COE–3001: Mechanics of deformable bodies

Chapter 1: strength of materials

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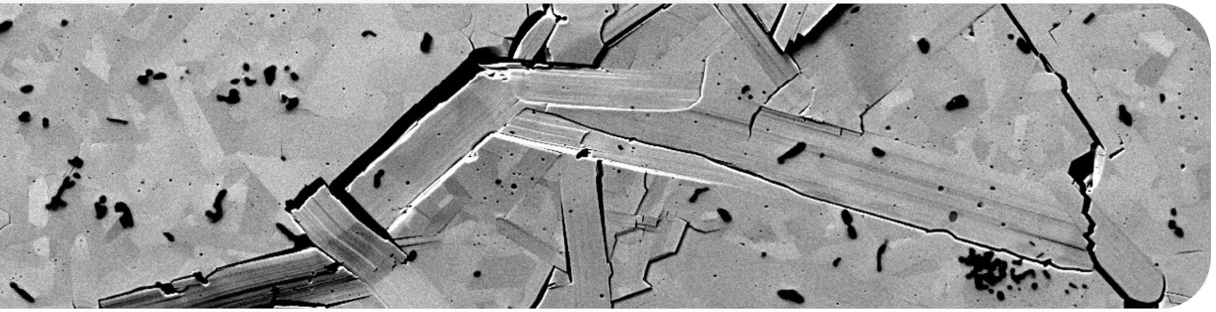


Some generalities

Deformable / non-deformable

Solid

Non-deformable (Brittle)

Deformable / non-deformable

Solid

Non-deformable (**Brittle**)	Deformable (**Ductile**)

Deformable / non-deformable

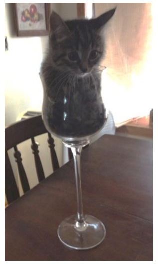
Solid		Liquid
Non-deformable (**Brittle**)	Deformable (**Ductile**)	Too deformable




Deformable / non-deformable

Solid | Liquid

Non-deformable (**Brittle**) | Deformable (**Ductile**) | Too deformable



Ig-Nobel 2017, Marc-Antoine FARDIN (ENS Lyon)

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  - Ability of a structure or material to sustain applied loads without reaching a failure state, such as yielding, fracture, or loss of integrity.

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- **Beam:**
  - A solid body for which one dimension is much larger than the other two.
  - It can be defined as a solid generated by a planar surface (S), called the cross section, which is constant or only slightly varying.
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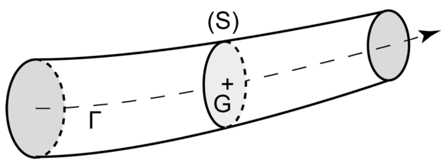
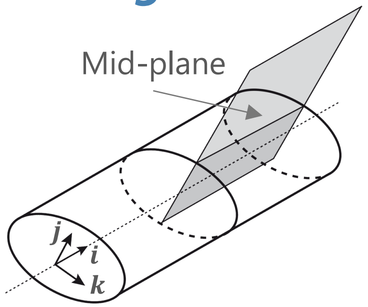
Arbitrary beam



Straight beam



Assumptions and principles of Strength of Materials

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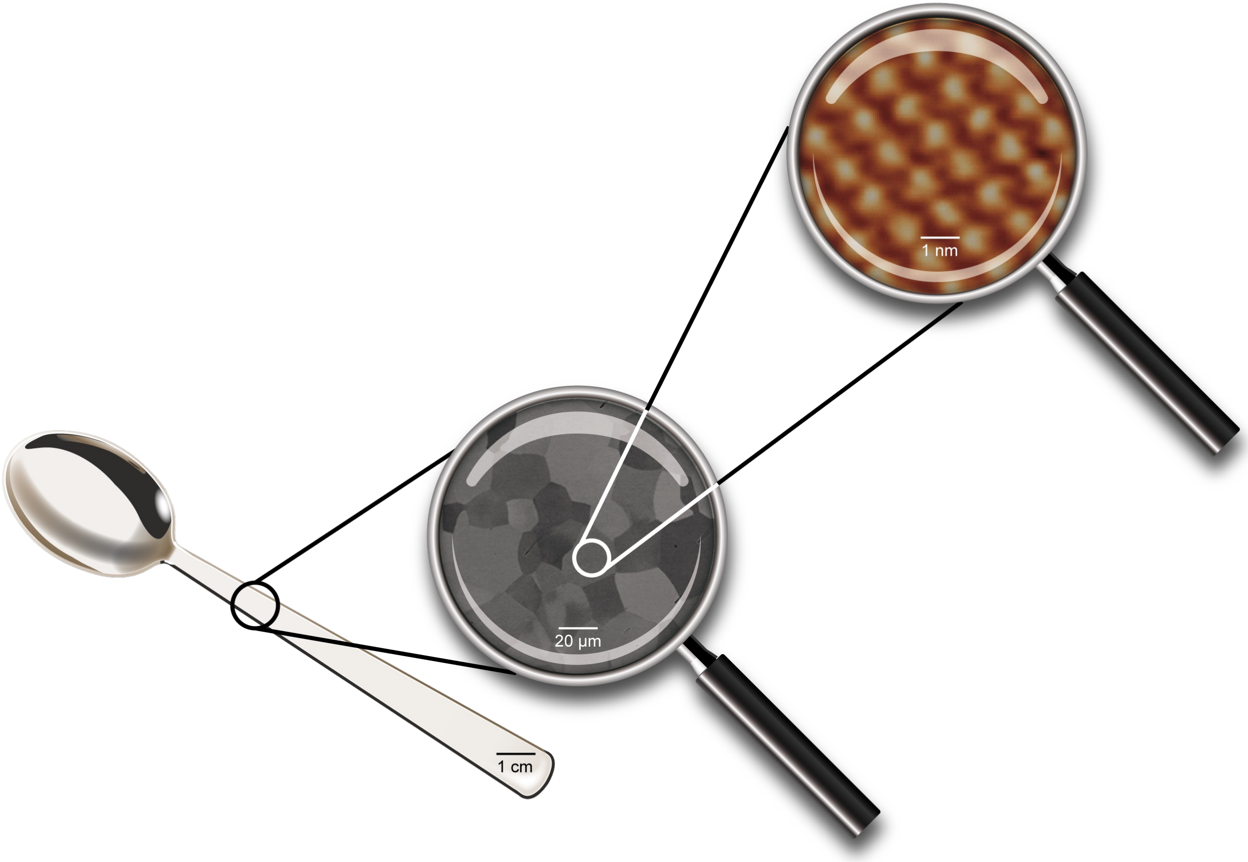
Millau Viaduct

In reality, no material perfectly satisfies these assumptions; it all depends on the scale at which the material is observed.

Micro-/nano-structures



L. Bragg & J.F. Nye, *Proc. R. Soc. Lond.* (1947)

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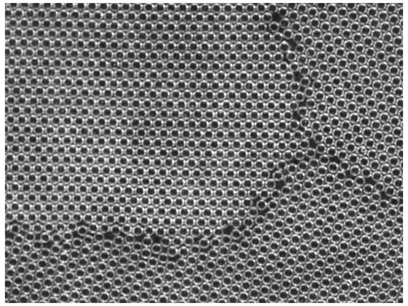
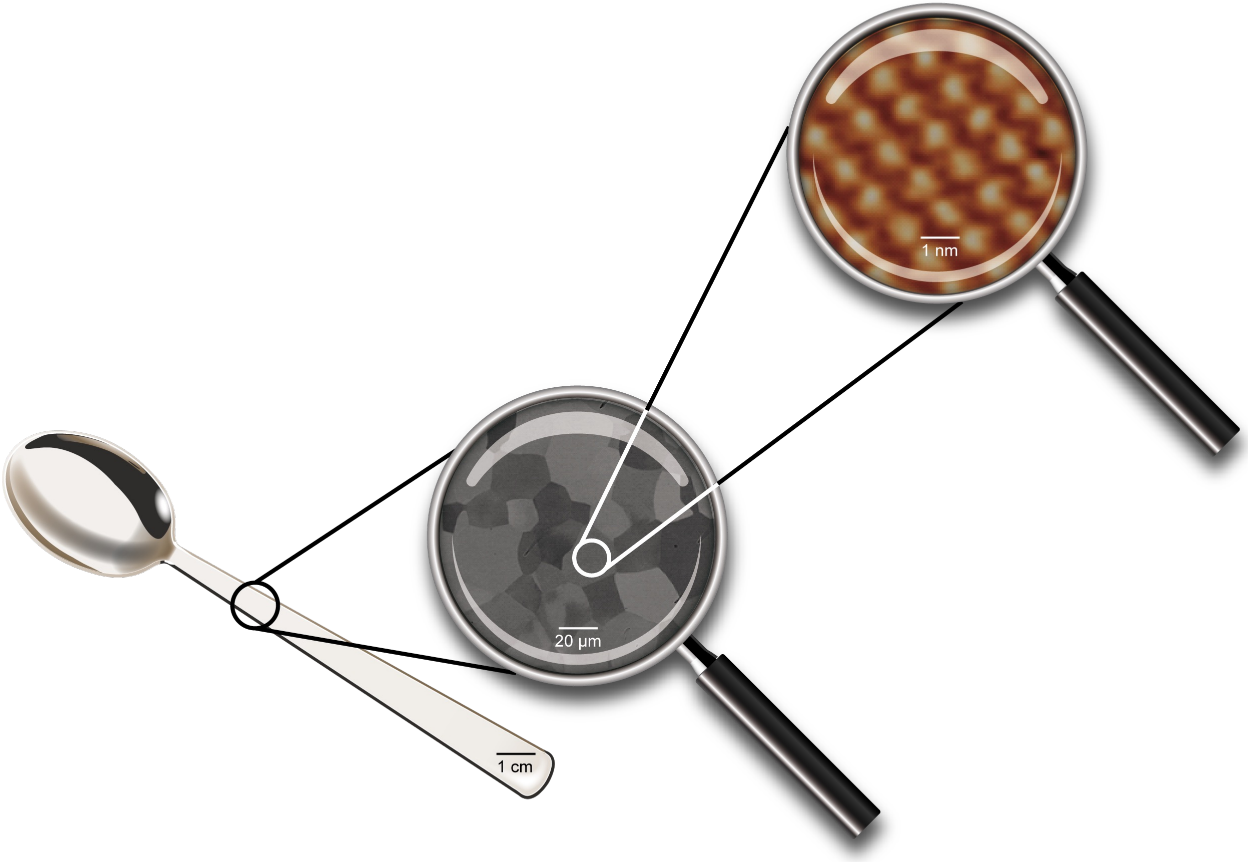


Grains:

- Size
- Shape
- Chemistry
- Distribution



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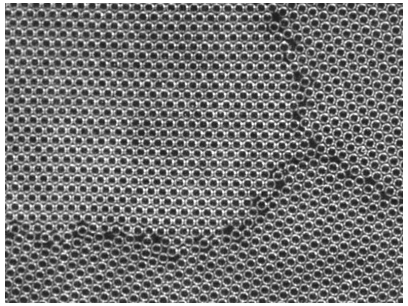

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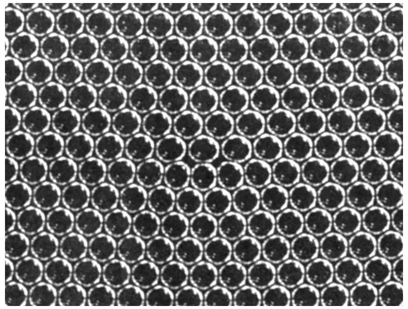
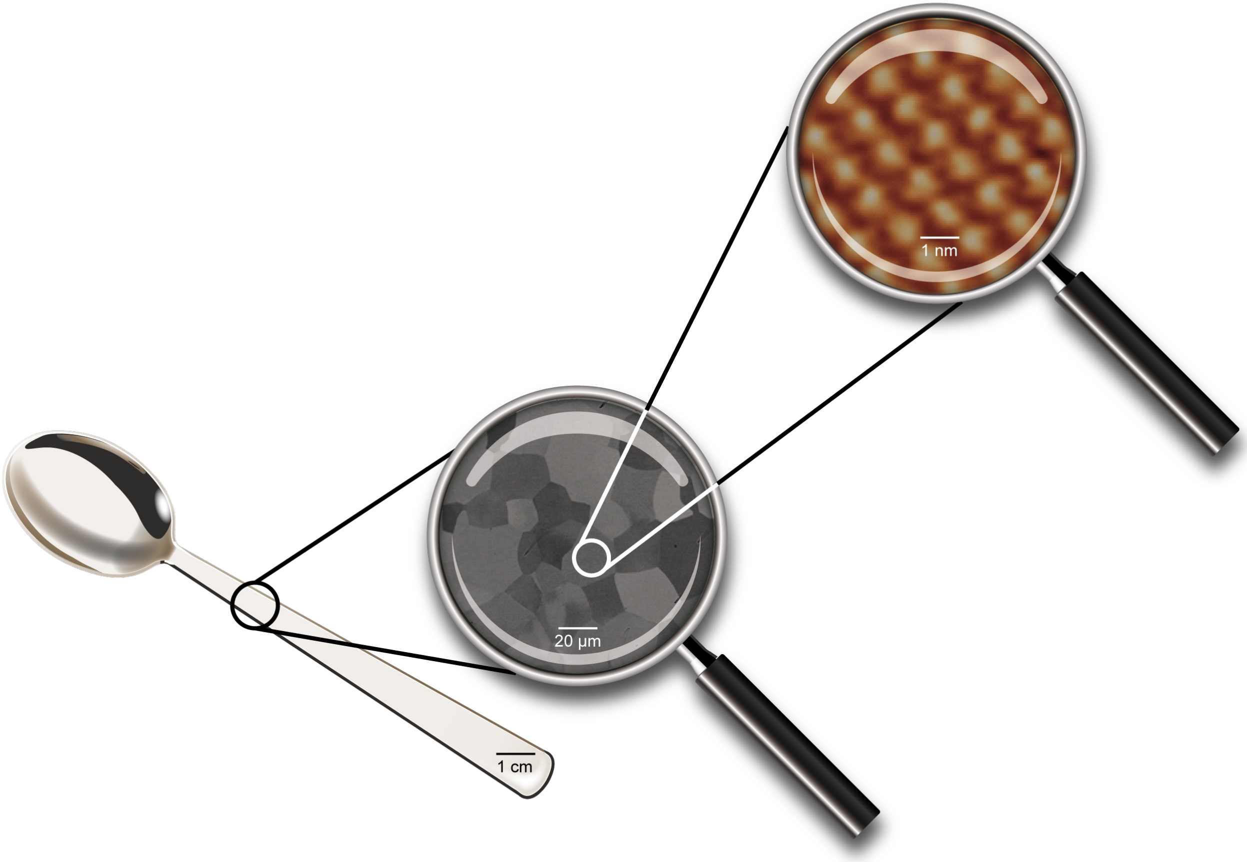


Dislocations:

- Types
- Density
- Distribution



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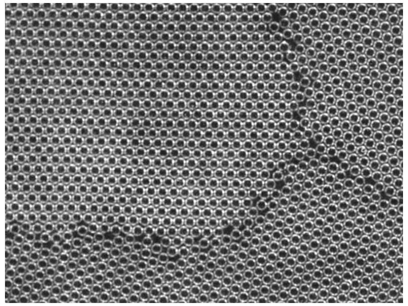

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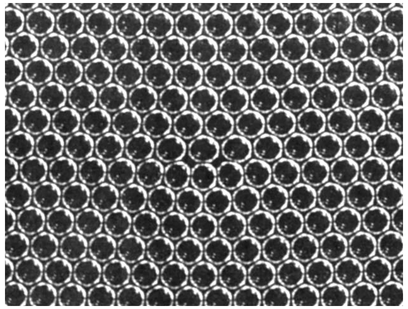

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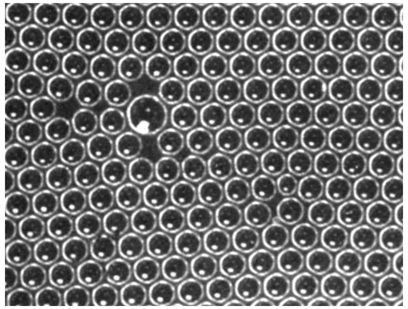
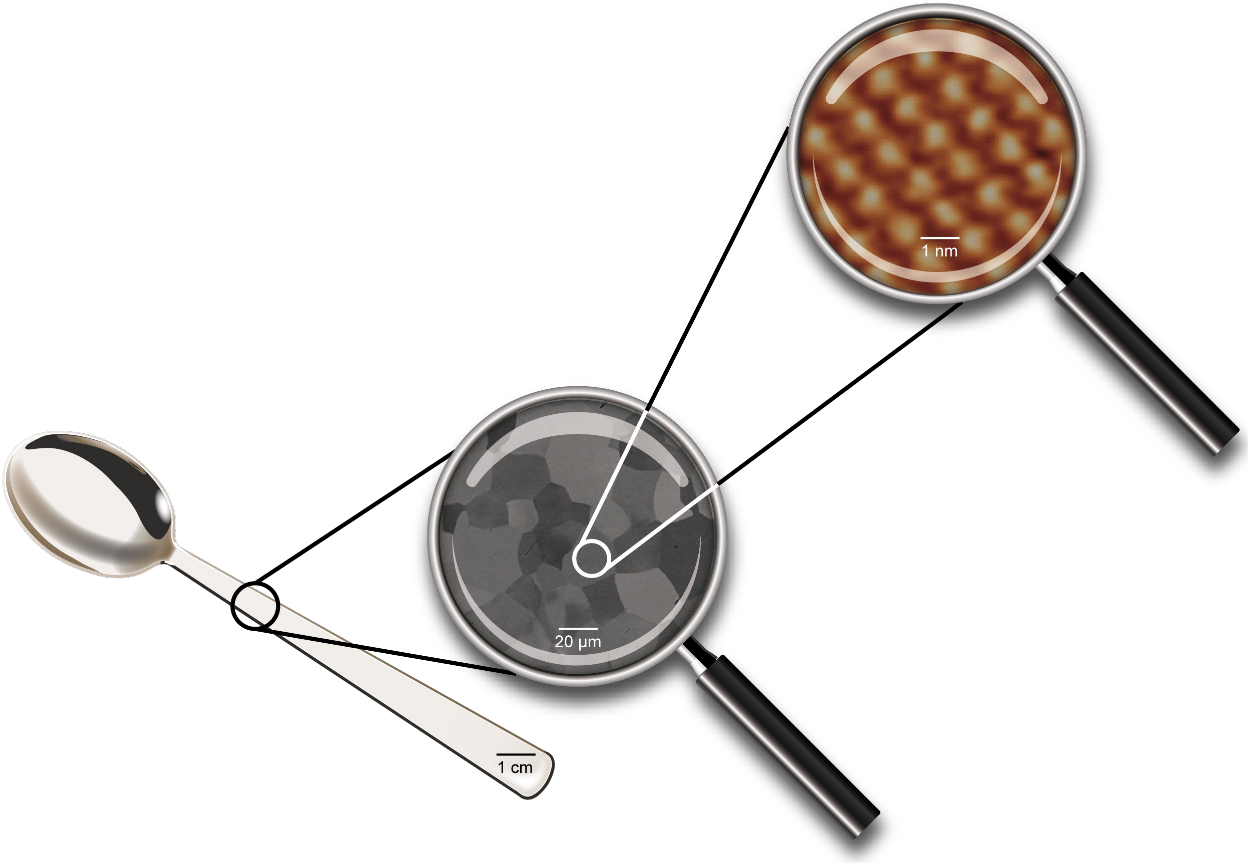


Solutes / Vacancies:
- Nature
- Number
- Distribution



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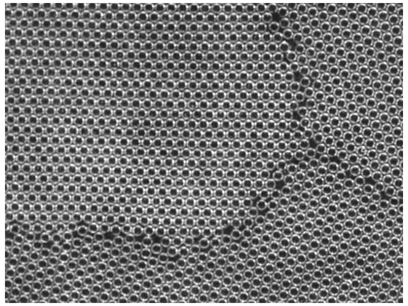

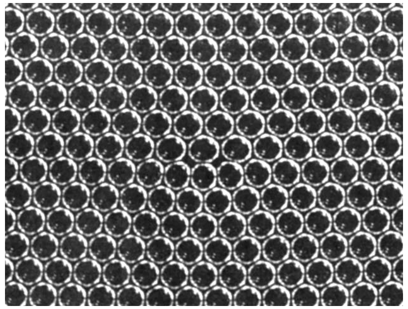

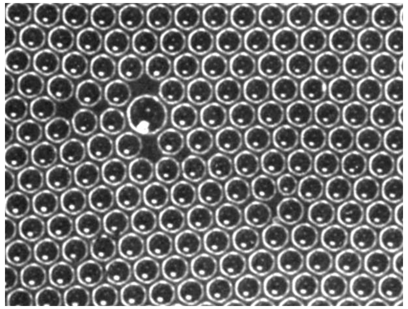

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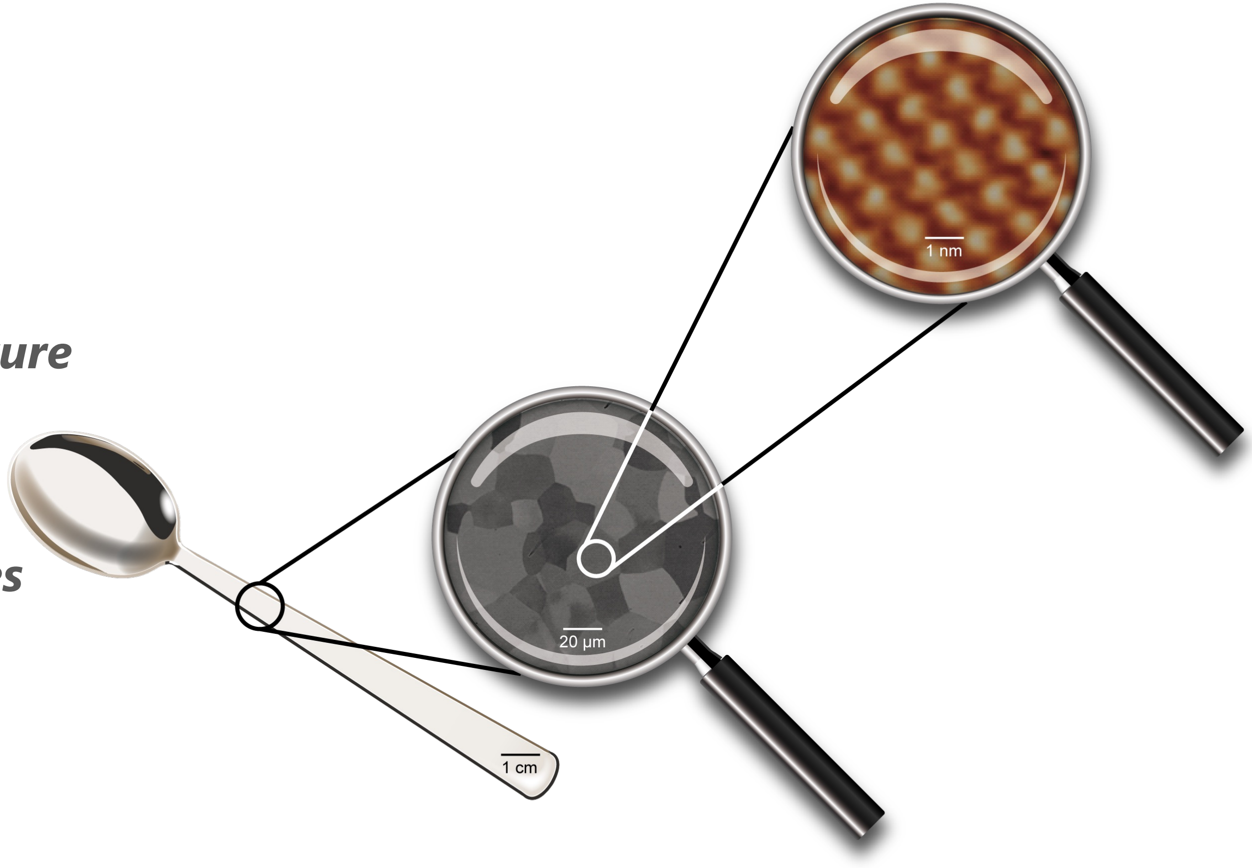
Microstructure

Properties



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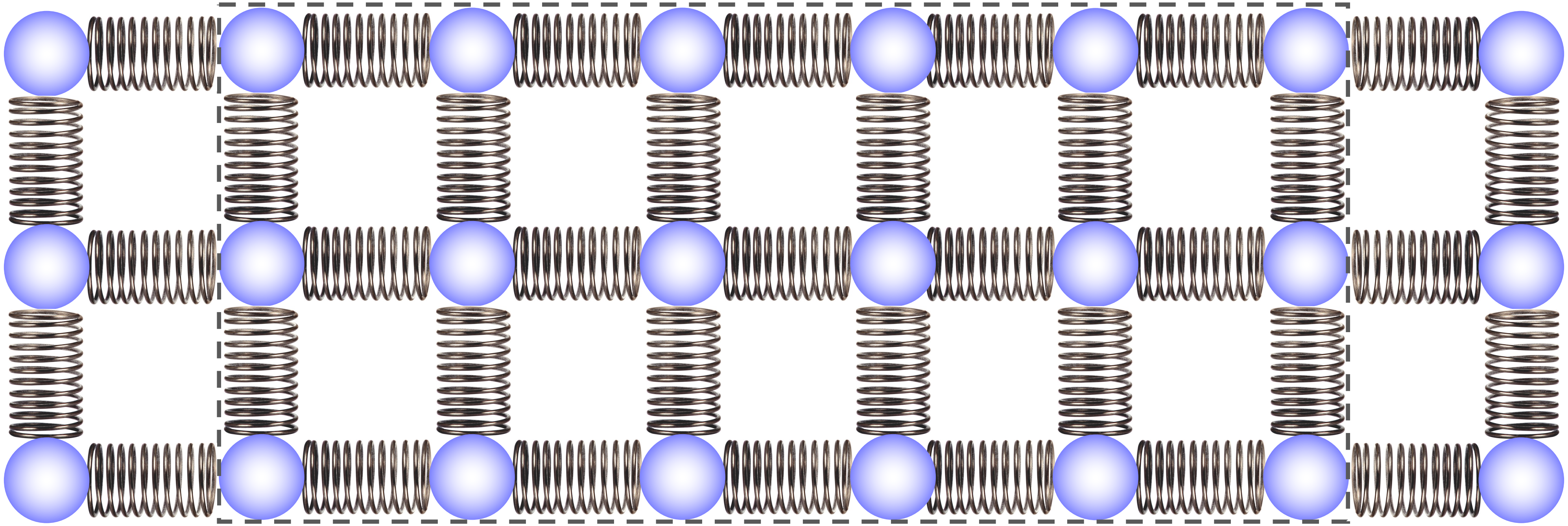
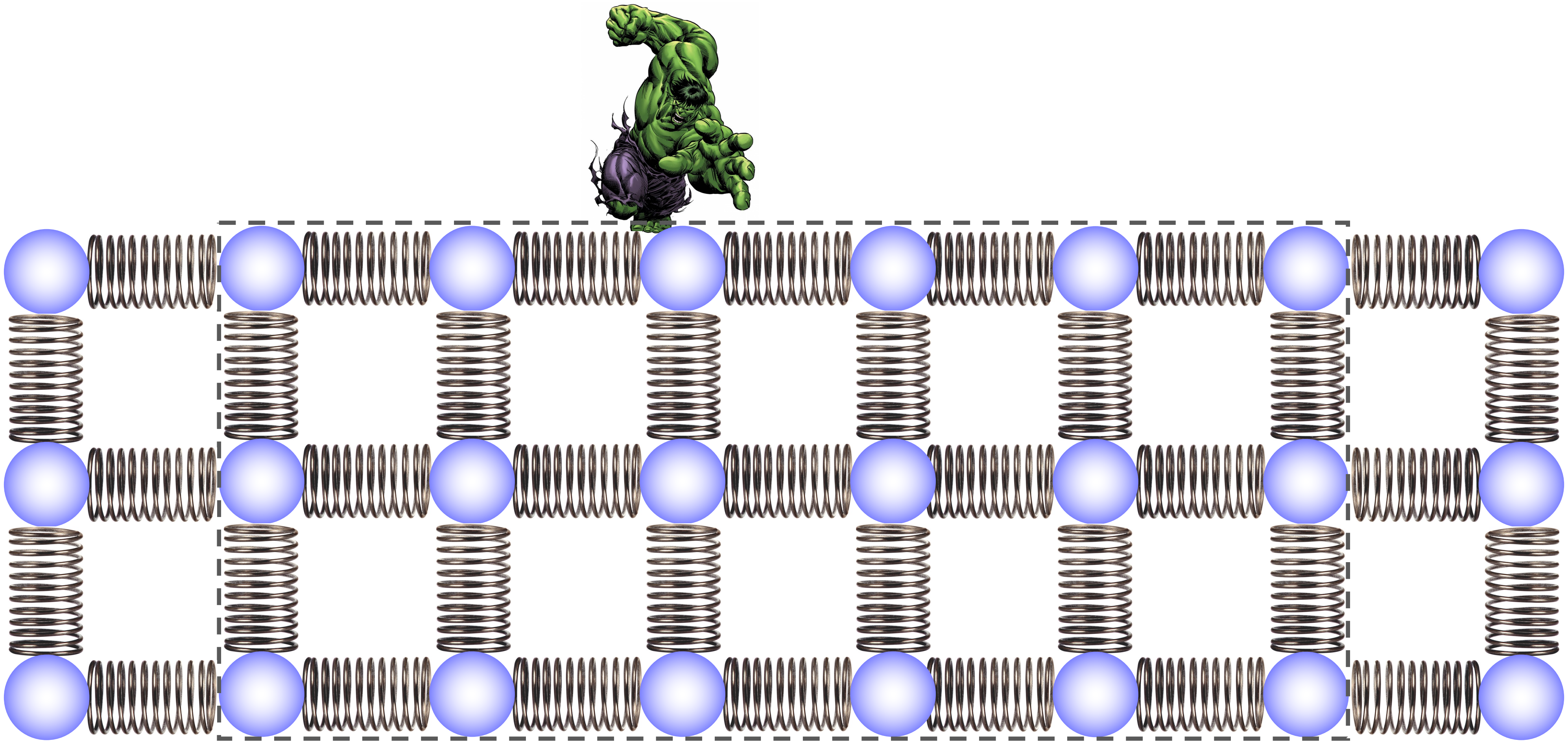
Deformation of materials

Initial size of the structure $L \times l \times h$



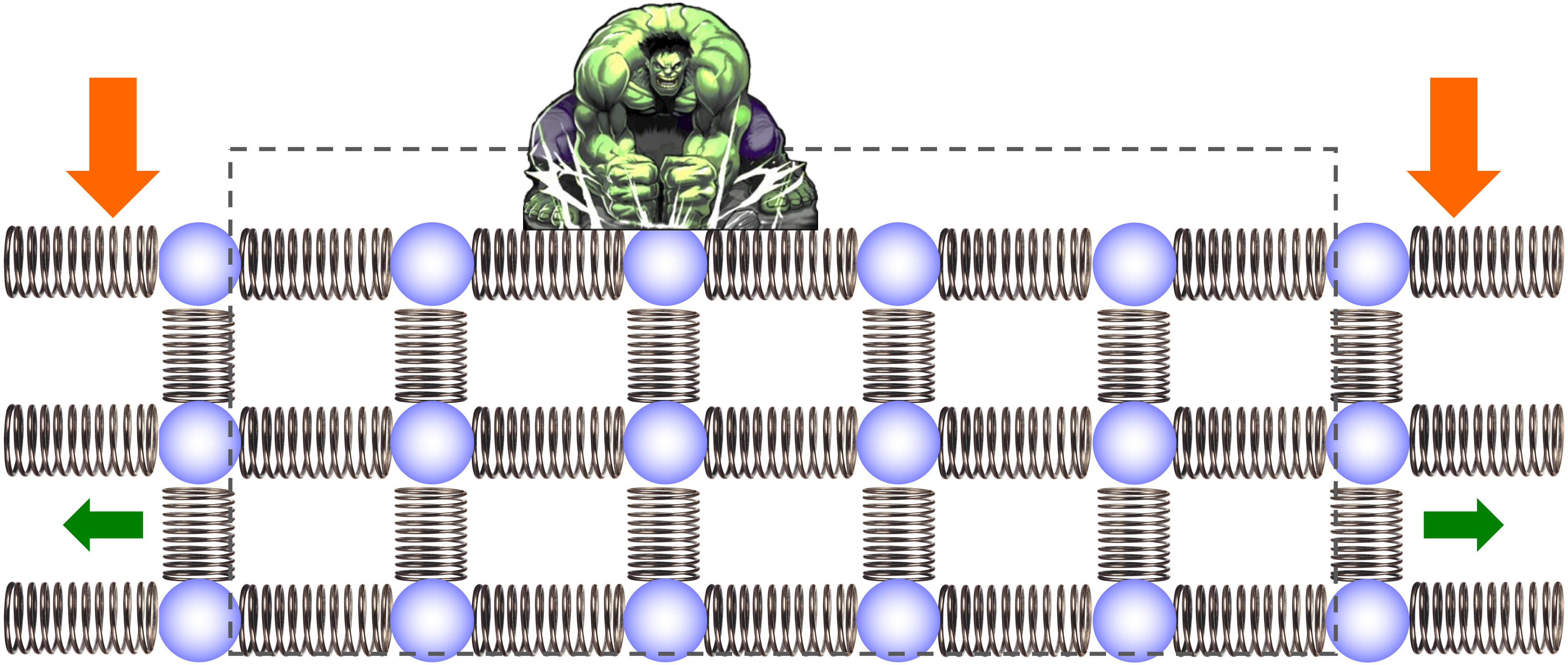
Deformation of materials

Application of a vertical load



Deformation of materials

Under the load, the vertical springs are compressed and the horizontal ones are stretched.



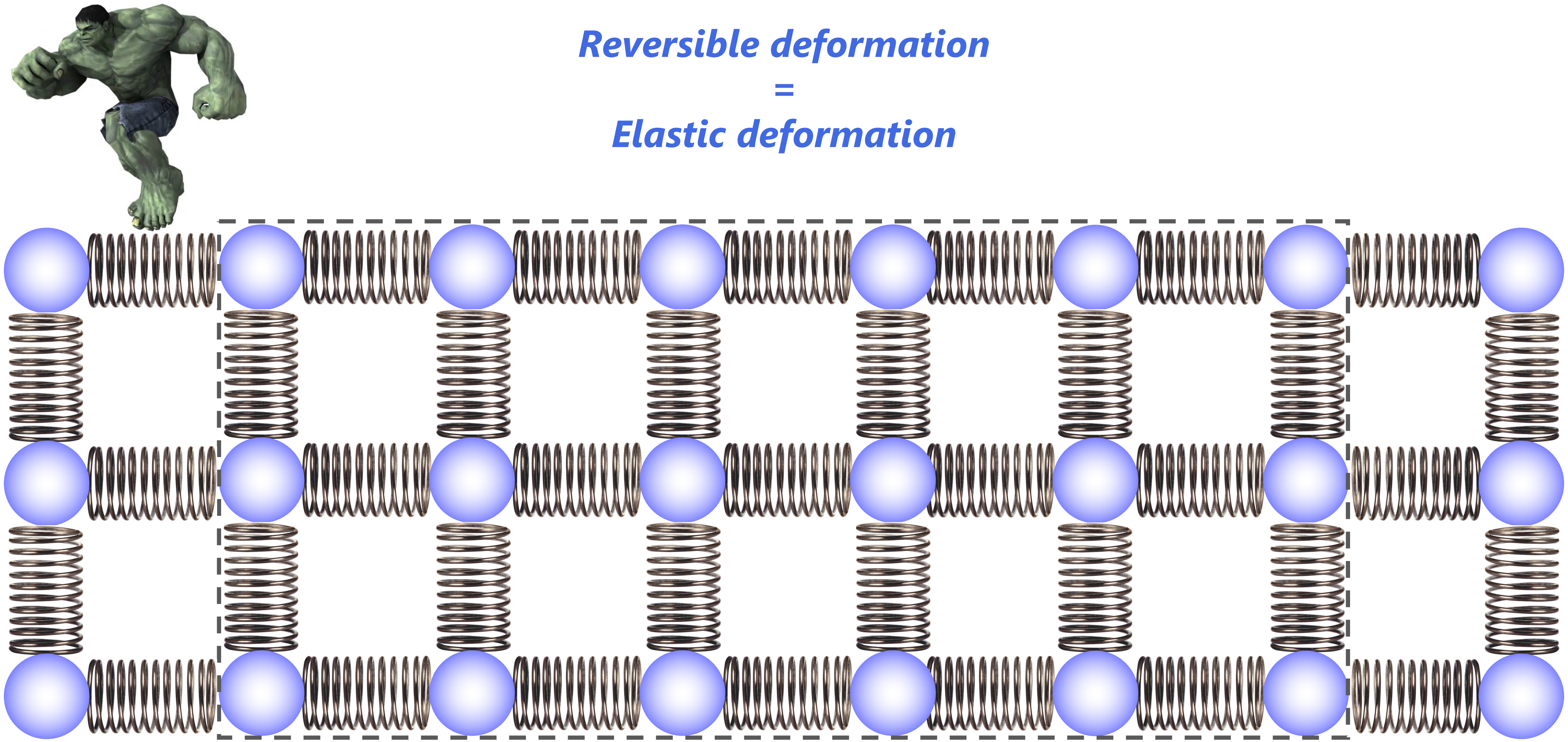
Deformation of materials

When the load is removed, the structure returns to its original dimensions.

Reversible deformation

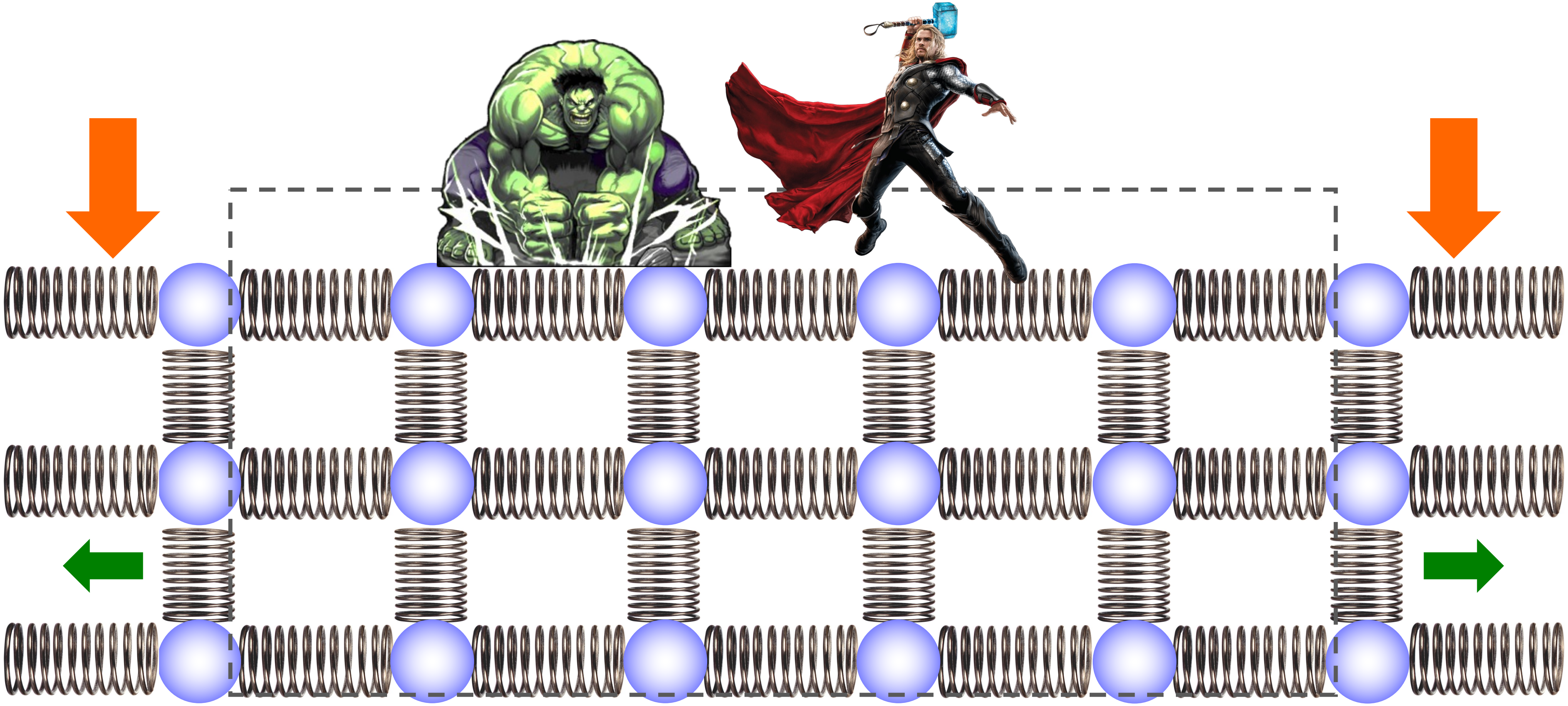
=

Elastic deformation



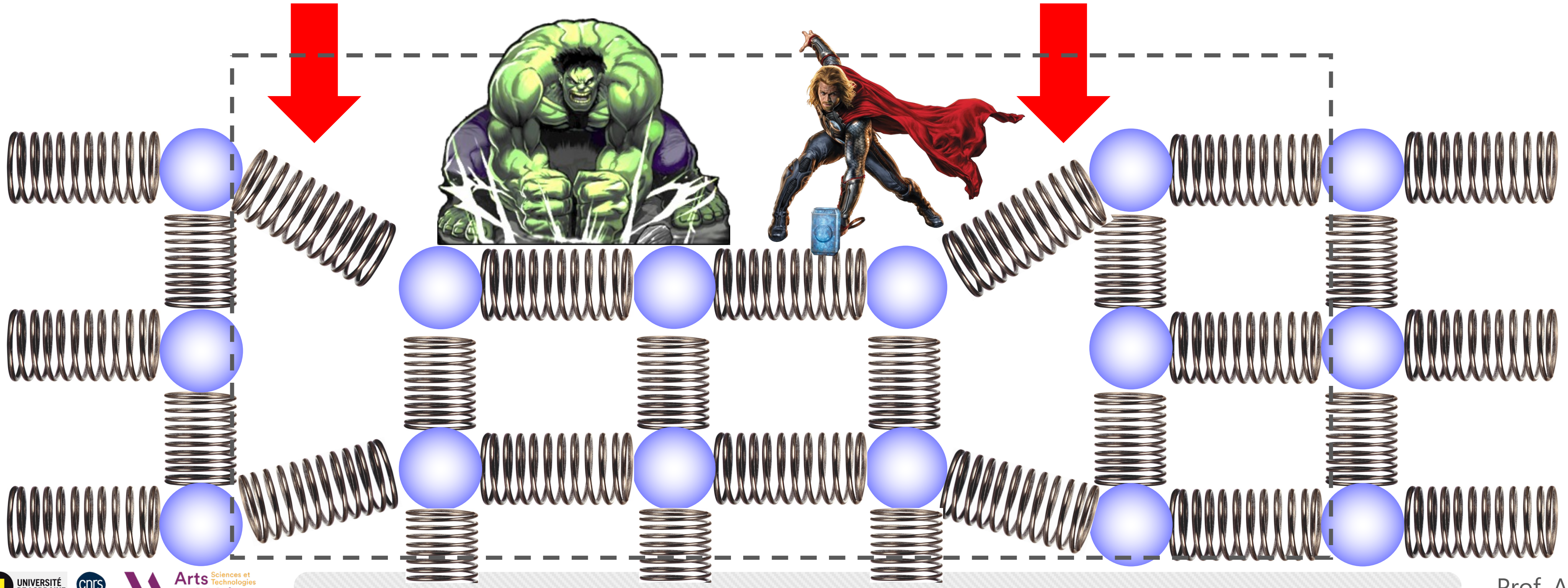
Deformation of materials

Under the load, the vertical springs are compressed and the horizontal ones are stretched.
A higher load is applied.



Deformation of materials

Under the load, the vertical springs are compressed and the horizontal ones are stretched.
Some springs are broken.



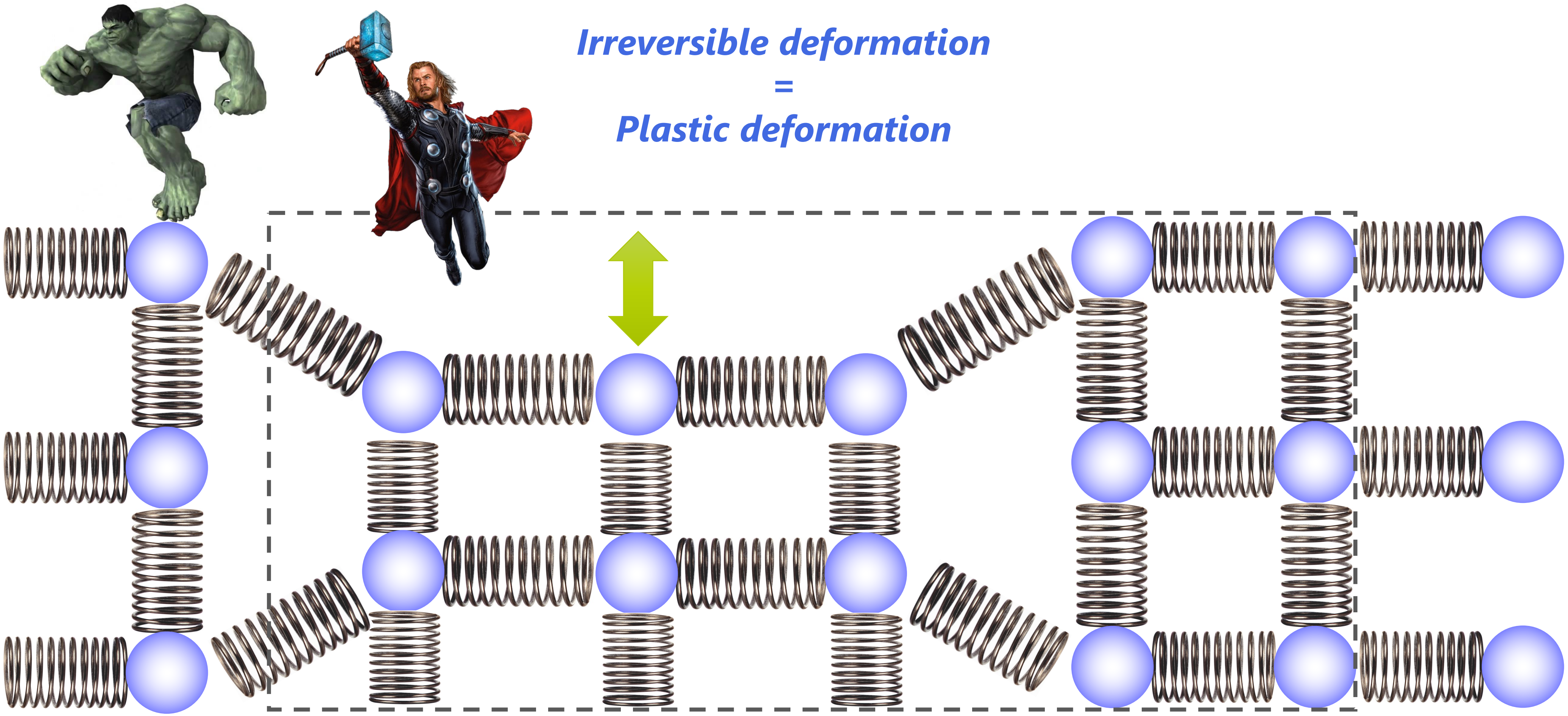
Deformation of materials

Since some springs are broken, the structure does not return to its original shape.

Irreversible deformation

=

Plastic deformation



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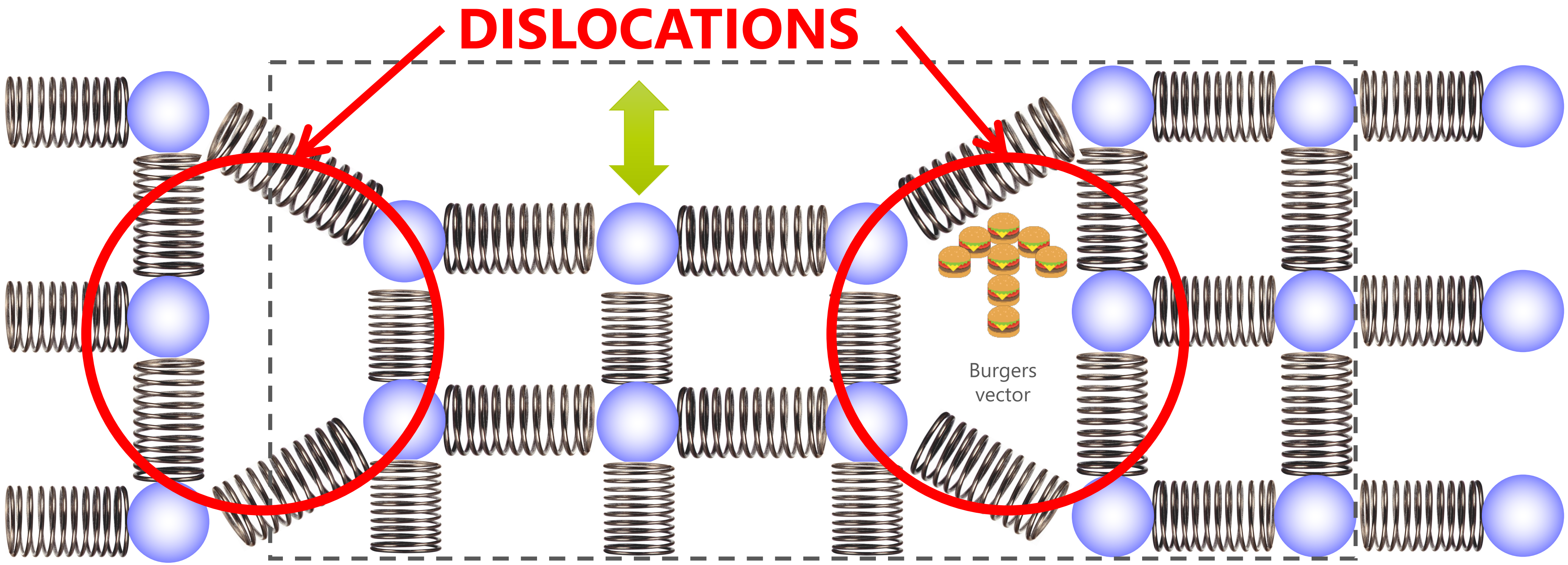

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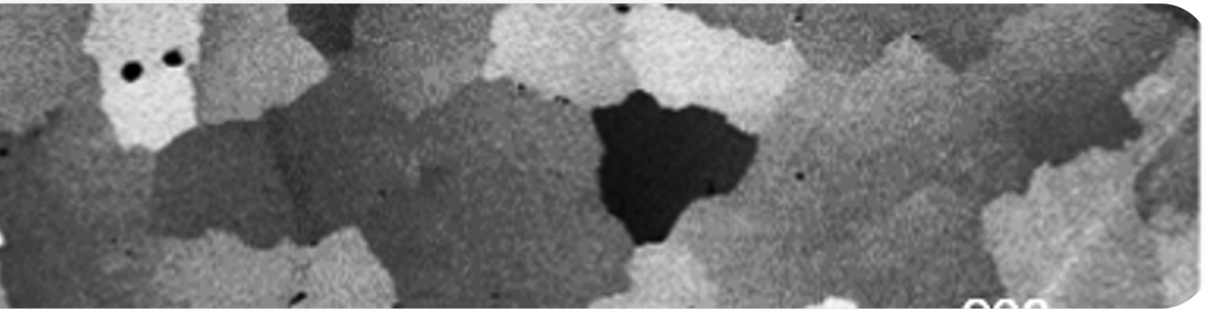
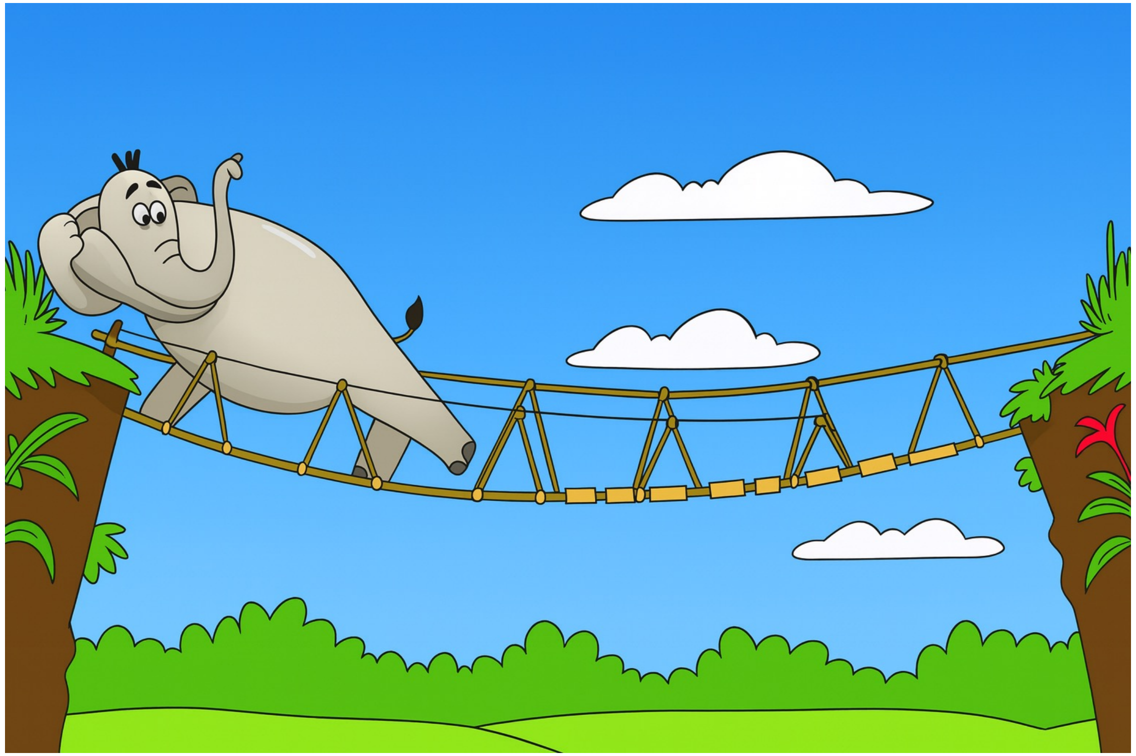
- **Navier-Bernoulli principle:**
  - Cross-sections remain plane and perpendicular to the beam's axis during bending.



External loads

External loads

- **Actions directly applied to the beam at points A_i: $\{\mathcal{T}_{A_i}\}$**
- Actions at the supports at points B_i: $\{\mathcal{T}_{B_i}\}$



Actions directly applied to the beam

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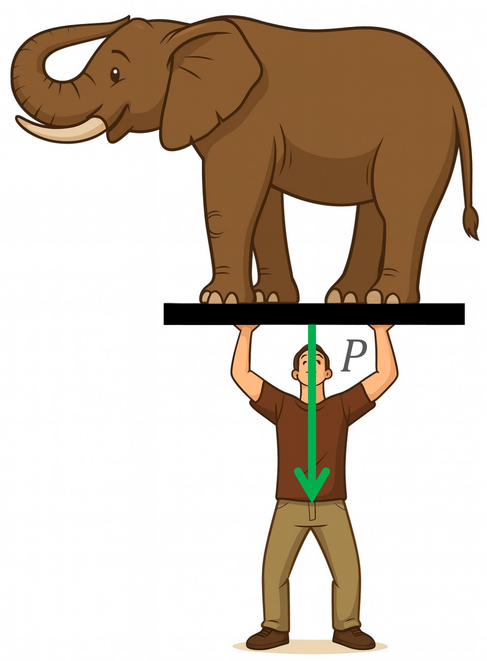
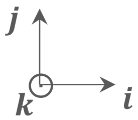
- Loads applied at a single point.





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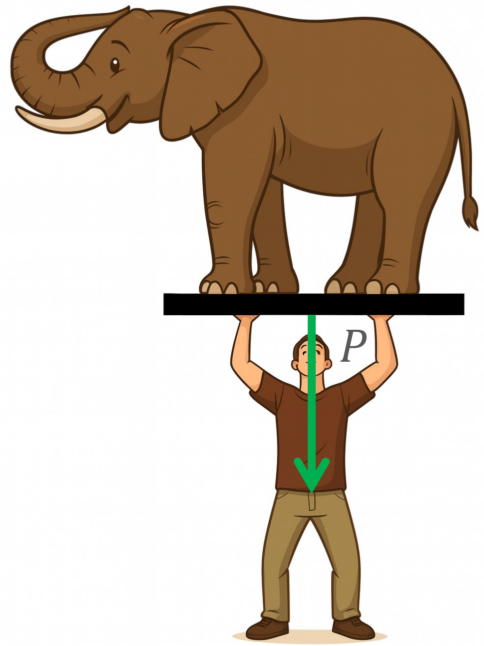

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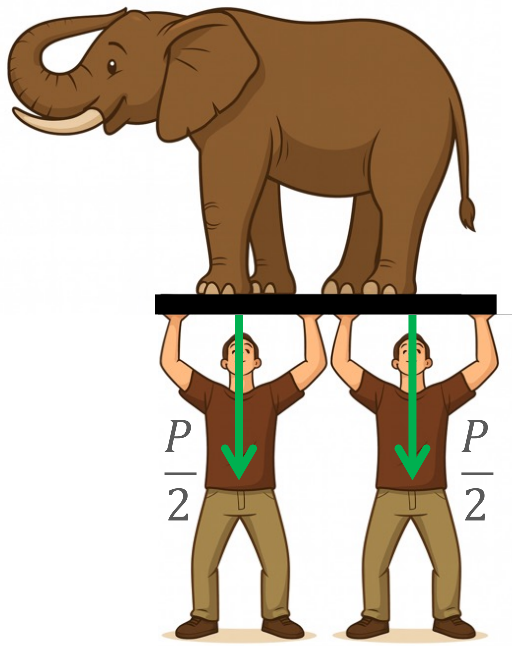

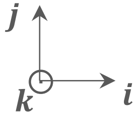

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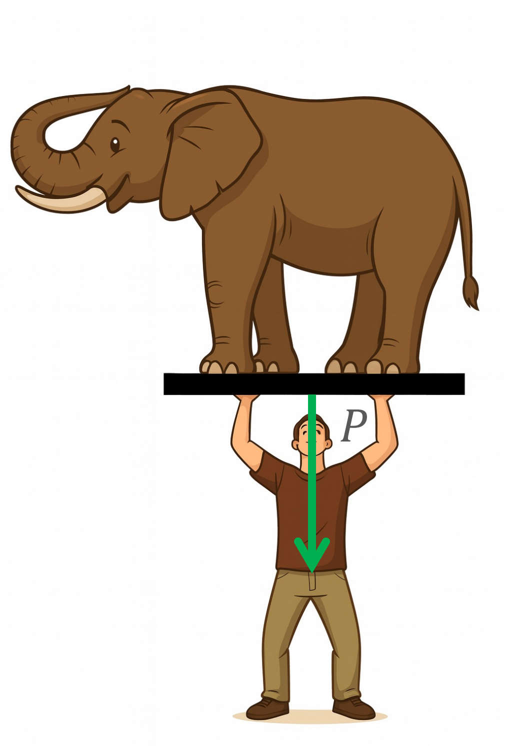

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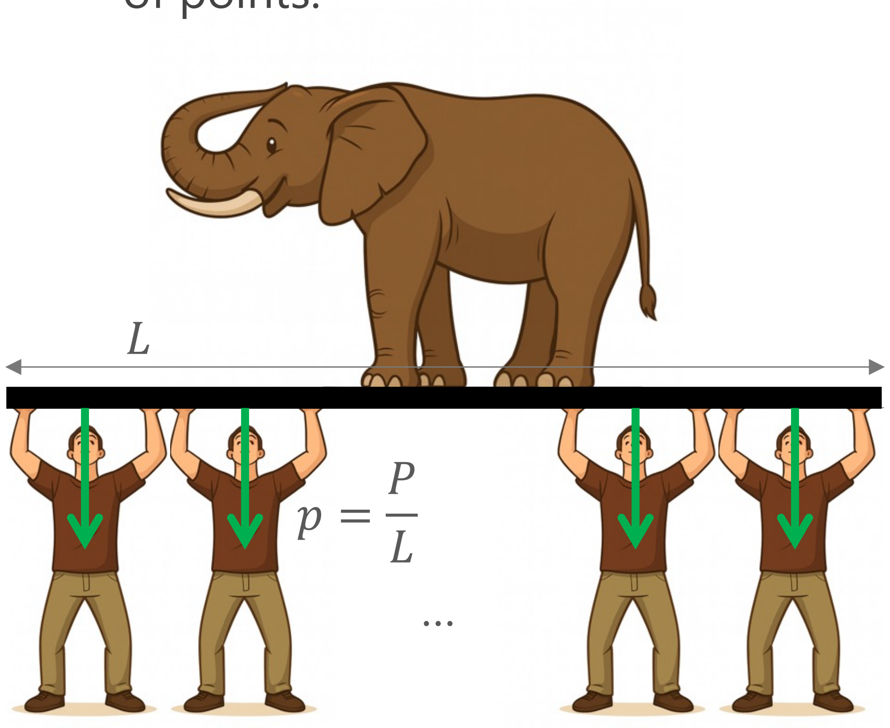

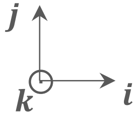

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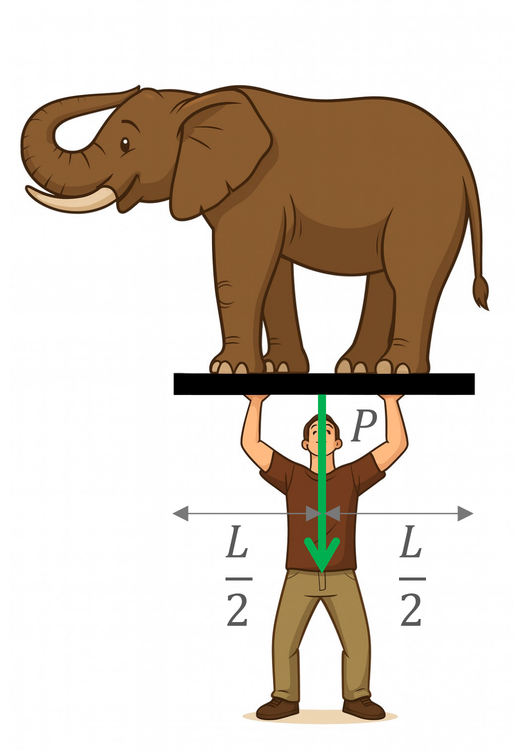

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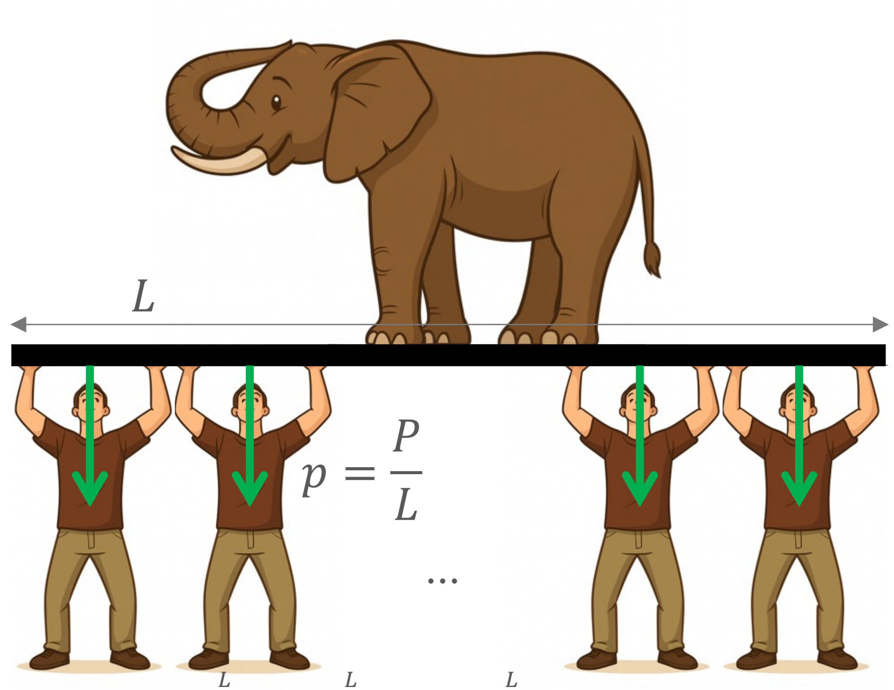
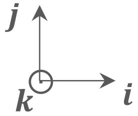


$$\int_0^L p\mathrm{d}x = \int_0^L \frac{P}{L}\mathrm{d}x = \frac{P}{L}\int_0^L \mathrm{d}x = P$$



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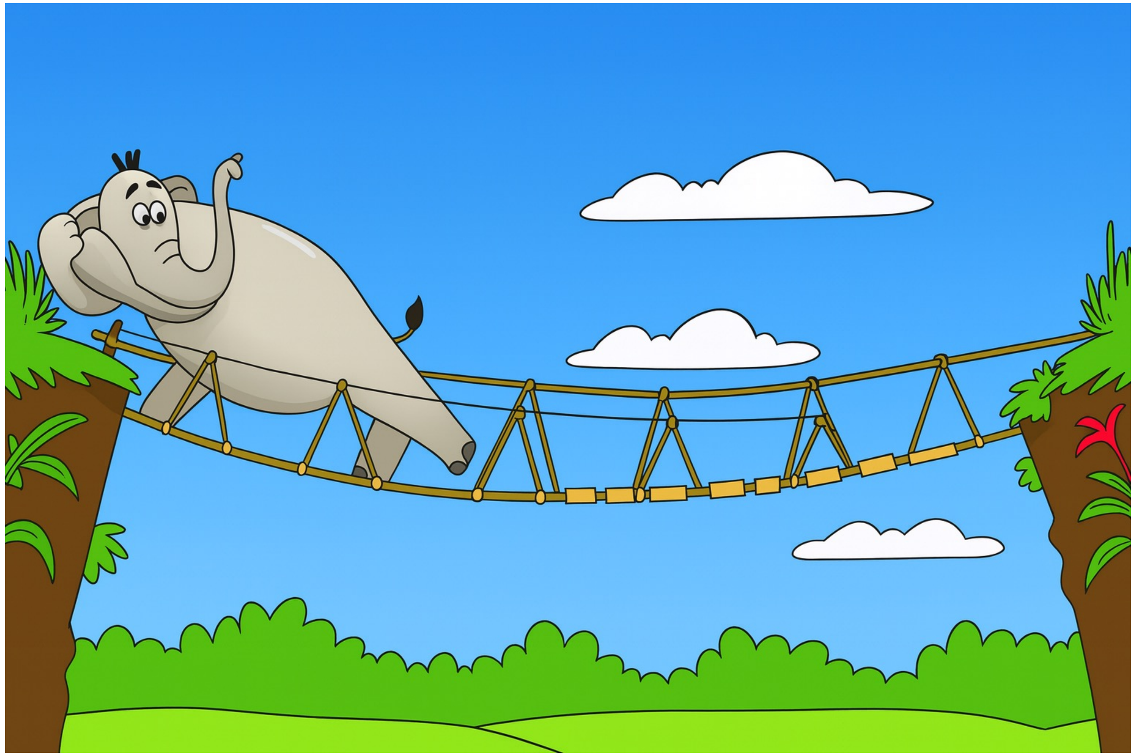


External loads: supports

Translation along $\boldsymbol{i}$

Translation along $\boldsymbol{j}$

Translation along $\boldsymbol{k}$

$$\{\mathcal{T}_A\} = \begin{Bmatrix} T_{A,x} & R_{A,x} \\ T_{A,y} & R_{A,y} \\ T_{A,z} & R_{A,z} \end{Bmatrix}$$

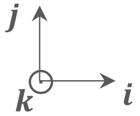
Rotation around $\boldsymbol{i}$

Rotation around $\boldsymbol{j}$

Rotation around $\boldsymbol{k}$

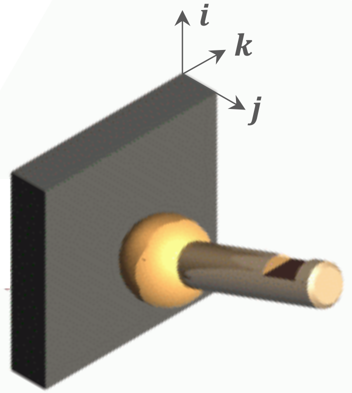
↳6 degrees of freedom (DOF).

↳Each support blocks some of them: blocked translations create reaction forces, and blocked rotations create reaction moments.



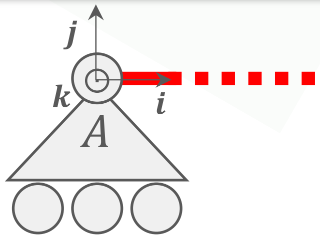
External loads: supports

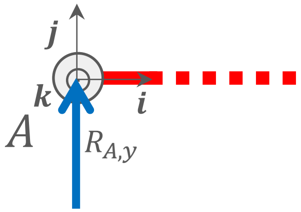
Simple support



$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

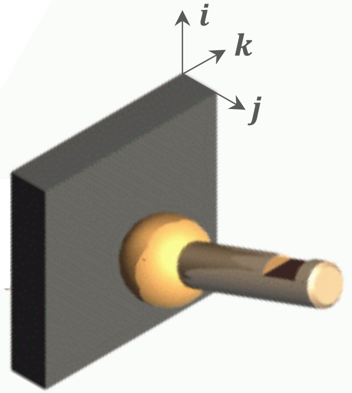
↳ 5 degrees of freedom.





External loads: supports

Simple support



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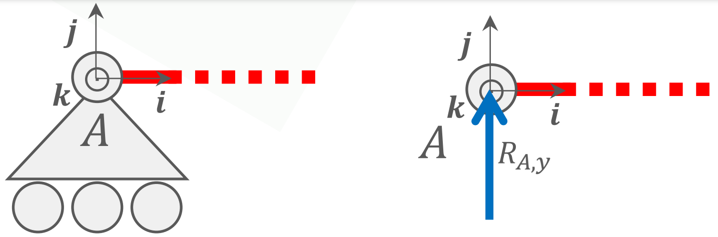
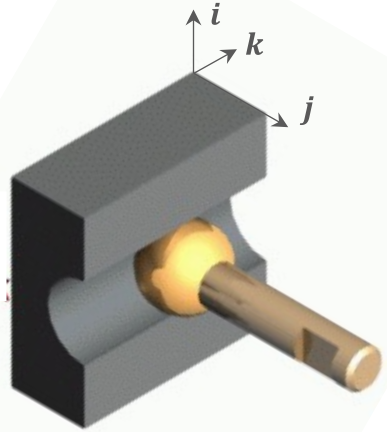
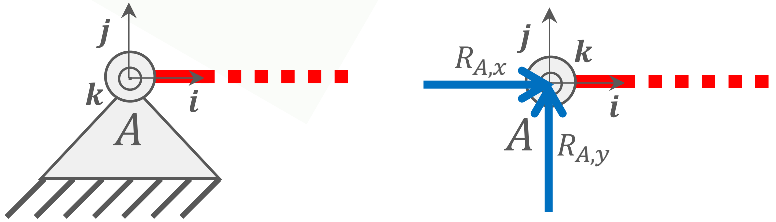


Pinned support



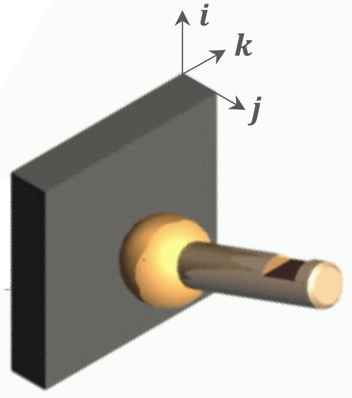
$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

↳ 4 degrees of freedom.



External loads: supports

Simple support



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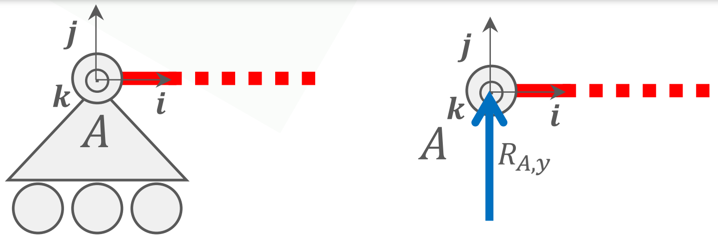
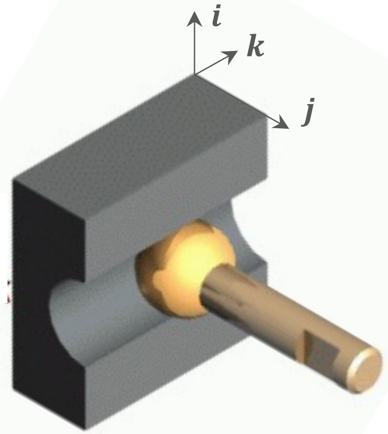


Pinned support



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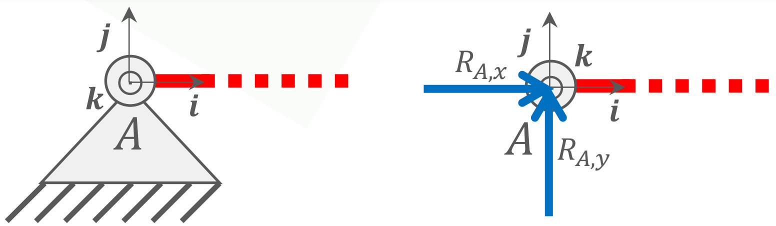
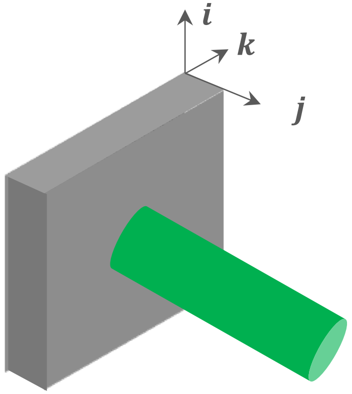


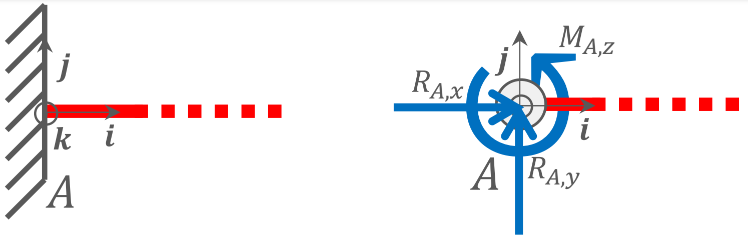
Fixed support



$$\{\mathcal{T}_A\}^{3D} = \begin{Bmatrix} R_{A,x} & M_{A,x} \\ R_{A,y} & M_{A,y} \\ R_{A,z} & M_{A,z} \end{Bmatrix}$$

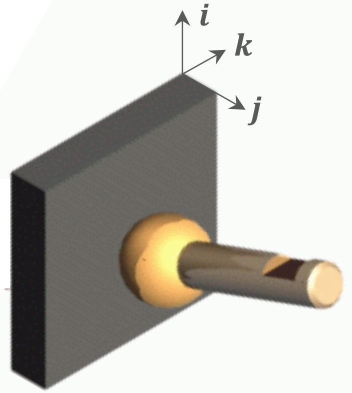
$$\{\mathcal{T}_A\}^{2D} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}$$

↳ 0 degree of freedom in 3D.



External loads: supports

Simple support



$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

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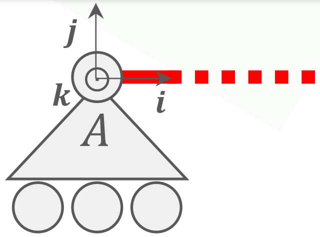
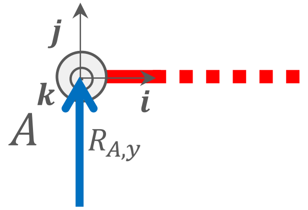
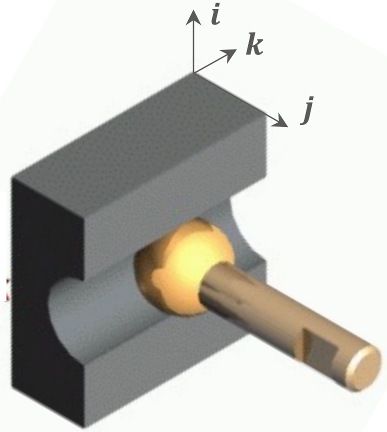


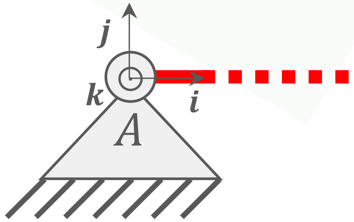


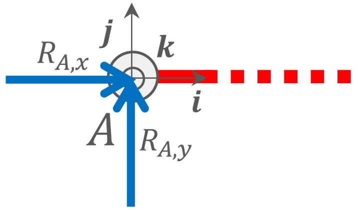
Pinned support



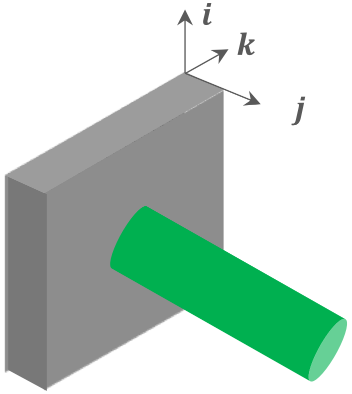
$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

↳ 4 degrees of freedom.





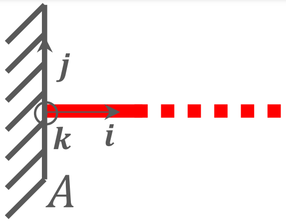
Fixed support

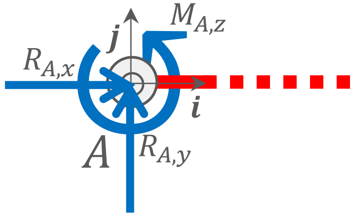


$$\{\mathcal{T}_A\}^{3D} = \begin{Bmatrix} R_{A,x} & M_{A,x} \\ R_{A,y} & M_{A,y} \\ R_{A,z} & M_{A,z} \end{Bmatrix}$$

$$\{\mathcal{T}_A\}^{2D} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}$$

↳ 0 degree of freedom in 3D.





↳In 2D $(\boldsymbol{i}, \boldsymbol{j})$ plane analysis only three support types are distinct

The fundamental principle of statics (FPS)

❖ **Statement:**

- A system is in equilibrium if and only if the resultant of all external forces at point A ($\boldsymbol{R_A}$) is zero and the resultant of all external moments at point A ($\boldsymbol{M_A}$) is zero.

$$\Rightarrow \boldsymbol{R_A} = \sum_{ext} \boldsymbol{f_{A,i}} = \boldsymbol{0} \; ; \boldsymbol{M_A} = \sum_{ext} \boldsymbol{m_{A,i}} = \boldsymbol{0}$$

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❖ Isostatic:

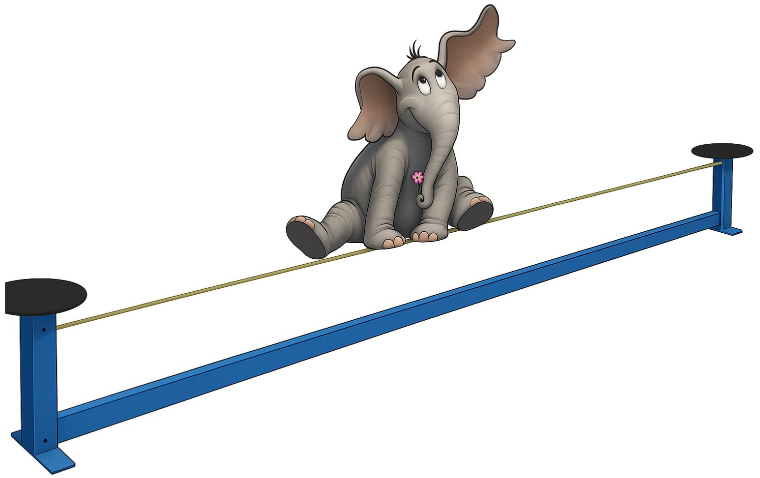
- If the number of unknown reaction components is equal to the number of equilibrium equations provided by the FPS.

❖ Hyperstatic:

- If the number of unknown reaction components is greater than the number of equilibrium equations provided by the FPS.

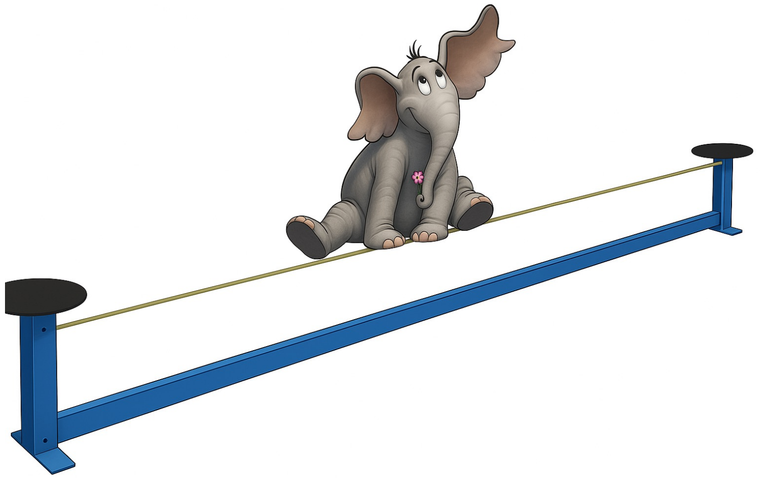
The free body diagram (FBD)

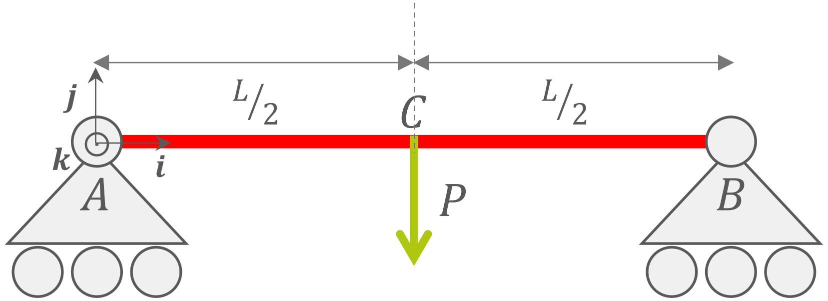
- **Isolate the system:**
  - Choose the object you want to analyze.



The free body diagram (FBD)

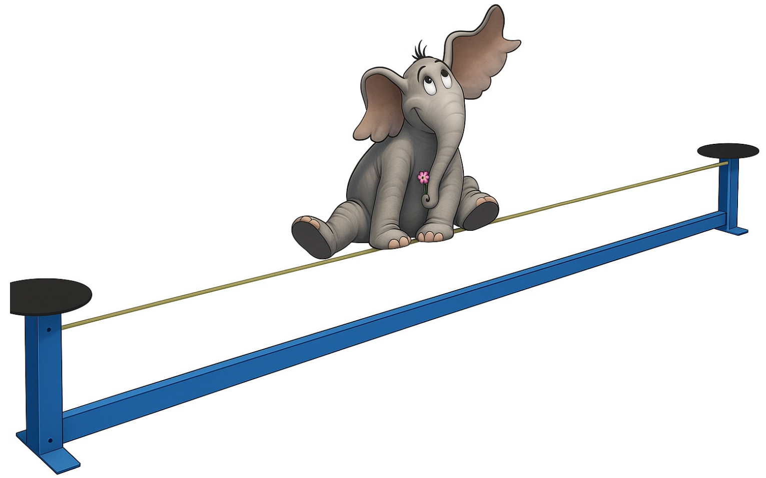
- **Isolate the system:**
  - Choose the object you want to analyze.
- **Model the system in connection with its environment:**
  - Use the most appropriate supports.

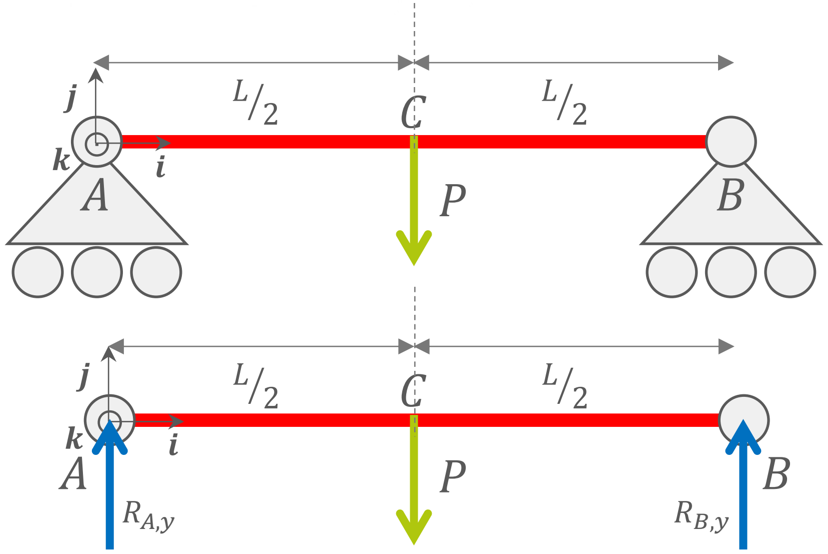




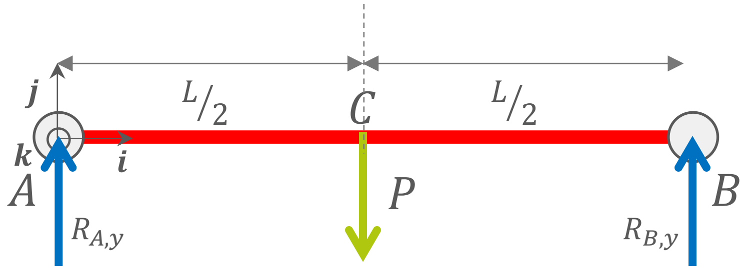
The free body diagram (FBD)

- **Isolate the system:**
  - Choose the object you want to analyze.
- **Model the system in connection with its environment:**
  - Use the most appropriate supports.
- **Replace each support by reaction forces:**
  - At each support, draw the appropriate reactions.
- **Add all external forces:**
  - The elephant's weight acting downward at its center of gravity.
- **Add geometric information if needed:**
  - Distances between supports, the position of the elephant on the line, lengths, etc.
- **Add a coordinate system:**
  - Draw axes.



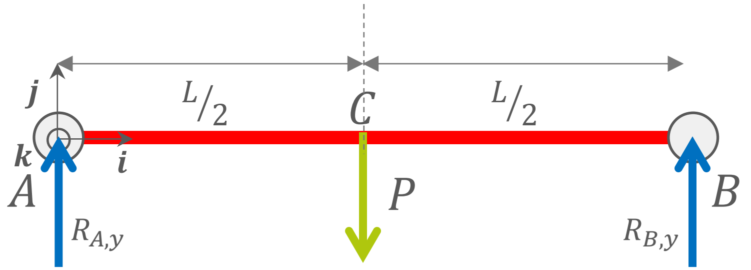


How to solve a statics problem?



- **Coordinate system:**
  - Origin: A; Basis: $(A, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$

How to solve a statics problem?



- **Coordinate system:**
  - Origin: A; Basis: $(A, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$
- **Writing the torsors:**
  - Supports

$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

  - External loads

$$\{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ -P = -mg & 0 \\ 0 & 0 \end{Bmatrix}$$

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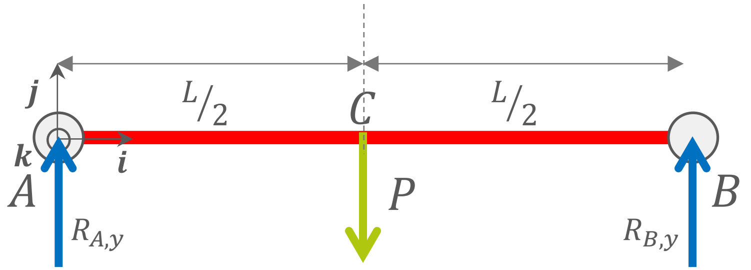


❖ **Coordinate system:**

- Origin: A; Basis: $(A, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$

❖ **Writing the torsors:**

- Supports

$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

- External loads

$$\{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ -P = -mg & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **Transporting the torsors to a common point (C for example):**

$$\{\mathcal{T}_{A\to C}\} = \begin{Bmatrix} \boldsymbol{R_C} = \boldsymbol{R_A} = R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_C} = \boldsymbol{M_A} + \boldsymbol{CA}\times\boldsymbol{R_A} \end{Bmatrix}$$

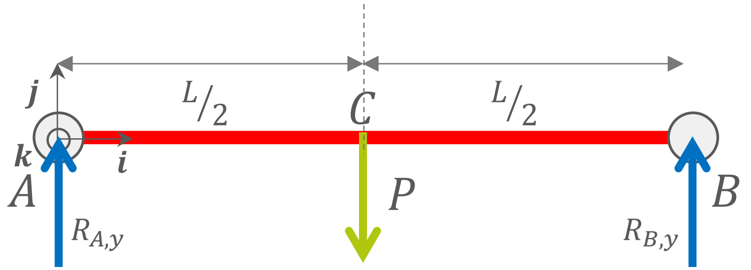
$$= \begin{Bmatrix} \boldsymbol{R_C} = R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_C} = \boldsymbol{0} + \left(-\frac{L}{2}\right)\boldsymbol{i}\times R_{A,y}\boldsymbol{j} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{R_C} = R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_C} = -\frac{L}{2}R_{A,y}\boldsymbol{k} \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & -\frac{L}{2}R_{A,y} \end{Bmatrix}$$

$$\{\mathcal{T}_{B\to C}\} = \begin{Bmatrix} \boldsymbol{R_C} = R_{B,y}\boldsymbol{j} \\ \boldsymbol{M_C} = \frac{L}{2}R_{B,y}\boldsymbol{k} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & \frac{L}{2}R_{B,y} \end{Bmatrix}$$

$$\{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ -P = -mg & 0 \\ 0 & 0 \end{Bmatrix}$$

How to solve a statics problem?



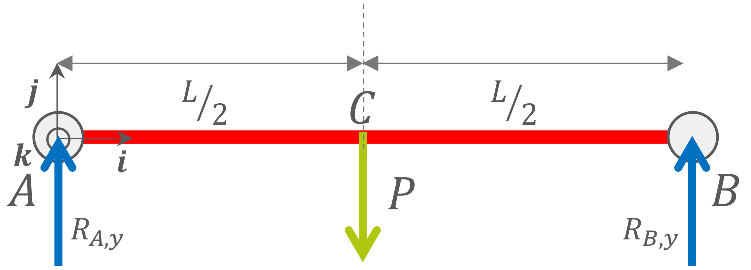
- **Let's apply the FPS in C:**

$$\{\mathcal{T}_{A\to C}\} + \{\mathcal{T}_{B\to C}\} + \{\mathcal{T}_C\} = \{0\}$$

$$\Rightarrow \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & -\frac{L}{2}R_{A,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & \frac{L}{2}R_{B,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ -P & 0 \\ 0 & 0 \end{Bmatrix} = \{0\}$$

$$\Leftrightarrow \begin{Bmatrix} 0+0+0=0 & 0+0+0=0 \\ R_{A,y} + R_{B,y} - P = 0 & 0+0+0=0 \\ 0+0+0=0 & -\frac{L}{2}R_{A,y} + \frac{L}{2}R_{B,y} + 0 = 0 \end{Bmatrix}$$

How to solve a statics problem?



- Obtaining a system of equations from the non zero components of the FPS torsor:

$$\begin{cases} R_{A,y} + R_{B,y} - P = 0 \\ -\frac{L}{2}R_{A,y} + \frac{L}{2}R_{B,y} = 0 \end{cases}$$

- Let's apply the FPS in C:

$$\{\mathcal{T}_{A\to C}\} + \{\mathcal{T}_{B\to C}\} + \{\mathcal{T}_{C}\} = \{0\}$$

$$\Rightarrow \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & -\frac{L}{2}R_{A,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & \frac{L}{2}R_{B,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ -P & 0 \\ 0 & 0 \end{Bmatrix} = \{0\}$$

$$\Leftrightarrow \begin{Bmatrix} 0+0+0=0 & 0+0+0=0 \\ R_{A,y} + R_{B,y} - P = 0 & 0+0+0=0 \\ 0+0+0=0 & -\frac{L}{2}R_{A,y} + \frac{L}{2}R_{B,y} + 0 = 0 \end{Bmatrix}$$

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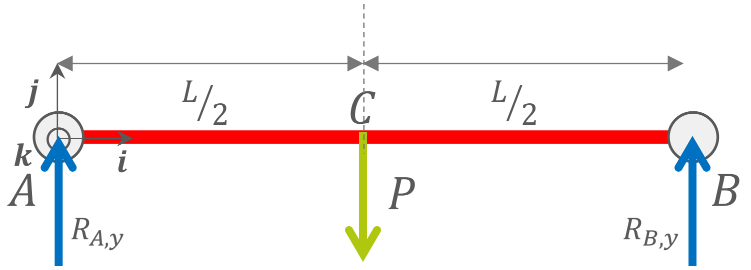

❖ **Let's apply the FPS in C:**

$$\{\mathcal{T}_{A\to C}\} + \{\mathcal{T}_{B\to C}\} + \{\mathcal{T}_{C}\} = \{0\}$$

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$$\Leftrightarrow \begin{Bmatrix} 0+0+0=0 & 0+0+0=0 \\ R_{A,y}+R_{B,y}-P=0 & 0+0+0=0 \\ 0+0+0=0 & -\frac{L}{2}R_{A,y}+\frac{L}{2}R_{B,y}+0=0 \end{Bmatrix}$$

❖ **Obtaining a system of equations from the non zero components of the FPS torsor:**

$$\begin{cases} R_{A,y} + R_{B,y} - P = 0 \\ -\frac{L}{2}R_{A,y} + \frac{L}{2}R_{B,y} = 0 \end{cases}$$

❖ **Solving the equation system:**

$$\begin{cases} R_{A,y} + R_{B,y} - P = 0 \\ -\frac{L}{2}R_{A,y} + \frac{L}{2}R_{B,y} = 0 \end{cases} \Leftrightarrow \begin{cases} R_{A,y} + R_{B,y} - P = 0 \\ \frac{L}{2}R_{B,y} = \frac{L}{2}R_{A,y} \end{cases}$$

$$\Leftrightarrow \begin{cases} R_{A,y} + R_{B,y} - P = 0 \\ R_{B,y} = R_{A,y} \end{cases} \Leftrightarrow \begin{cases} 2R_{A,y} = P \\ R_{B,y} = R_{A,y} \end{cases} \Leftrightarrow \begin{cases} R_{A,y} = \frac{P}{2} \\ R_{B,y} = R_{A,y} \end{cases}$$

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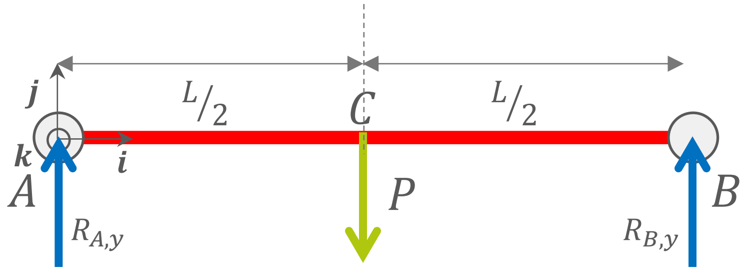

❖ **Let's apply the FPS in C:**

$$\{\mathcal{T}_{A\to C}\} + \{\mathcal{T}_{B\to C}\} + \{\mathcal{T}_C\} = \{0\}$$

$$\Rightarrow \begin{Bmatrix} 0 & 0 \\ R_{A,y} & 0 \\ 0 & -\frac{L}{2}R_{A,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ R_{B,y} & 0 \\ 0 & \frac{L}{2}R_{B,y} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ -P & 0 \\ 0 & 0 \end{Bmatrix} = \{0\}$$

$$\Leftrightarrow \begin{Bmatrix} 0+0+0=0 & 0+0+0=0 \\ R_{A,y}+R_{B,y}-P=0 & 0+0+0=0 \\ 0+0+0=0 & -\frac{L}{2}R_{A,y}+\frac{L}{2}R_{B,y}+0=0 \end{Bmatrix}$$

❖ **Obtaining a system of equations from the non zero components of the FPS torsor:**

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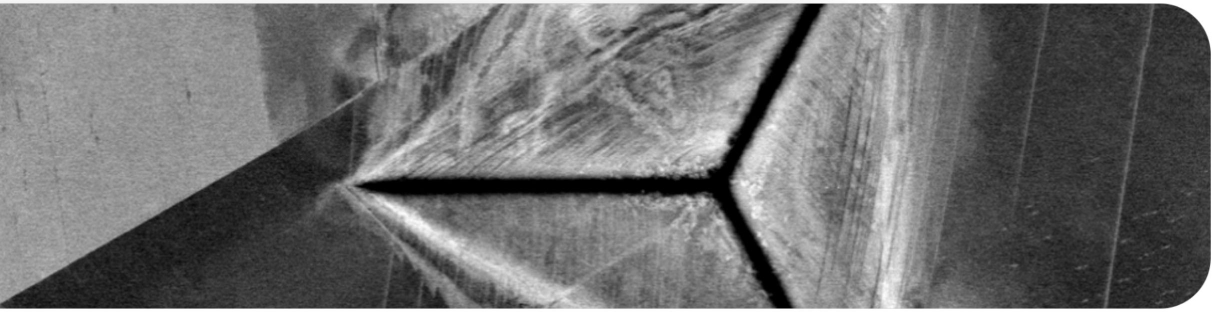
❖ **Solving the equation system:**

$$\begin{cases} R_{A,y}+R_{B,y}-P=0 \\ -\frac{L}{2}R_{A,y}+\frac{L}{2}R_{B,y}=0 \end{cases} \Leftrightarrow \begin{cases} R_{A,y}+R_{B,y}-P=0 \\ \frac{L}{2}R_{B,y}=\frac{L}{2}R_{A,y} \end{cases}$$

$$\Leftrightarrow \begin{cases} R_{A,y}+R_{B,y}-P=0 \\ R_{B,y}=R_{A,y} \end{cases} \Leftrightarrow \begin{cases} 2R_{A,y}=P \\ R_{B,y}=R_{A,y} \end{cases} \Leftrightarrow \begin{cases} R_{A,y}=\frac{P}{2} \\ R_{B,y}=R_{A,y} \end{cases}$$

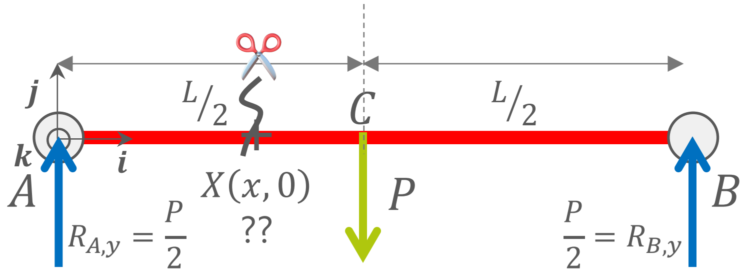
$$\Leftrightarrow \begin{cases} R_{A,y}=\frac{P}{2} \\ R_{B,y}=\frac{P}{2} \end{cases}$$

Check whether the result is physically consistent!



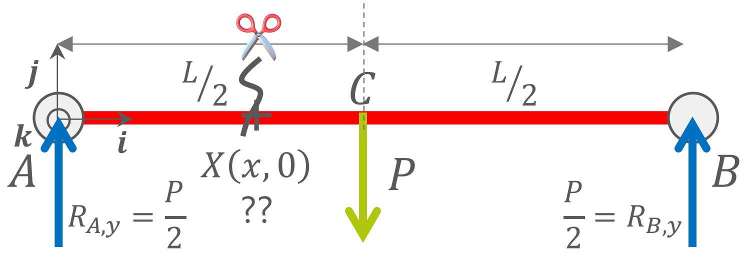
Material cohesion and internal forces

Materials cohesion



- The cohesion of the material must be checked along the beam, that is, for every point $X(x,0)$ with $X \in [0;L]$.
- A lack of cohesion will lead to failure.

Materials cohesion



- **The cohesion of the material must be checked along the beam, that is, for every point $X(x, 0)$ with $X \in [0; L]$.**
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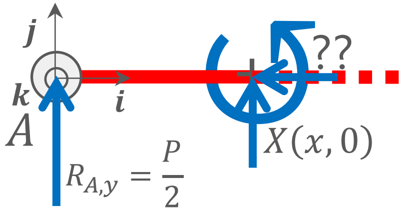
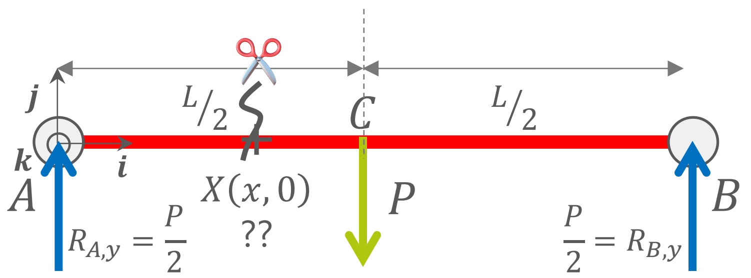
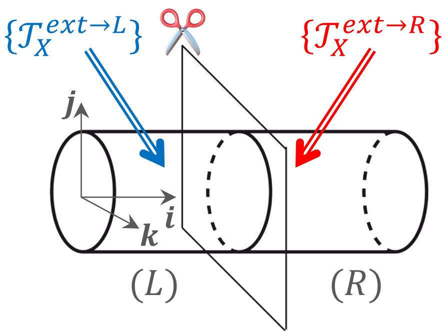


Materials cohesion





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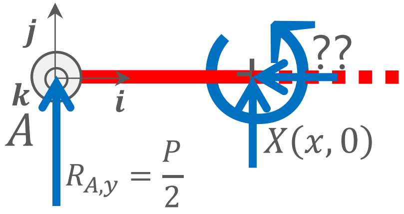
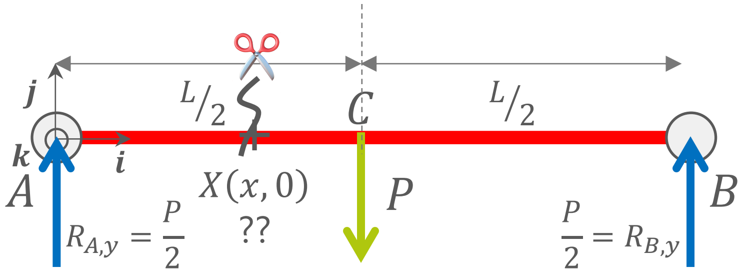
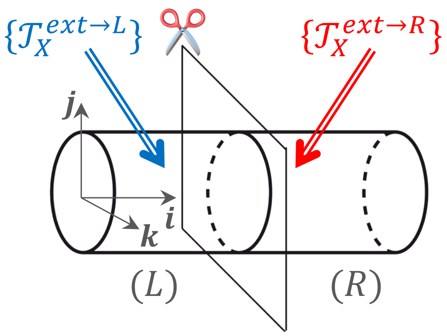


- **External loads acting on the beam at X:**
  - From the exterior acting on the left part: $\{\mathcal{T}_X^{ext\to L}\}$
  - From the exterior acting on the right part: $\{\mathcal{T}_X^{ext\to R}\}$

Materials cohesion





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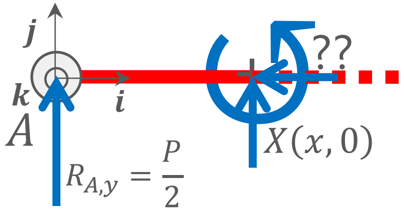

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  - From the exterior acting on the left part: $\{\mathcal{T}_X^{ext \to L}\}$
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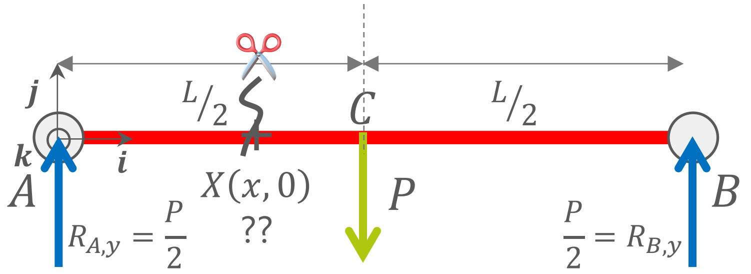
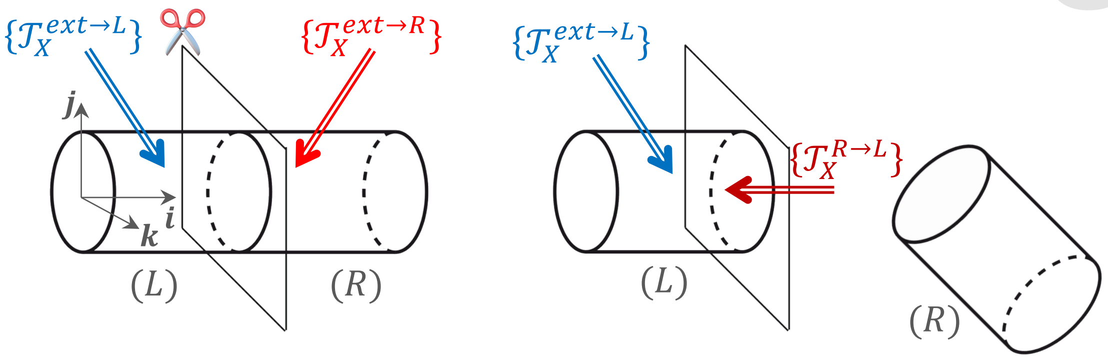
- ❖ **We are at the equilibrium:**

$$\{\mathcal{T}_X^{ext \to L}\} + \{\mathcal{T}_X^{ext \to R}\} = \{0\} \Rightarrow \{\mathcal{T}_X^{ext \to L}\} = -\{\mathcal{T}_X^{ext \to R}\}$$

Materials cohesion





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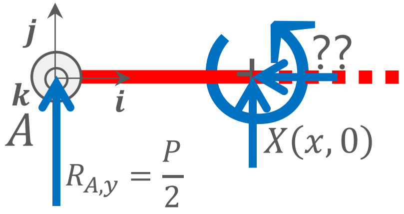

- **External loads acting on the beam at X:**
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$$\{\mathcal{T}_X^{ext\to L}\} + \{\mathcal{T}_X^{ext\to R}\} = \{0\} \Rightarrow \{\mathcal{T}_X^{ext\to L}\} = -\{\mathcal{T}_X^{ext\to R}\}$$

- **We cut and isolate the left part (L), which is also in equilibrium.**

$$\{\mathcal{T}_X^{tot\to L}\} = \{\mathcal{T}_X^{ext\to L}\} + \{\mathcal{T}_X^{R\to L}\} = 0$$

$$\Leftrightarrow \{\mathcal{T}_X^{R\to L}\} = -\{\mathcal{T}_X^{ext\to L}\} = \{\mathcal{T}_X^{ext\to R}\} = \{\mathcal{T}_X^{coh}\}$$

↳ $\{\mathcal{T}_X^{R\to L}\}$ **is the cohesion tensor.**

Cohesion torsor

Normal force

Torque

Shear force along $\boldsymbol{j}$

$$\left\{\mathcal{T}_X^{coh}\right\} = \begin{Bmatrix} N_X & T_X \\ S_{X,y} & B_{X,y} \\ S_{X,z} & B_{X,z} \end{Bmatrix}$$

Bending moment about $\boldsymbol{j}$

Bending moment about $\boldsymbol{k}$

Shear force along $\boldsymbol{k}$

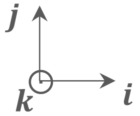
↳ $\left\{\mathcal{T}_X^{coh}\right\}$ **contains all internal actions at section** X**: normal force, shear forces and internal moments**

↳ **It fully characterizes how the beam resists loads.**

Simple loading modes

❖ **Tension/compression:**

$$\left\{\mathcal{T}_X^{coh,tract}\right\} = \begin{Bmatrix} N_X(>0) & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} ; \left\{\mathcal{T}_X^{coh,comp}\right\} = \begin{Bmatrix} N_X(<0) & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}$$



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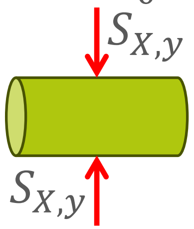
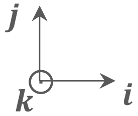
❖ Pure shear:

$$\{\mathcal{T}_X^{coh}\} = \begin{Bmatrix} 0 & 0 \\ S_{X,y} & 0 \\ 0 & 0 \end{Bmatrix}$$





Simple loading modes

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❖ Torsion:

$$\left\{\mathcal{T}_X^{coh}\right\} = \begin{Bmatrix} 0 & T_X \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}$$

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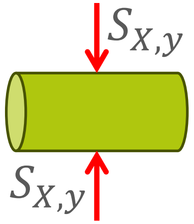
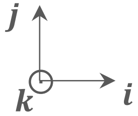
$$\left\{\mathcal{T}_X^{coh}\right\} = \begin{Bmatrix} 0 & 0 \\ S_{X,y} & 0 \\ 0 & 0 \end{Bmatrix}$$





Simple loading modes

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$$\left\{\mathcal{T}_X^{coh,tract}\right\} = \begin{Bmatrix} N_X(>0) & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix};\ \left\{\mathcal{T}_X^{coh,comp}\right\} = \begin{Bmatrix} N_X(<0) & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}$$

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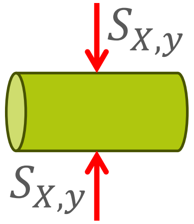
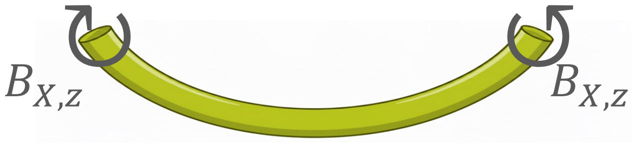
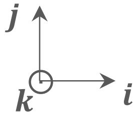


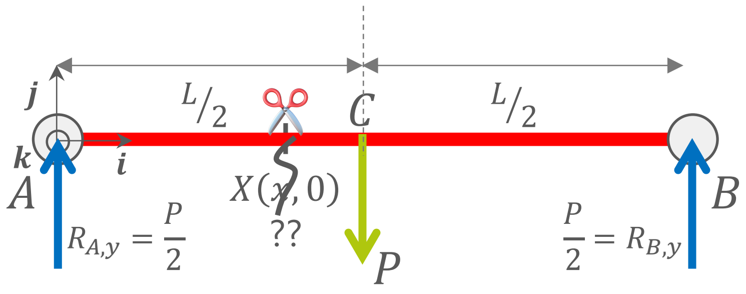
❖ Pure bending:

$$\left\{\mathcal{T}_X^{coh}\right\} = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & B_{X,z} \end{Bmatrix}$$





How to to obtain the cohesion torsor?



❖ **Let's transform the torsor into cohesion torsor:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} = \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

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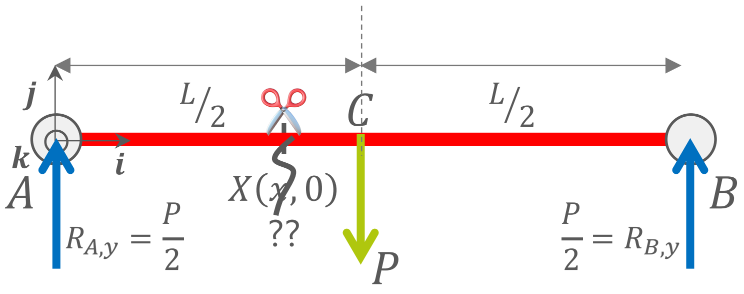

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$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_X} = \boldsymbol{R_A} = -R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_X} = \boldsymbol{M_A} + \boldsymbol{XA}\times\boldsymbol{R_A} \end{Bmatrix}$$

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}$$

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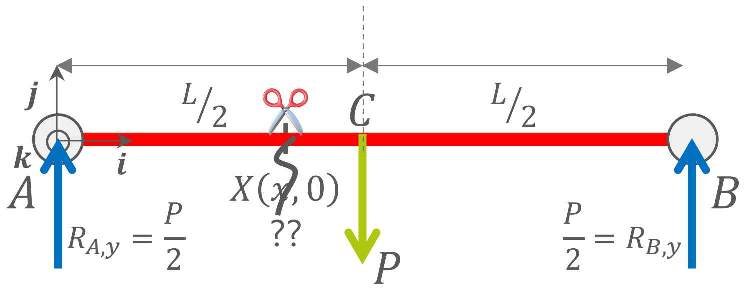

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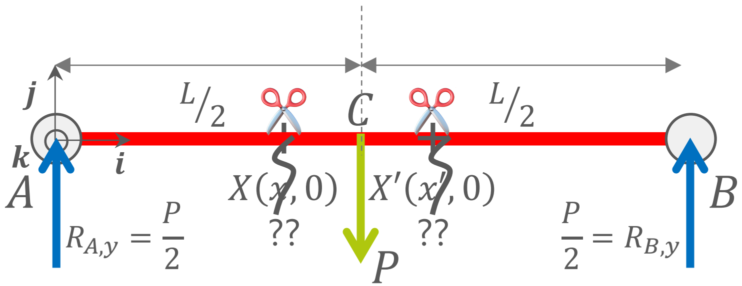
$$\hookrightarrow \frac{\mathrm{d}B_{X,z}}{\mathrm{d}x} = -S_{X,y}$$

How to to obtain the cohesion torsor?



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$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}$$

$$\Rightarrow \frac{\mathrm{d}B_{X,z}}{\mathrm{d}x} = -S_{X,y}$$

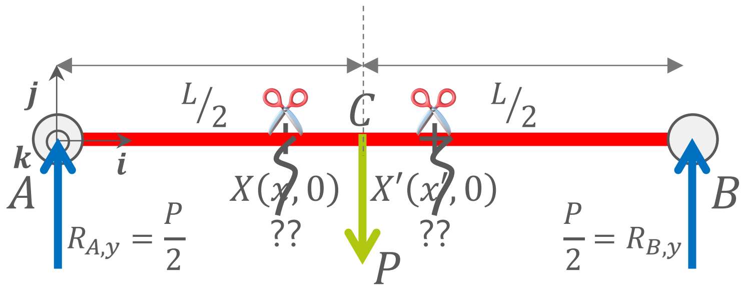
❖ **We express the torsor at point $C(\frac{L}{2}, 0)$ subjected to the external force $-P\boldsymbol{j}$:**

$$\{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}_C + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix}$$

External action $-P\boldsymbol{j}$; internal reaction $+P\boldsymbol{j}$
⇒ Newton's 3rd law

$$\Rightarrow \{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -\frac{P}{2} & 0 \\ 0 & \frac{P}{2} \times \frac{L}{2} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{PL}{4} \end{Bmatrix}$$

How to to obtain the cohesion torsor?



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$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} = \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **We transport the torsor from A to $X \in [A; C[$:**

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_X} = \boldsymbol{R_A} = -R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_X} = \boldsymbol{M_A} + \boldsymbol{XA}\times\boldsymbol{R_A} \end{Bmatrix}$$

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$$\looparrowright \frac{\mathrm{d}B_{X,z}}{\mathrm{d}x} = -S_{X,y}$$

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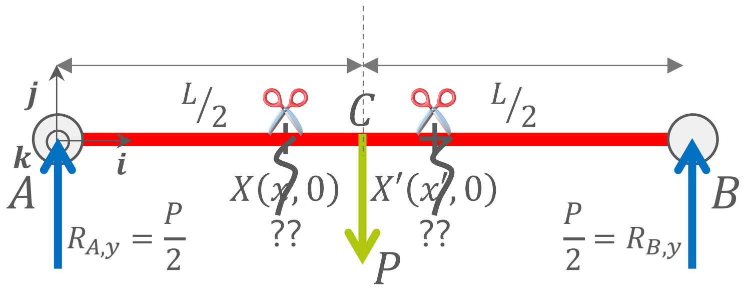
External action $-P\boldsymbol{j}$; internal reaction $+P\boldsymbol{j}$
⇒ Newton's 3rd law

$$\Rightarrow \{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -\frac{P}{2} & 0 \\ 0 & \frac{P}{2}\times\frac{L}{2} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{PL}{4} \end{Bmatrix}$$

❖ **We transport the torsor from C to $X' \in [C; B[$:**

$$\Rightarrow \{\mathcal{T}_{C\to X'}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_{X'}} = \frac{P}{2}\boldsymbol{j} \\ \boldsymbol{M_{X'}} = \frac{PL}{4}\boldsymbol{k} + \left(\frac{L}{2} - x'\right)\boldsymbol{i}\times\frac{P}{2}\boldsymbol{j} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{P}{2}(L - x') \end{Bmatrix}$$

How to to obtain the cohesion torsor?



❖ **Let's transform the torsor into cohesion torsor:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} = \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **We transport the torsor from A to $X \in [A; C[$:**

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_X} = \boldsymbol{R_A} = -R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_X} = \boldsymbol{M_A} + \boldsymbol{XA} \times \boldsymbol{R_A} \end{Bmatrix}$$

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}$$

$$\Rightarrow \frac{\mathrm{d}B_{X,z}}{\mathrm{d}x} = -S_{X,y}$$

❖ **We express the torsor at point $C(\frac{L}{2}, 0)$ subjected to the external force $-P\boldsymbol{j}$:**

$$\{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}_C + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix}$$

External action $-P\boldsymbol{j}$; internal reaction $+P\boldsymbol{j}$
⇒ Newton's 3rd law

$$\Rightarrow \{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -\frac{P}{2} & 0 \\ 0 & \frac{P}{2} \times \frac{L}{2} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{PL}{4} \end{Bmatrix}$$

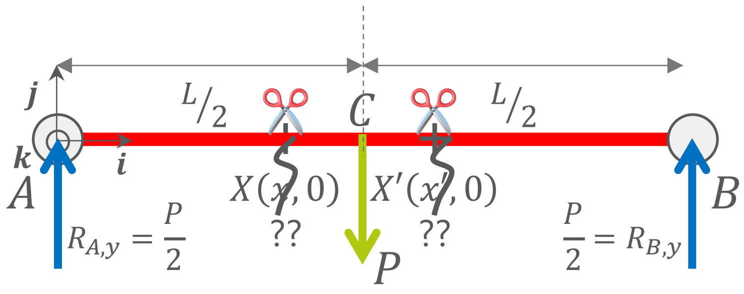
❖ **We transport the torsor from $\boldsymbol{C}$ to $\boldsymbol{X'} \in [\boldsymbol{C}; \boldsymbol{B}[$:**

$$\Rightarrow \{\mathcal{T}_{C\to X'}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_{X'}} = \frac{P}{2}\boldsymbol{j} \\ \boldsymbol{M_{X'}} = \frac{PL}{4}\boldsymbol{k} + \left(\frac{L}{2} - x'\right)\boldsymbol{i} \times \frac{P}{2}\boldsymbol{j} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{P}{2}(L - x') \end{Bmatrix}$$

❖ **And the torsor in $B(L; 0)$:**

$$\{\mathcal{T}_B^{coh}\} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix}$$

How to to obtain the cohesion torsor?



❖ **Let's transform the torsor into cohesion torsor:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} 0 & 0 \\ R_{A,y} = \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix} \Rightarrow \{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **We transport the torsor from A to $X \in [A; C[$:**

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_X} = \boldsymbol{R_A} = -R_{A,y}\boldsymbol{j} \\ \boldsymbol{M_X} = \boldsymbol{M_A} + \boldsymbol{XA}\times\boldsymbol{R_A} \end{Bmatrix}$$

$$\Rightarrow \{\mathcal{T}_{A\to X}^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}$$

$$\Rightarrow \frac{\mathrm{d}B_{X,z}}{\mathrm{d}x} = -S_{X,y}$$

❖ **We express the torsor at point $C(\frac{L}{2}, 0)$ subjected to the external force $-P\boldsymbol{j}$:**

$$\{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -R_{A,y} & 0 \\ 0 & R_{A,y}x \end{Bmatrix}_C + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix}$$

External action $-P\boldsymbol{j}$; internal reaction $+P\boldsymbol{j}$
⇒ Newton's 3rd law

$$\Rightarrow \{\mathcal{T}_C^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -\frac{P}{2} & 0 \\ 0 & \frac{P}{2}\times\frac{L}{2} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ P & 0 \\ 0 & 0 \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{PL}{4} \end{Bmatrix}$$

❖ **We transport the torsor from C to $X' \in [C; B[$:**

$$\Rightarrow \{\mathcal{T}_{C\to X'}^{coh}\} = \begin{Bmatrix} \boldsymbol{R_{X'}} = \frac{P}{2}\boldsymbol{j} \\ \boldsymbol{M_{X'}} = \frac{PL}{4}\boldsymbol{k} + \left(\frac{L}{2} - x'\right)\boldsymbol{i}\times\frac{P}{2}\boldsymbol{j} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & \frac{P}{2}(L-x') \end{Bmatrix}$$

❖ **And the torsor in $B(L; 0)$:**

$$\{\mathcal{T}_B^{coh}\} = \begin{Bmatrix} 0 & 0 \\ \frac{P}{2} & 0 \\ 0 & 0 \end{Bmatrix}$$

$$\Rightarrow \{\mathcal{T}_B^{coh}\} = \{\mathcal{T}_B\}$$

Evolution of of non-zero internal loads

Cohesion torsors:

$X \in [A;C[$

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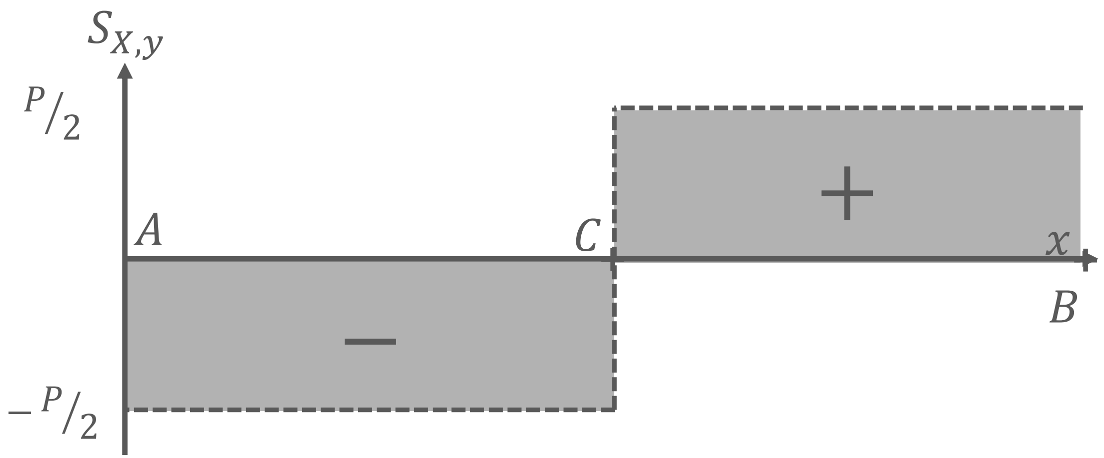
❖ Diagram of shear force along *j*



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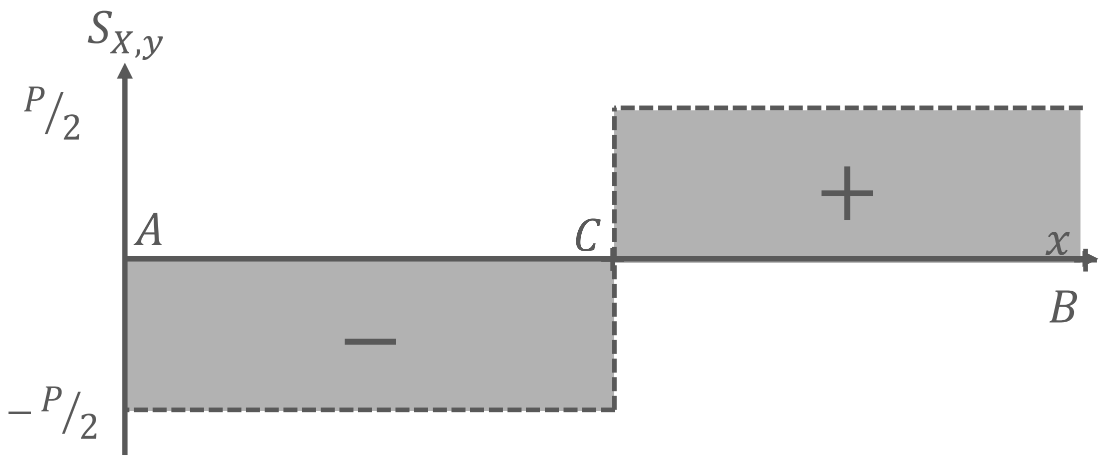

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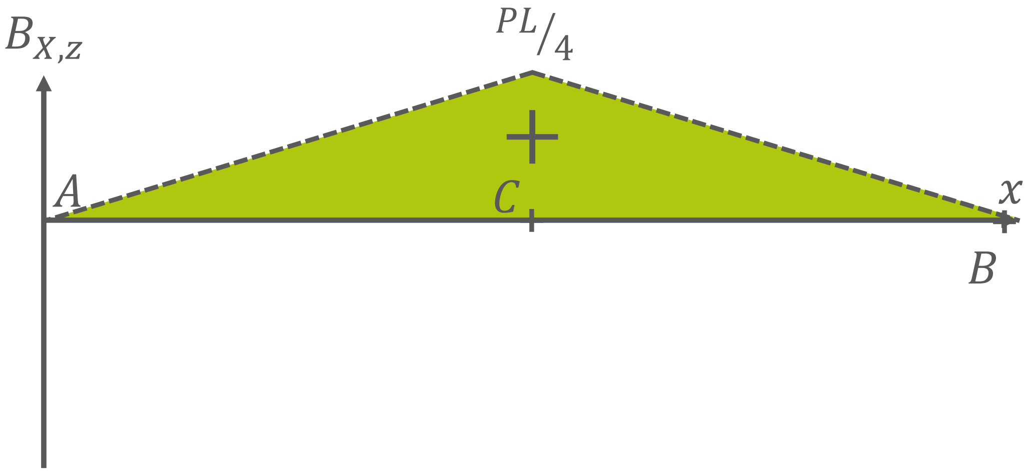

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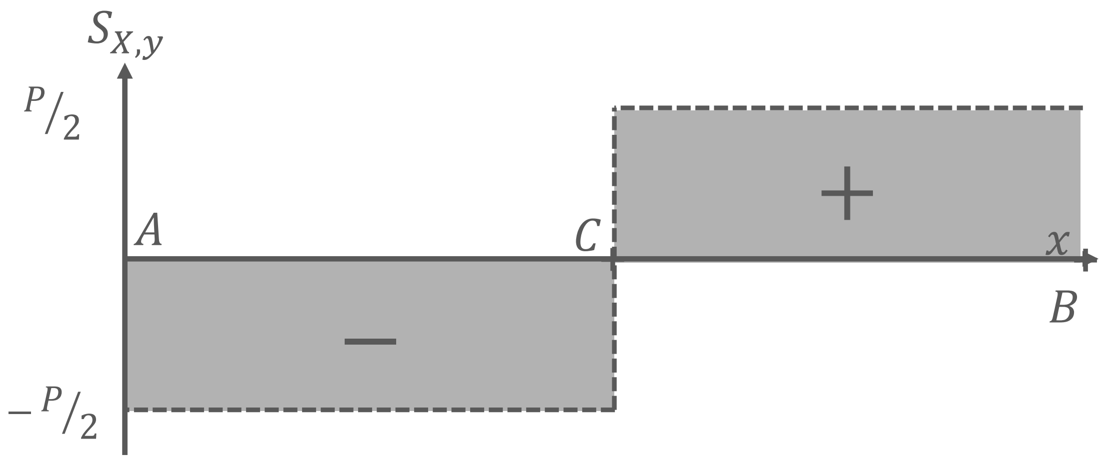

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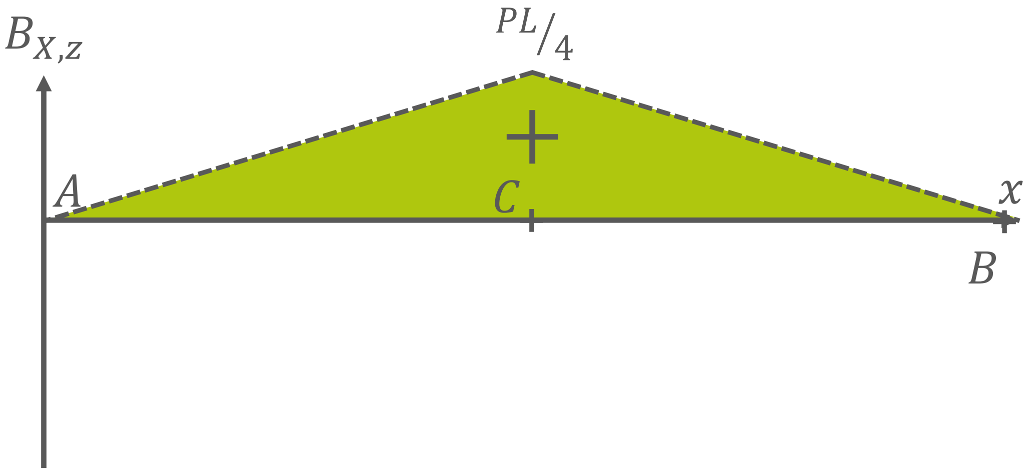
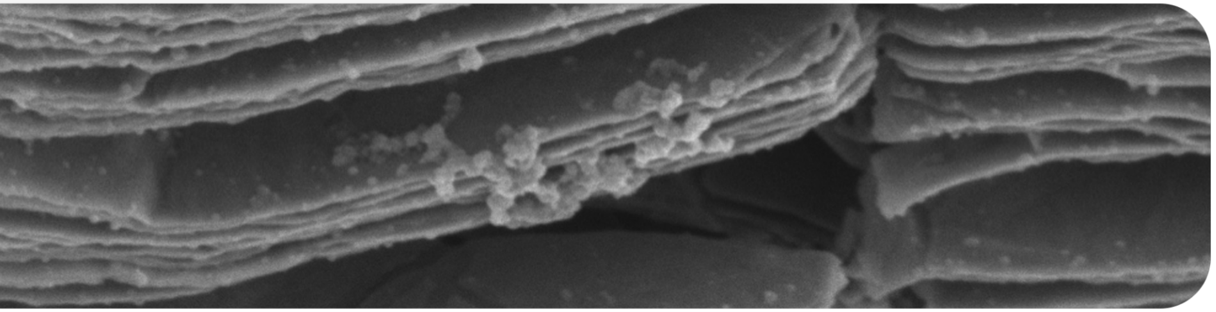


↳ **Large variations of internal loads show where the beam is working the most.**
↳ **These locations are typically the most critical and the most likely to fail.**



Stresses

What is stress?

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- The set of responses produced by an organism when subjected to pressures from its environment.
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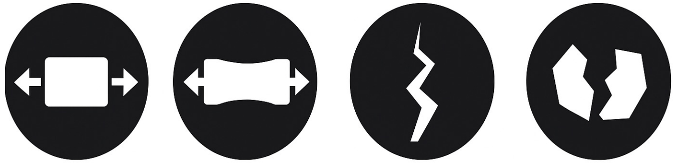

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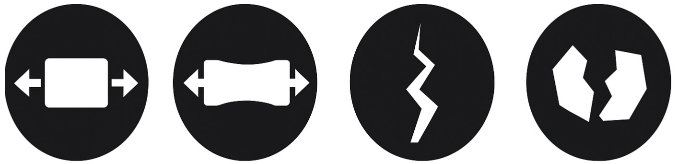

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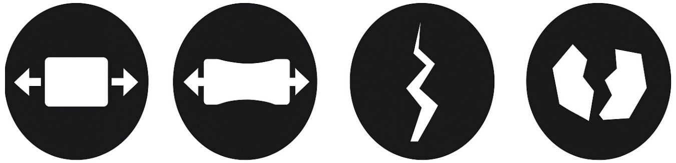

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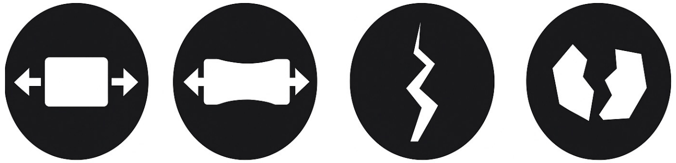

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↳ Stress is the response to pressure, in organisms as in materials.

↳ **It is not a measurement!**

How to use the stress?

⇒ *Designing structures*

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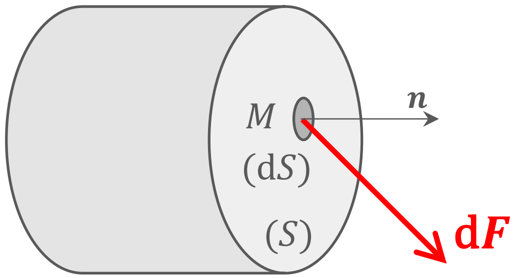
Design must use the allowable stress to avoid any unpextec failure.

The stress vector

❖ What happens locally?



- M is a point on the surface (S).
- $\mathrm{d}S$ is an infinitesimally small surface around M.
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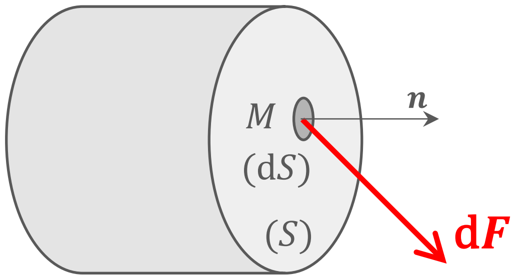

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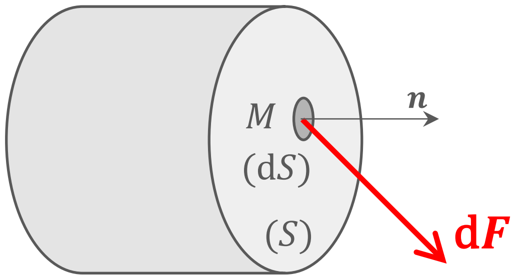

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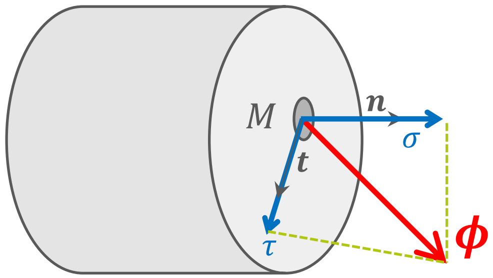

$$\boldsymbol{\phi}(M,\boldsymbol{n}) = \sigma\boldsymbol{n} + \tau\boldsymbol{t}$$

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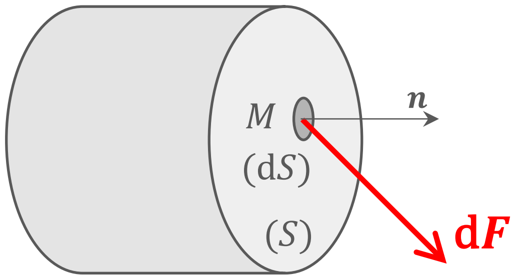

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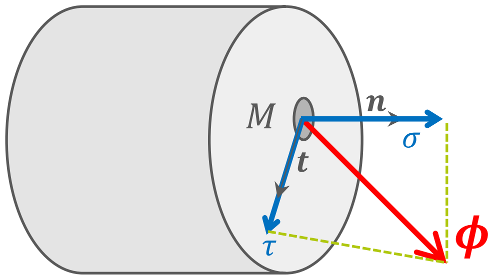

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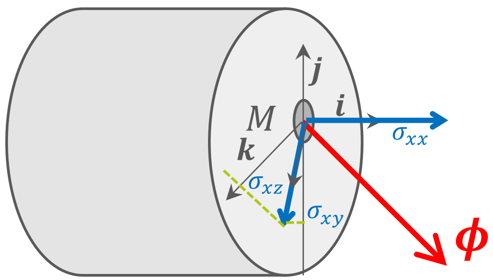
$$\boldsymbol{\phi}(M,\boldsymbol{n}=\boldsymbol{i}) = \sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k}$$

σ_{ij}: Stress on the face whose normal is $\boldsymbol{i}$ in direction $\boldsymbol{j}$.



Relationship between ϕ and the internal forces

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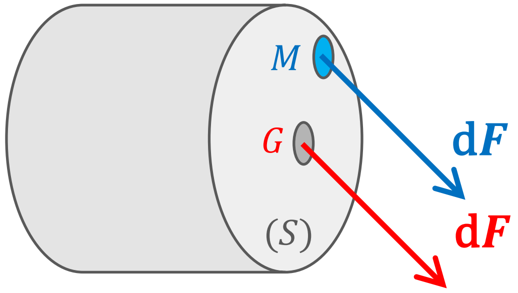

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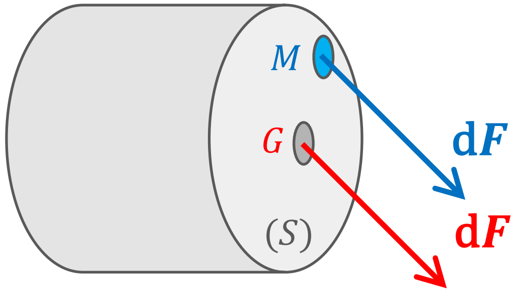

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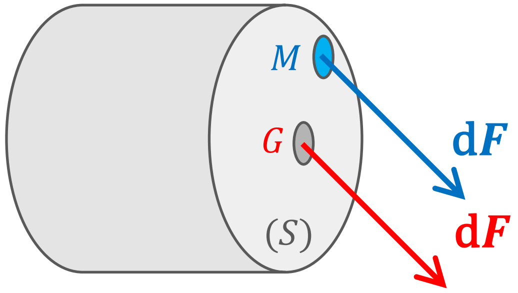

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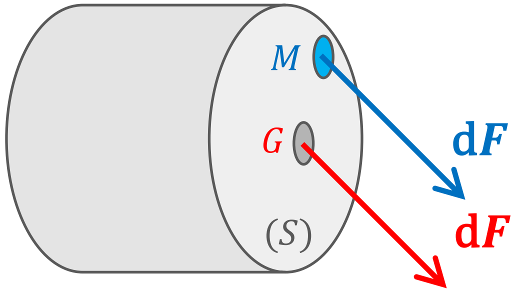

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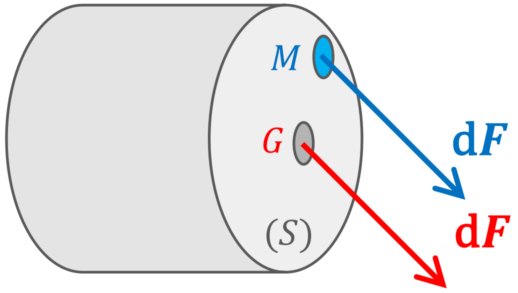

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BUT $\boldsymbol{\phi}(M,\boldsymbol{n}=\boldsymbol{i}) = \sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k}$ AND $\boldsymbol{GM} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

- **After integration over (S):**

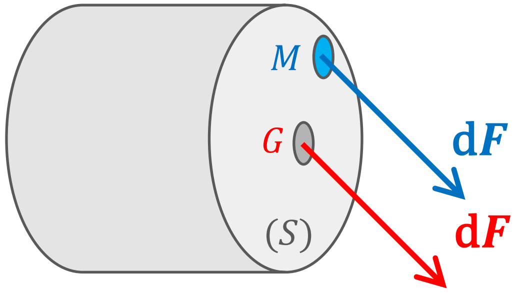
$$\{\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \boldsymbol{R}_G = \iint (\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \\ \boldsymbol{M}_G = \iint \boldsymbol{GM}\times(\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \end{Bmatrix} = \begin{Bmatrix} N_G = \iint \sigma_{xx}\mathrm{d}S & T_G = \iint (y\sigma_{xz} - z\sigma_{xy})\mathrm{d}S \\ S_{G,y} = \iint \sigma_{xy}\mathrm{d}S & B_{G,y} = \iint (z\sigma_{xz} - x\sigma_{xz})\mathrm{d}S \\ S_{G,z} = \iint \sigma_{xz}\mathrm{d}S & B_{G,z} = \iint (x\sigma_{xy} - y\sigma_{xx})\mathrm{d}S \end{Bmatrix}$$

➯ Very complex. Additional assumptions are needed about how stresses are distributed over (S).

Relationship between ϕ and the internal forces

- **Cohesion torsor in M:**

$$\{\mathcal{T}_M^{coh}\} = \begin{Bmatrix} \boldsymbol{R}_M = \mathrm{d}\boldsymbol{F} \\ \boldsymbol{M}_M = \boldsymbol{0} \end{Bmatrix}$$



- **Let's transport this torsor into G near M:**

$$\{\mathrm{d}\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R}_G = \boldsymbol{R}_M = \mathrm{d}\boldsymbol{F} \\ \mathrm{d}\boldsymbol{M}_G = \boldsymbol{0} + \boldsymbol{GM}\times\boldsymbol{R}_M \end{Bmatrix} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R}_G = \mathrm{d}\boldsymbol{F} \\ \mathrm{d}\boldsymbol{M}_M = \boldsymbol{GM}\times\mathrm{d}\boldsymbol{F} \end{Bmatrix}$$

BUT $\boldsymbol{\phi}(M,\boldsymbol{n}) = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}S} \Rightarrow \mathrm{d}\boldsymbol{F} = \boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S$

$$\Rightarrow \{\mathrm{d}\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \mathrm{d}\boldsymbol{R}_G = \boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S \\ \mathrm{d}\boldsymbol{M}_G = \boldsymbol{GM}\times\boldsymbol{\phi}(M,\boldsymbol{n})\mathrm{d}S \end{Bmatrix}$$

BUT $\boldsymbol{\phi}(M,\boldsymbol{n}=\boldsymbol{i}) = \sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k}$ AND $\boldsymbol{GM} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

- **After integration over (S):**

$$\{\mathcal{T}_G^{coh}\} = \begin{Bmatrix} \boldsymbol{R}_G = \iint (\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \\ \boldsymbol{M}_G = \iint \boldsymbol{GM}\times(\sigma_{xx}\boldsymbol{i} + \sigma_{xy}\boldsymbol{j} + \sigma_{xz}\boldsymbol{k})\mathrm{d}S \end{Bmatrix} = \begin{Bmatrix} N_G = \iint \sigma_{xx}\mathrm{d}S & T_G = \iint (y\sigma_{xz} - z\sigma_{xy})\mathrm{d}S \\ S_{G,y} = \iint \sigma_{xy}\mathrm{d}S & B_{G,y} = \iint (z\sigma_{xz} - x\sigma_{xz})\mathrm{d}S \\ S_{G,z} = \iint \sigma_{xz}\mathrm{d}S & B_{G,z} = \iint (x\sigma_{xy} - y\sigma_{xx})\mathrm{d}S \end{Bmatrix}$$

(Chap #2: N_G; Chap #4: T_G; Chap #3: $S_{G,y}$, $S_{G,z}$; Chap #5: $B_{G,y}$, $B_{G,z}$)

Very complex. Additional assumptions are needed about how stresses are distributed over (S).

Thanks for your listening!

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