COE–3001: Mechanics of deformable bodies

Chapter 6: buckling

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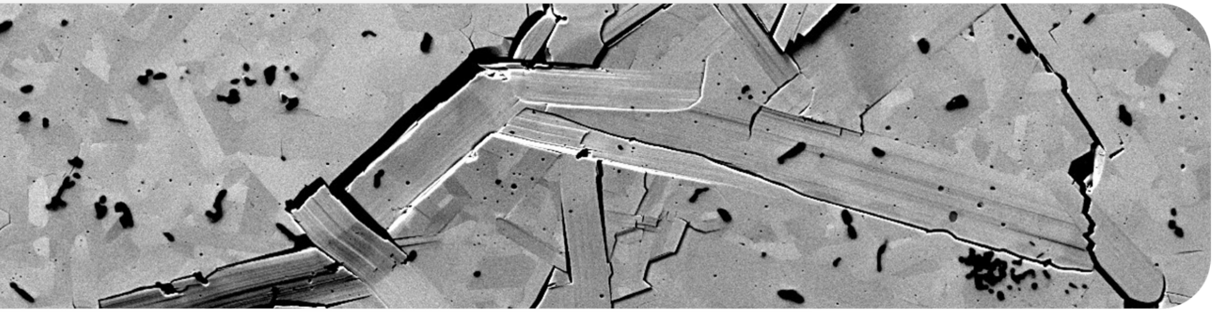
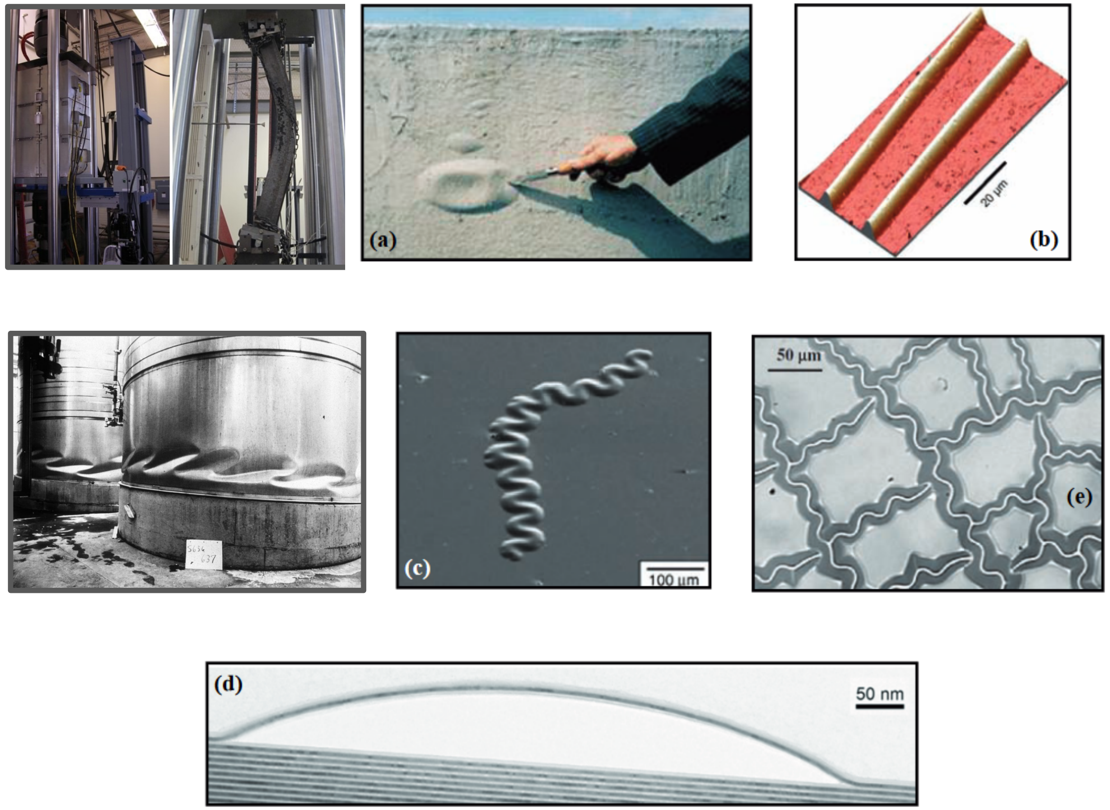


Introduction

Definition

- Instability phenomenon characterized by a sudden lateral deformation of a structure under compressive loading when a critical load is exceeded.
- Buckling may be global (columns) or local (plates, shells).
- Structural collapse may occur.
- Failure by instability can occur well before material yielding.



Geometry often matters more than material strength.

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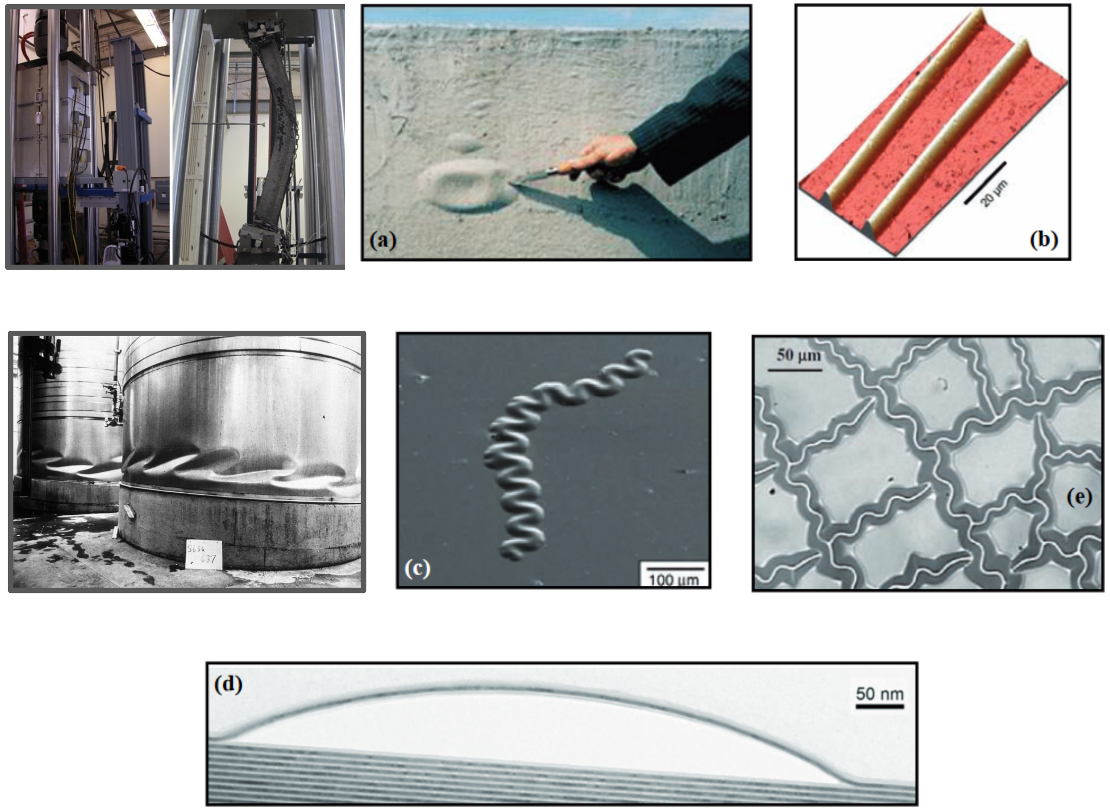
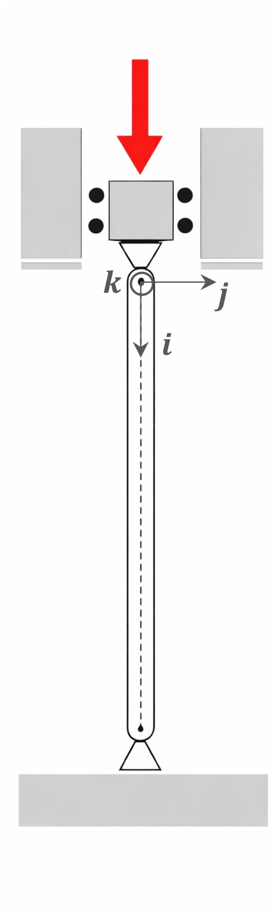
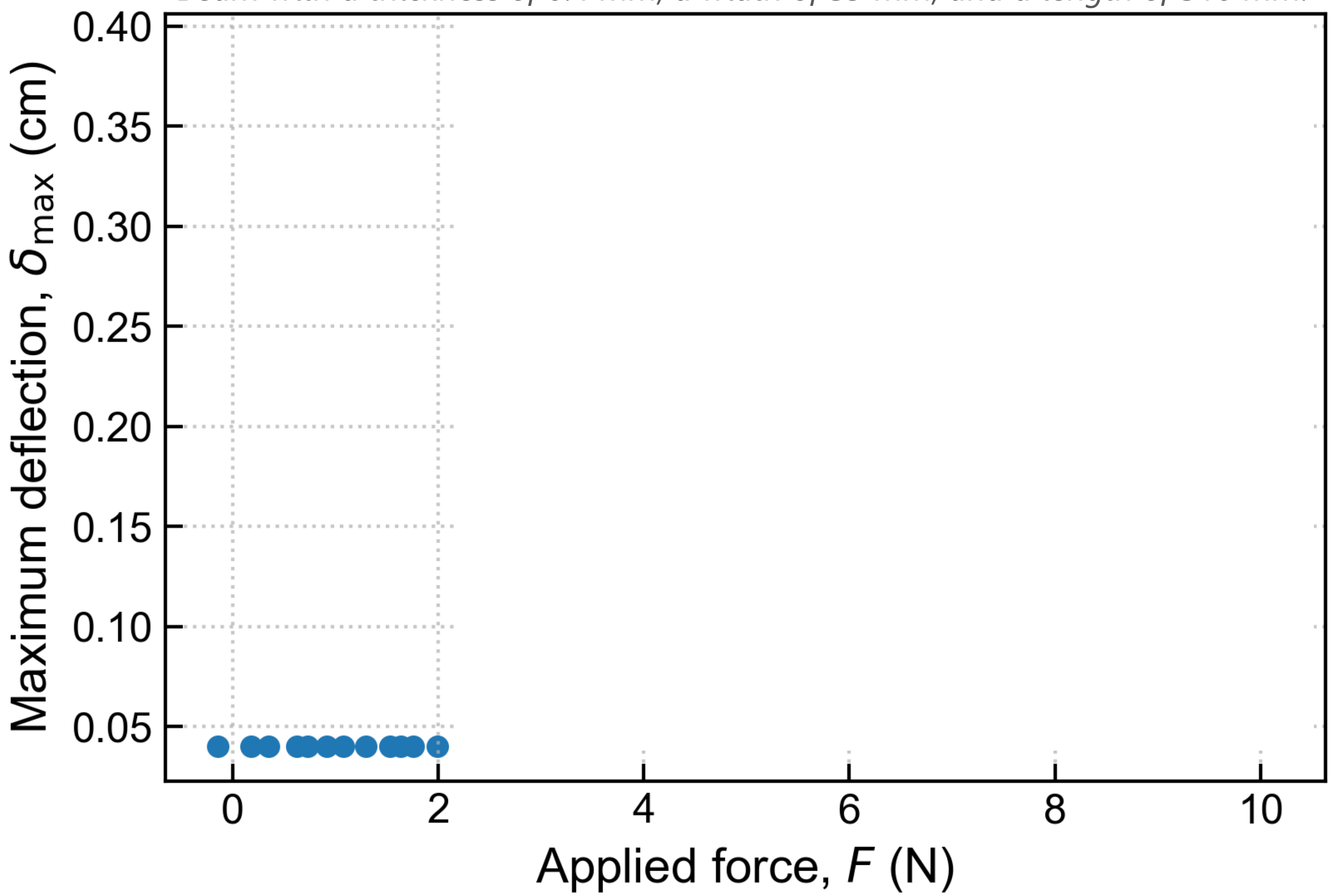


Measure of the deflection

Beam with a thickness of 0.4 mm, a width of 39 mm, and a length of 310 mm.





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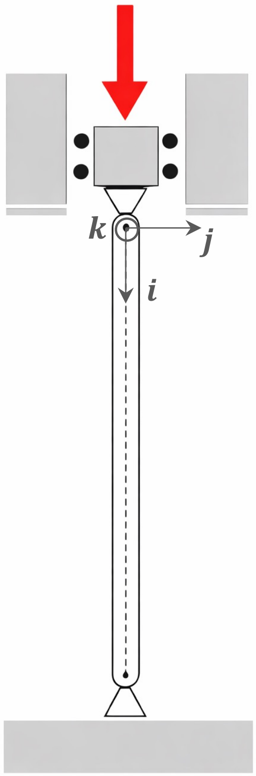
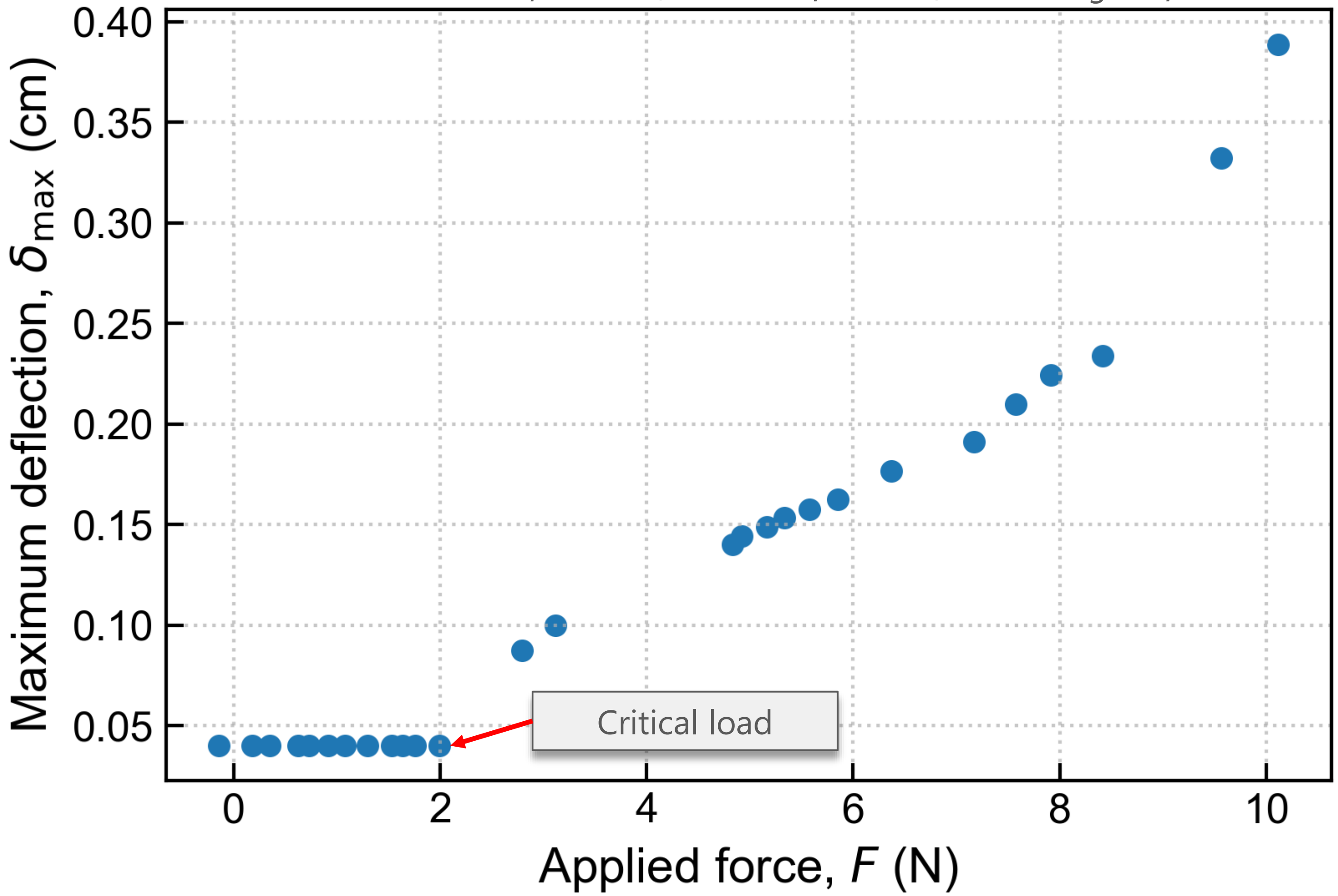
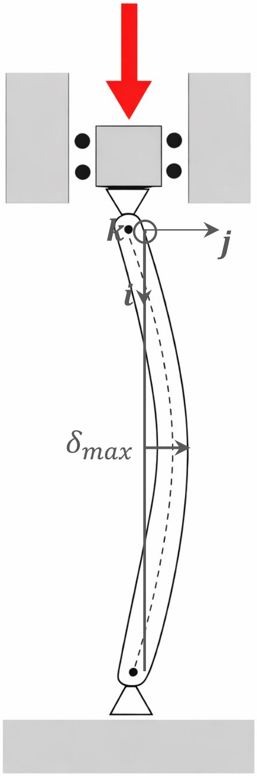
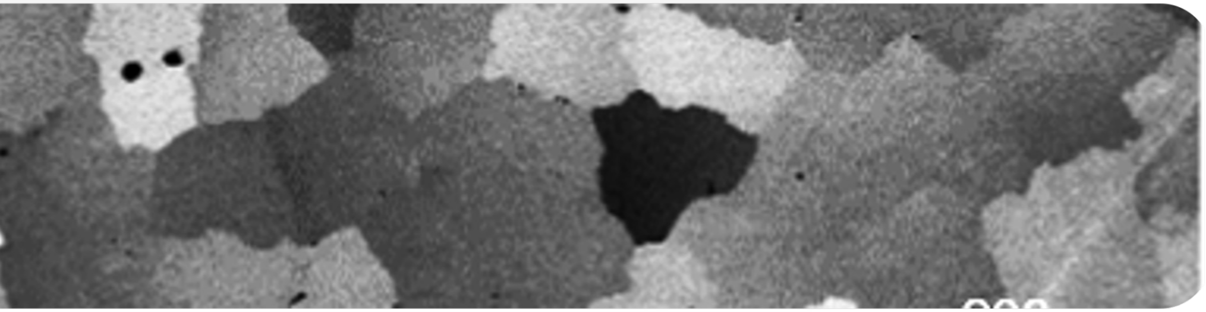


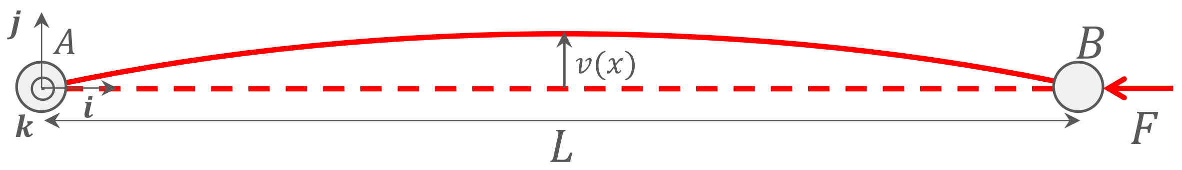






The Euler theory of buckling

The Euler theory



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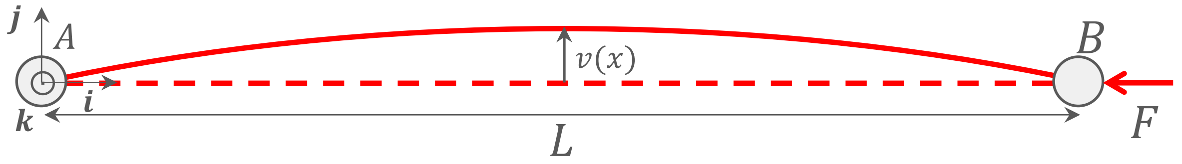

❖ **Cohesion torsor at $X(x, v(x), 0) \in [AB]$:**

$$\{\mathcal{T}_B\} = \{\mathcal{T}_A\} = \begin{Bmatrix} -F & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} F & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_{A\rightarrow X}^{coh}\} = \begin{Bmatrix} F & 0 \\ 0 & 0 \\ 0 & -v(x)F \end{Bmatrix}$$

↳ Combined compressive loading and pure bending.

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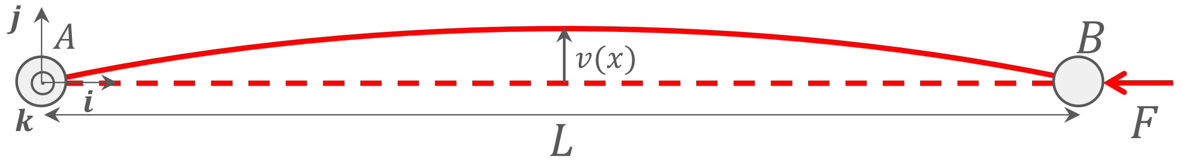

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$$\frac{\mathrm{d}^2 v(x)}{\mathrm{d}x^2} = \frac{B_{X,z}(x)}{EI_x} = \frac{-v(x)F}{EI_x}$$

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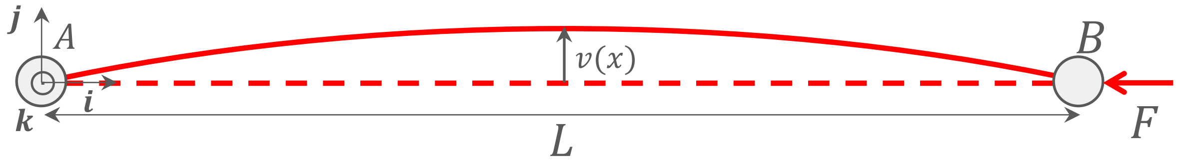

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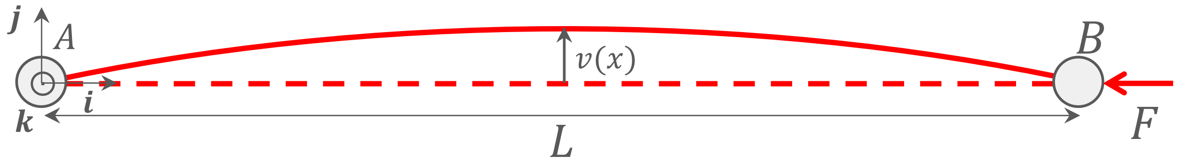
(Second-order differential equation with constant coefficients)

The Euler theory



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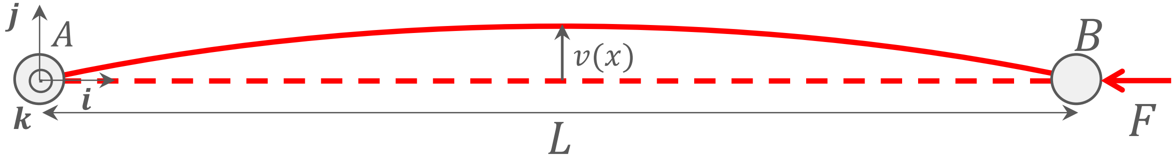

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$$v(x) = A \cos \omega x + B \sin \omega x$$

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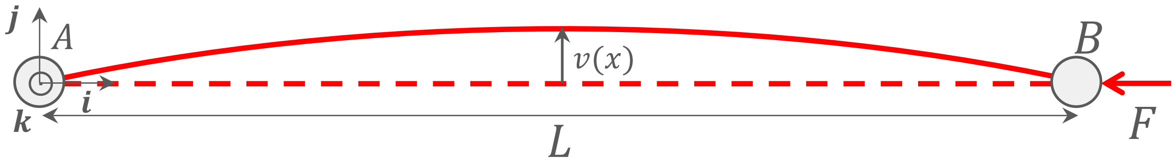

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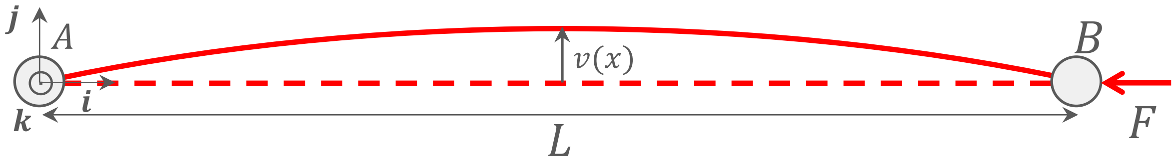
❖ **Limit conditions:**

$$\begin{cases} v(0) = 0 \\ v(L) = 0 \end{cases} \Rightarrow \begin{cases} A = 0 \\ \sin\omega L = 0 \Rightarrow \omega L = k\pi : k \in \mathbb{N}^+ \end{cases}$$

$$\Rightarrow v(x) = B\sin\left(\frac{k\pi}{L}x\right)$$

The Euler theory



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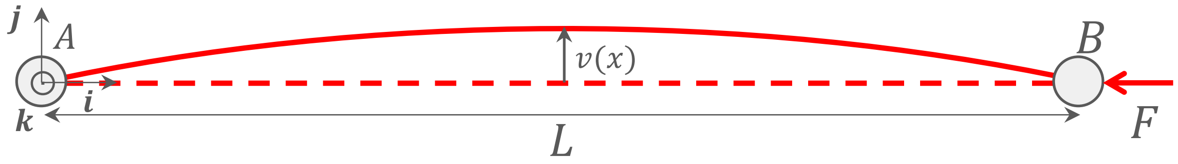

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- The solution is substituted back into the differential equation.

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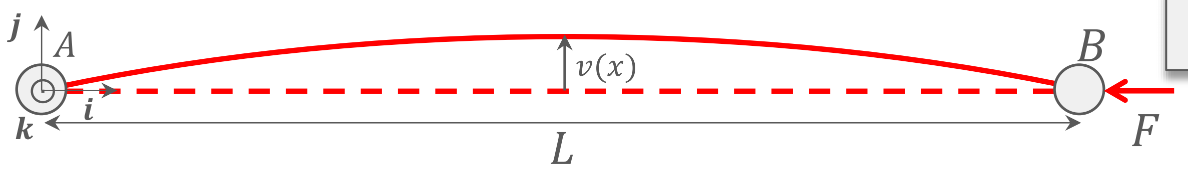

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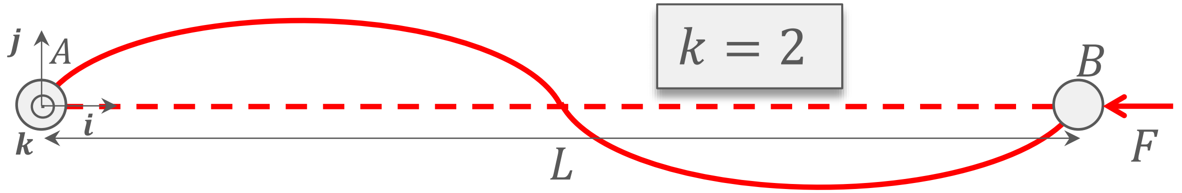

Thanks for your listening!

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