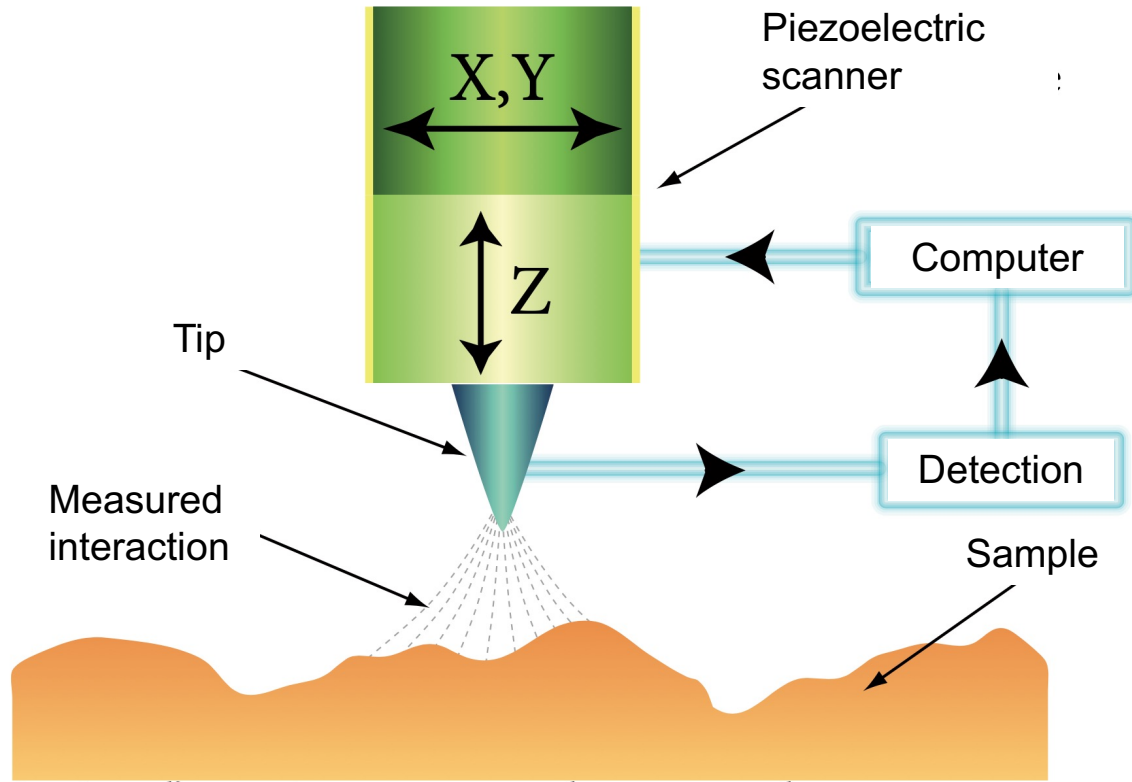
- Burgers vector b / Fault vector R
- Line direction ξ

Nevertheless, near the surface
(\approx hundred nanometers below the surface)



Near-field scanning microscopy

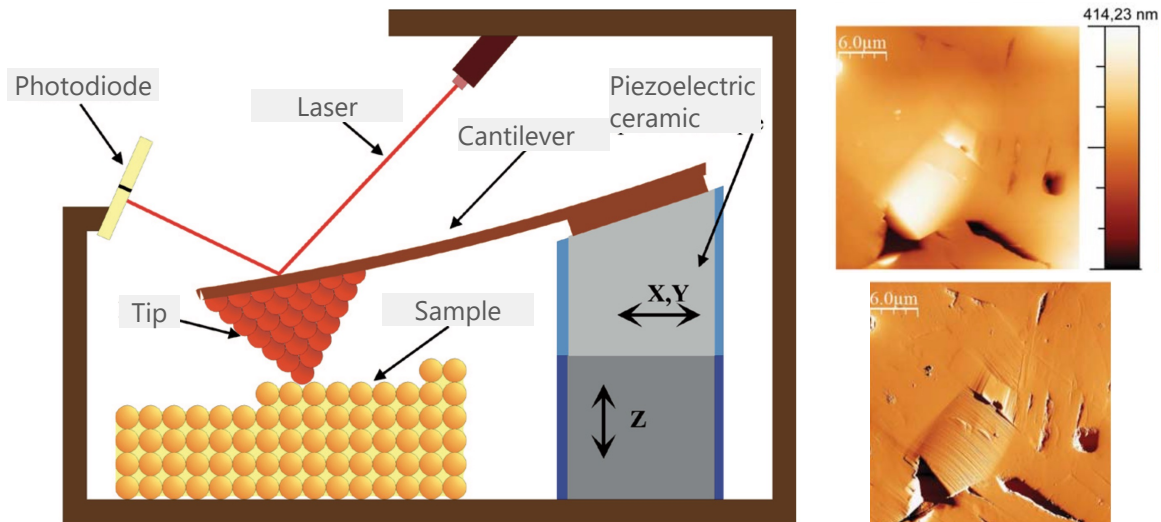
Nanometer scale



- ❖ Bypass the diffraction limit of wave microscopy
- ❖ Rely on evanescent waves captured in the near-field (within $\sim 20\text{--}100$ nm of the sample surface)
- ❖ Use a sharp tip to scan very close to the sample
- ❖ Topographic information
- ❖ Enable imaging of non-conductive or biological samples
- ❖ Require precise tip-sample distance control

Two types of near-field microscopes

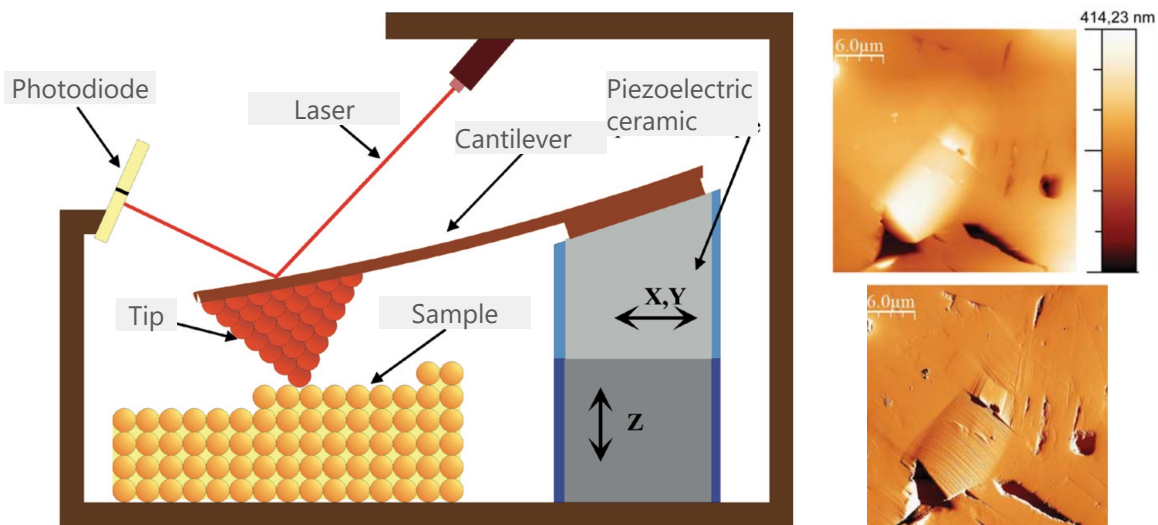
Atomic force Microscope (AFM)



- ↪ Interaction forces between the tip and the surface
 - ↪ All materials
 - ↪ nm resolution
 - ↪ Quantification in z

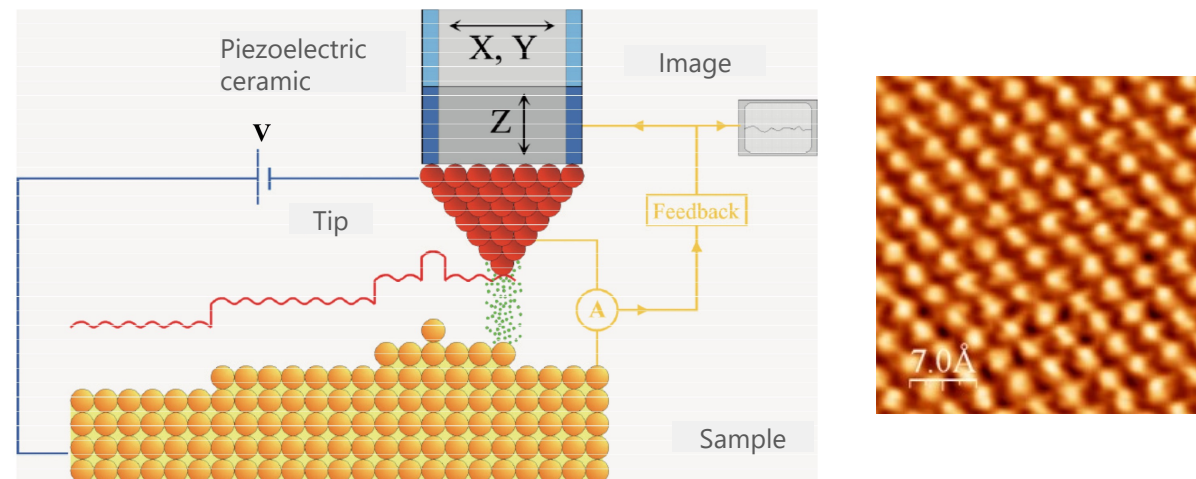
Two types of near-field microscopes

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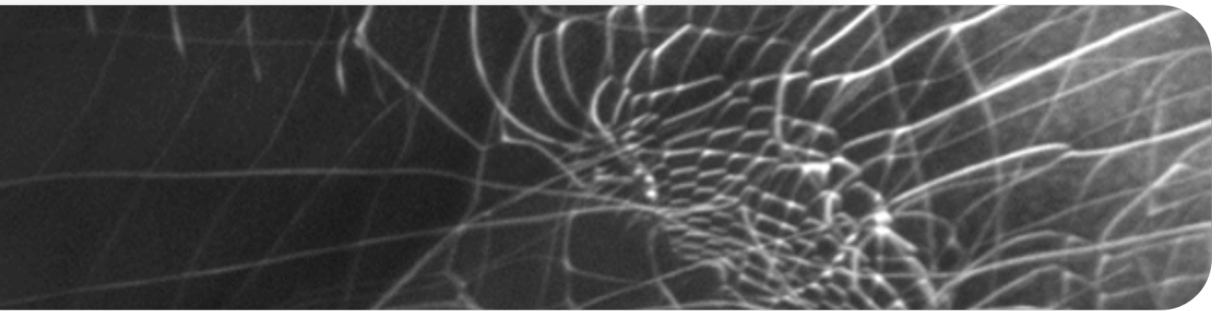


- ↪ Interaction forces between the tip and the surface
 - ↪ All materials
 - ↪ nm resolution
 - ↪ Quantification in z

Scanning Tunneling Microscope (STM)



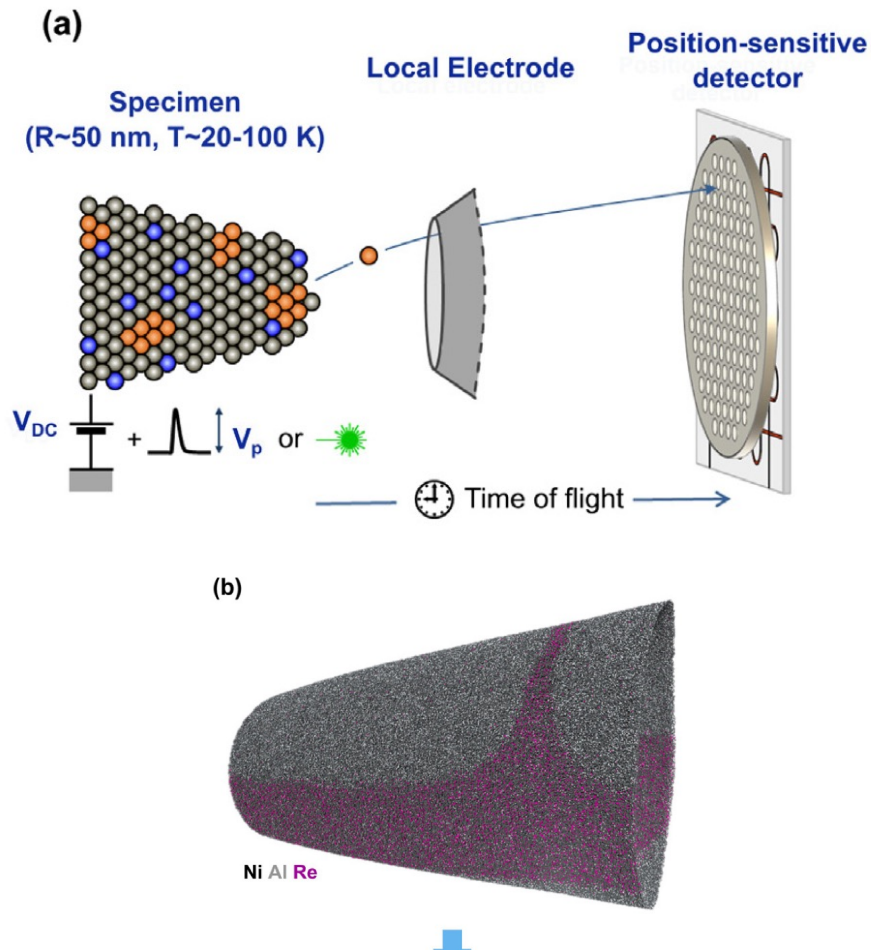
- ↪ Tunnel current
 - ↪ Only electrical conductors
 - ↪ Atomic resolution
 - ↪ Quantification in z



Atom Probe Tomography (APT)

Atomic scale

Basics of APT



❖ Definition:

- APT is a microanalytical technique that provides 3D compositional imaging and quantitative analysis at the atomic scale.

❖ Principle:

- Utilizes field evaporation induced by a high electric field to remove atoms from a sharp specimen tip.
- These atoms are then identified based on their time of flight.

❖ Spatial resolution:

- Offers near-atomic spatial resolution, enabling the visualization of individual atoms within a material.



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Thanks for your listening!

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