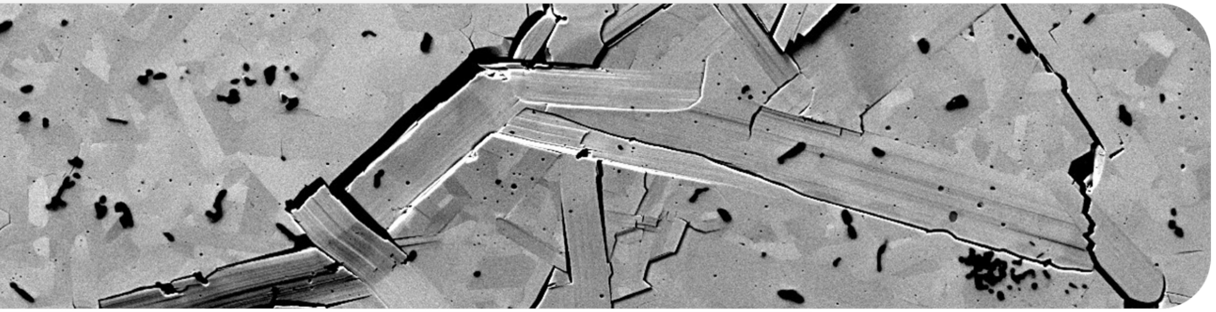
COE–3001: Mechanics of deformable bodies

Chapter 3: shear

Prof. Antoine GUITTON

Université de Lorraine, CNRS, Arts et Métiers Institute of Technology, LEM3, F-57000 Metz,
antoine.guitton@univ-Lorraine.fr

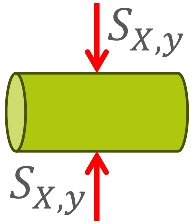


Deformation measurement

Deformation measure: shear angle/strain

❖ **Reminder:**

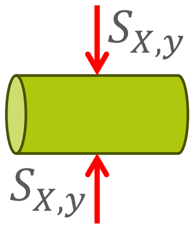
$$\{\mathcal{T}_X^{coh,shear}\} = \begin{Bmatrix} 0 & 0 \\ S_{X,y} & 0 \\ 0 & 0 \end{Bmatrix}$$



Deformation measure: shear angle/strain

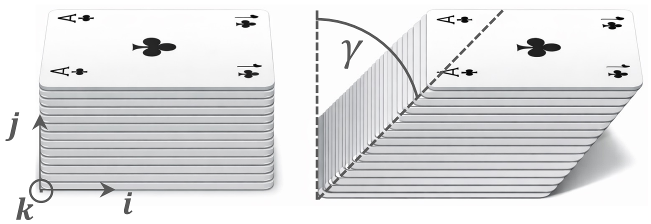
❖ Reminder:

$$\left\{\mathcal{T}_X^{coh,shear}\right\} = \begin{Bmatrix} 0 & 0 \\ S_{X,y} & 0 \\ 0 & 0 \end{Bmatrix}$$



❖ Shear angle:

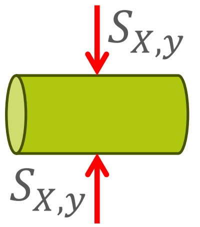
- It is the angular deformation produced by a shear load.



Deformation measure: shear angle/strain

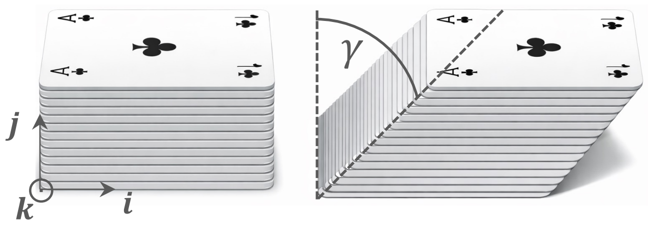
❖ Reminder:

$$\{\mathcal{T}_X^{coh,shear}\} = \begin{Bmatrix} 0 & 0 \\ S_{X,y} & 0 \\ 0 & 0 \end{Bmatrix}$$



❖ Shear angle:

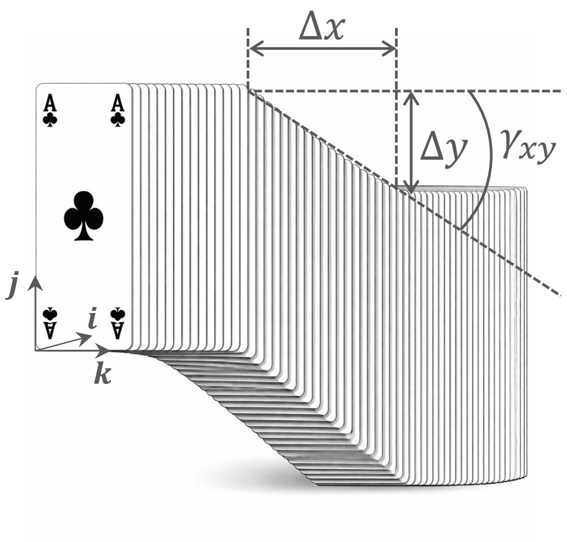
- It is the angular deformation produced by a shear load.

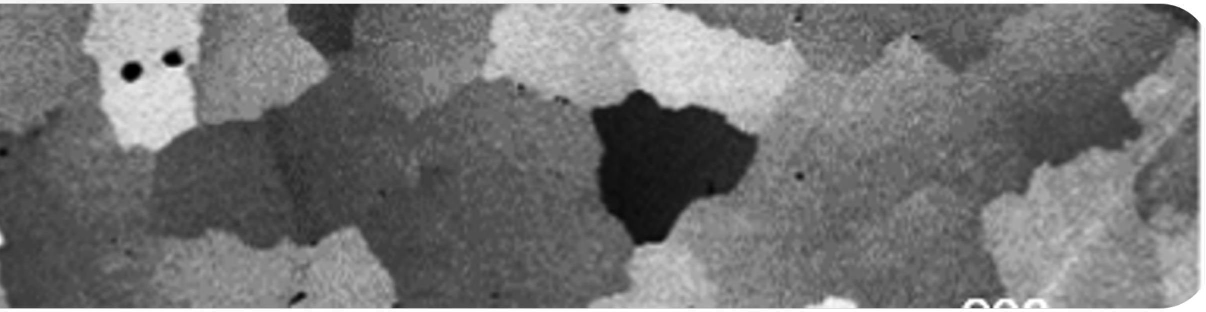


❖ Shear strain:

$$\tan\gamma_{xy} = \frac{\Delta y}{\Delta x} \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\tan\gamma_{xy} \approx \gamma_{xy} = \frac{\mathrm{d}y}{\mathrm{d}x}$$





Stress

Relationship between ϕ and the internal forces

- **Cohesion torsor in M:**

$$\{\mathcal{T}_M^{coh}\} = \begin{Bmatrix} \boldsymbol{R_M} = \mathrm{d}\boldsymbol{F} \\ \boldsymbol{M_M} = \boldsymbol{0} \end{Bmatrix}$$

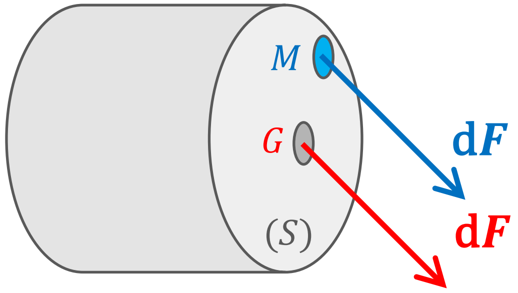
- **Let's transport this torsor into G near M:**

$$\{\mathrm{d}\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R_G} = \boldsymbol{R_M} = \mathrm{d}\boldsymbol{F} \\ \mathrm{d}\boldsymbol{M_G} = \boldsymbol{0} + \boldsymbol{GM}\times\boldsymbol{R_M} \end{Bmatrix} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R_G} = \mathrm{d}\boldsymbol{F} \\ \mathrm{d}\boldsymbol{M_M} = \boldsymbol{GM}\times\mathrm{d}\boldsymbol{F} \end{Bmatrix}$$

BUT $\boldsymbol{\phi}(M,\boldsymbol{n}) = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}S} \Rightarrow \mathrm{d}\boldsymbol{F} = \boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S$

$$\Rightarrow \{\mathrm{d}\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R_G} = \boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S \\ \mathrm{d}\boldsymbol{M_G} = \boldsymbol{GM}\times\boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S \end{Bmatrix}$$

BUT $\boldsymbol{\phi}(M,\boldsymbol{n}=\boldsymbol{i}) = \sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k}$ AND $\boldsymbol{GM} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

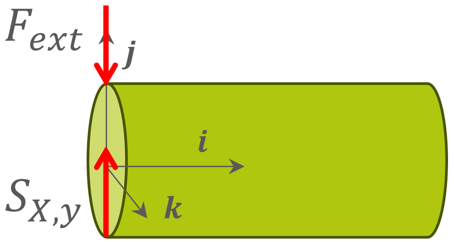


- **After integration over (S):**

$$\{\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \boldsymbol{R_G} = \iint (\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \\ \boldsymbol{M_G} = \iint \boldsymbol{GM}\times(\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \end{Bmatrix} = \begin{Bmatrix} N_G = \iint \sigma_{xx}\mathrm{d}S & T_G = \iint (y\sigma_{xz} - z\sigma_{xy})\mathrm{d}S \\ S_{G,y} = \iint \sigma_{xy}\mathrm{d}S & B_{G,y} = \iint (z\sigma_{xz} - x\sigma_{xz})\mathrm{d}S \\ S_{G,z} = \iint \sigma_{xz}\mathrm{d}S & B_{G,z} = \iint (x\sigma_{xy} - y\sigma_{xx})\mathrm{d}S \end{Bmatrix}$$

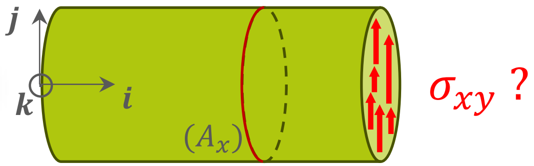
Very complex. Additional assumptions are needed about how stresses are distributed over (S).

How to calculate the stress?

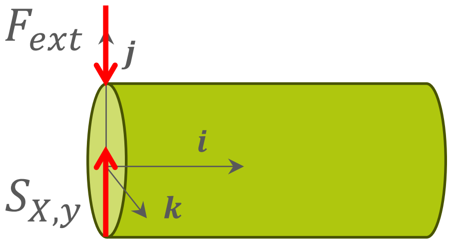


Equilibrium $\Rightarrow F_{ext}\boldsymbol{j} = -S_{X,y}\boldsymbol{j}$

$$S_{X,y} = \iint \sigma_{xy} \, \mathrm{d}S$$

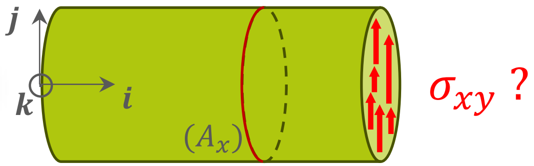


How to calculate the stress?



Equilibrium $\Rightarrow F_{ext}\boldsymbol{j} = -S_{X,y}\boldsymbol{j}$

$$S_{X,y} = \iint \sigma_{xy} \mathrm{d}S$$

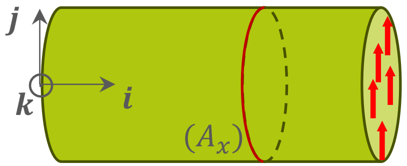


❖ **We assume that σ_{xy} is constant.**

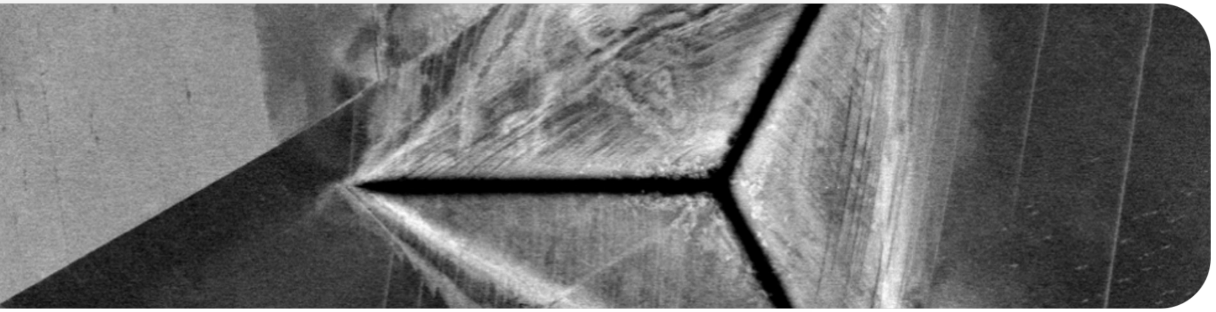
- Uniform cross-section: the area (A) does not vary along x.
- Constant axial force: the shear force F_{ext} is the same at every cross-section (no distributed axial loads).
- Pure shearing (no bending, no torsion).
- Homogeneous material: material properties are identical everywhere.
- Small deformations: geometry changes are negligible.

$$S_{X,y} = \iint \sigma_{xy} \mathrm{d}S \Leftrightarrow F_{ext} = \iint \sigma_{xy} \mathrm{d}S = \sigma_{xy} \iint \mathrm{d}S = \sigma_{xy} A_x$$

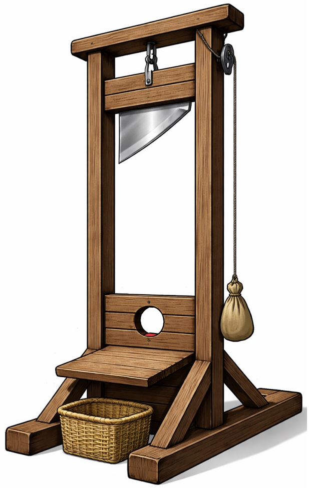
$$\Leftrightarrow \sigma_{xy} = \frac{F_{ext}}{A_x}$$



$$\sigma_{xy} = \frac{F_{ext}}{A_x} = \tau$$



Shear testing



(the French shear test)

Principle of shear test

❖ **Objective**

- Characterize the material response under shear loading
- Determine the maximum shear stress and shear strength
- Identify failure modes (ductile vs brittle)

Principle of shear test

❖ Objective

- Characterize the material response under shear loading
- Determine the maximum shear stress and shear strength
- Identify failure modes (ductile vs brittle)

❖ Principle

- Application of opposite, parallel forces on a cross-section
- Relative sliding of two parts of the specimen
- Ideally produces pure shear deformation

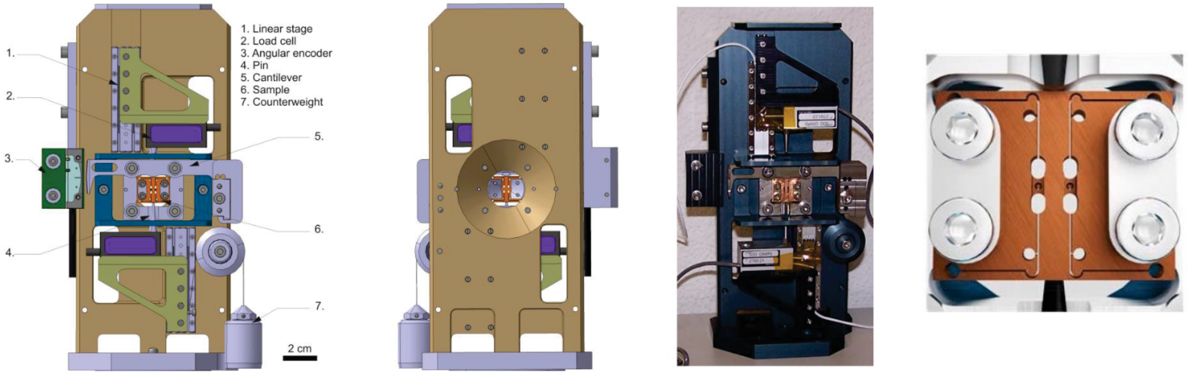
Principle of shear test

❖ Objective

- Characterize the material response under shear loading
- Determine the maximum shear stress and shear strength
- Identify failure modes (ductile vs brittle)

❖ Principle

- Application of opposite, parallel forces on a cross-section
- Relative sliding of two parts of the specimen
- Ideally produces pure shear deformation



A. Irastrza Landa, PhD Thesis, *EPFL* (2016)

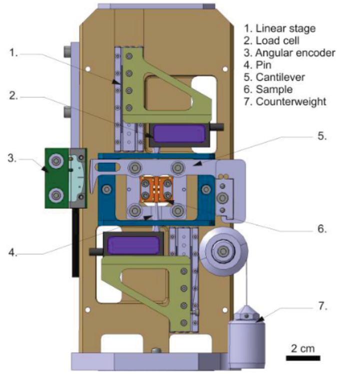
Principle of shear test

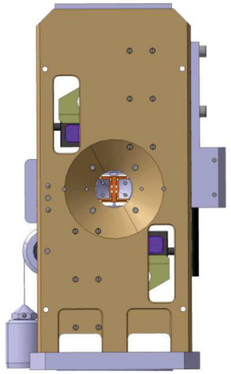
❖Objective

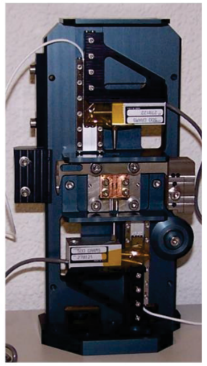
- Characterize the material response under shear loading
- Determine the maximum shear stress and shear strength
- Identify failure modes (ductile vs brittle)

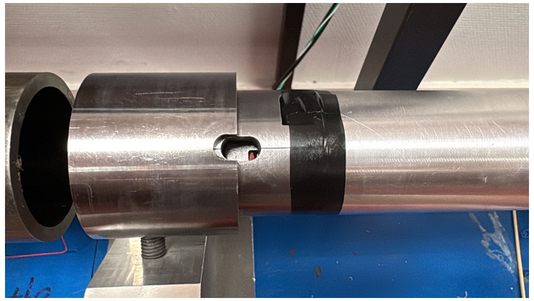
❖Principle

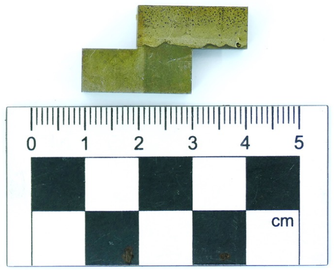
- Application of opposite, parallel forces on a cross-section
- Relative sliding of two parts of the specimen
- Ideally produces pure shear deformation





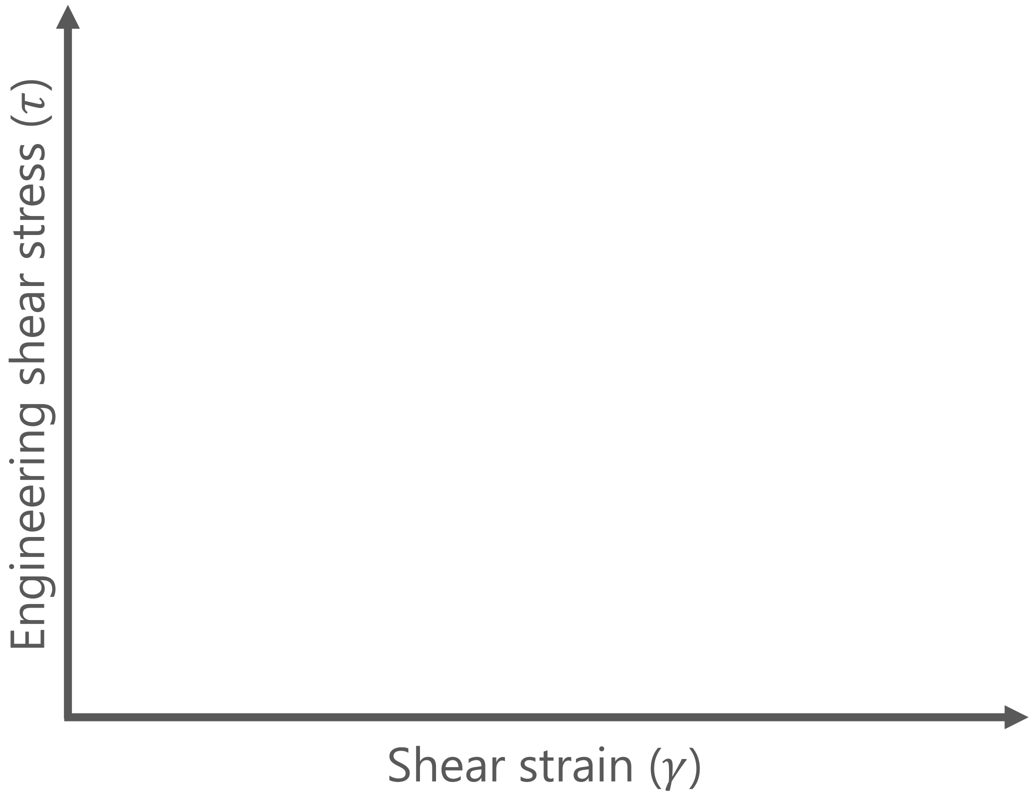






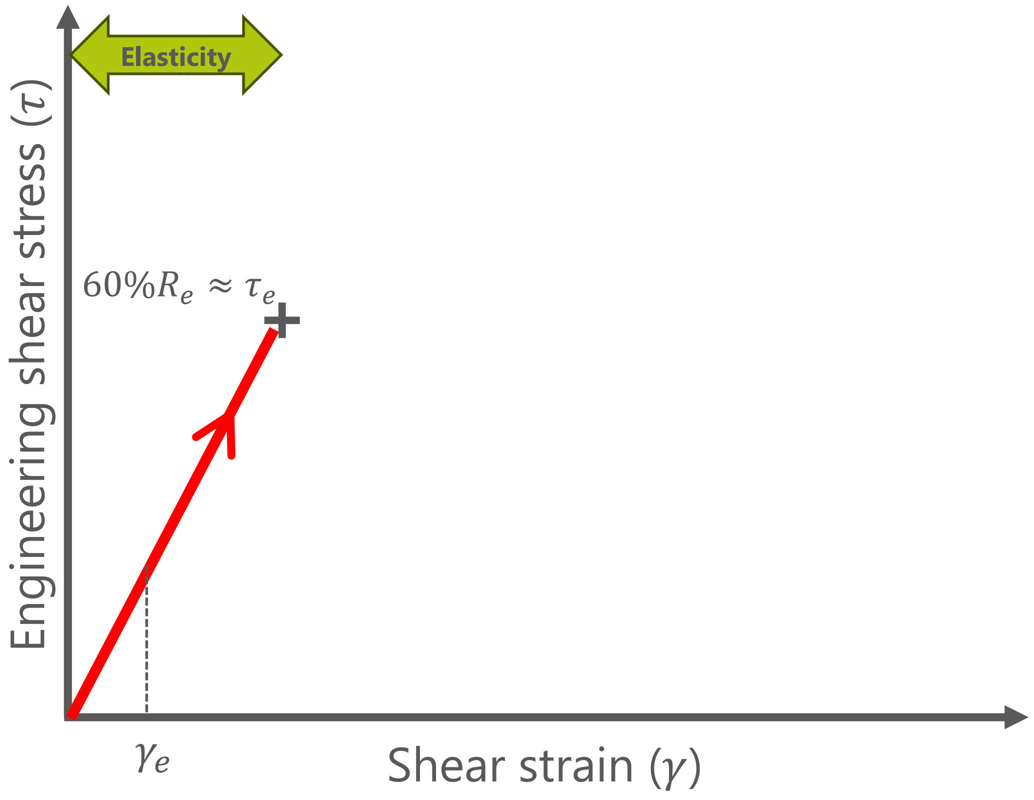
A. Irastrza Landa, PhD Thesis, *EPFL* (2016)
A. Monnet, PhD Thesis, *Université de Lorraine* (2026)

Use of results



- **Engineering shear stress-shear strain curve (ductile material):**
  - The vertical axis represents the engineering shear stress (τ), defined using the initial shear area.
    $$\tau = \frac{F_{applied}}{A_{x0}}$$
  - The horizontal axis represents the shear strain (γ), defined using the initial geometry.
    $$\gamma = \frac{\Delta u}{h}$$
    (u: tangential displacement; h: distance between the faces)

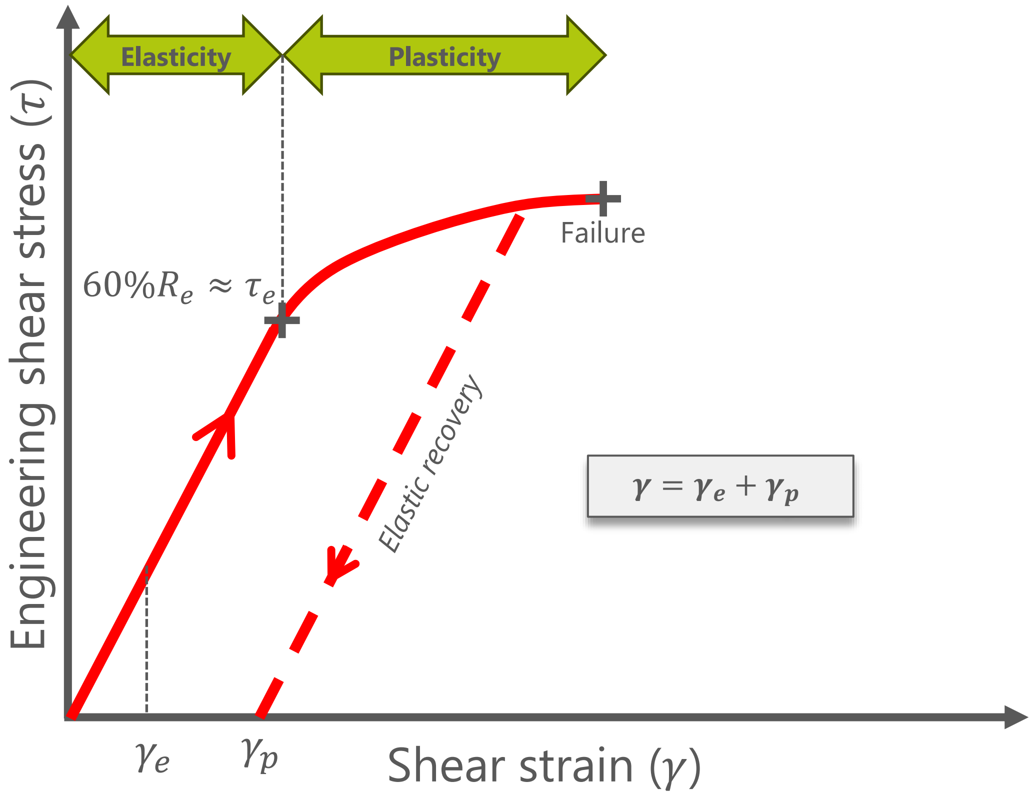
Use of results



- **Engineering shear stress-shear strain curve (ductile material):**
  - The vertical axis represents the engineering shear stress (τ), defined using the initial shear area.
  $$\tau = \frac{F_{applied}}{A_{x0}}$$
  - The horizontal axis represents the shear strain (γ), defined using the initial geometry.
  $$\gamma = \frac{\Delta u}{h}$$
  (u: tangential displacement; h: distance between the faces)

- **Elastic region ($\tau < \tau_e \approx 60\%R_e$):**
  - The initial linear part corresponds to the elastic regime.
  - Shear stress and strain are proportional (Hooke's law):
  $$\tau = G\gamma_e$$
  - Deformation is fully reversible upon unloading.
  - The slope of this region is the shear modulus ($G = \frac{E}{2(1+\nu)}$).
  - The elastic limit is reached at the yield stress, τ_e.

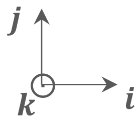
Use of results



- **Engineering shear stress-shear strain curve (ductile material):**
  - The vertical axis represents the engineering shear stress (τ), defined using the initial shear area.
  $$\tau = \frac{F_{applied}}{A_{x0}}$$
  - The horizontal axis represents the shear strain (γ), defined using the initial geometry.
  $$\gamma = \frac{\Delta u}{h}$$
  (u: tangential displacement; h: distance between the faces)
- **Elastic region ($\tau < \tau_e \approx 60\% R_e$):**
  - The initial linear part corresponds to the elastic regime.
  - Shear stress and strain are proportional (Hooke's law):
  $$\tau = G\gamma_e$$
  - Deformation is fully reversible upon unloading.
  - The slope of this region is the shear modulus ($G = \frac{E}{2(1+\nu)}$).
  - The elastic limit is reached at the yield stress, τ_e.
- **Plastic region ($\tau > \tau_e \approx 60\% R_e$):**
  - Deformation becomes irreversible.
  - Unloading from this region follows a linear elastic path.
  - After unloading, a permanent plastic strain (γ_p) remains.
  - The recovered part is the elastic shear angle.
  - Plastic deformation in shear tends to localize early into shear bands.

Summary

Loadings	Deformation	Stress	Governing equations	Coefficients	Elasticity limit
Tension	$\varepsilon_{xx} > 0$	$\sigma_{xx} = \dfrac{N}{A_x} > 0$	$\sigma_{xx} = E\varepsilon_{xx}$ $\varepsilon_{zz} = \varepsilon_{yy} = -\nu\varepsilon_{xx}$	E, ν	R_e
Compression	$\varepsilon_{xx} < 0$	$\sigma_{xx} = \dfrac{N}{A_x} < 0$	$\sigma = E\varepsilon_{xx}$ $\varepsilon_{zz} = \varepsilon_{yy} = -\nu\varepsilon_{xx}$	E, ν	R_e
Shear	γ_{xy}	$\tau_{xy} = \dfrac{S_y}{A_x}$	$\tau_{xy} = G\gamma_{xy}$ $G = \dfrac{E}{2(1+\nu)}$	G	τ_e



Thanks for your listening!

If you need further information:

Prof. Antoine GUITTON

Full Professor at Université de Lorraine

Phone (LEM3): +33 372 747 826

Email: antoine.guitton@univ-lorraine.fr

Website: www.antoine-guitton.fr