



COE–3001: Mechanics of deformable bodies

Chapter 0: introduction

Prof. Antoine GUITTON

Université de Lorraine, CNRS, Arts et Métiers Institute of Technology, LEM3, F-57000 Metz,
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Disclaimer

❖ I am neither English nor American...



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- [th] = [z]
- [h] = ?
- [r] = ?
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- It's amazing! = C'est pas mauvais (that's not bad)



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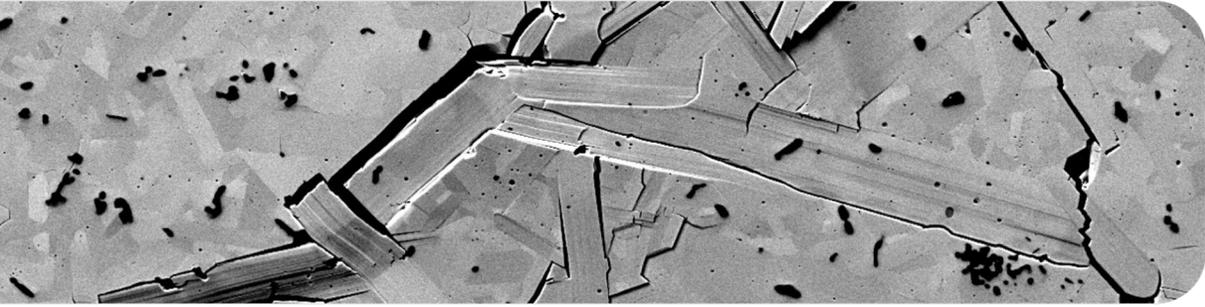
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❖ Exams:

- 1 partial class exam: 60 min; 33%
- 1 final exam: 2h50; 66%





Who am I?

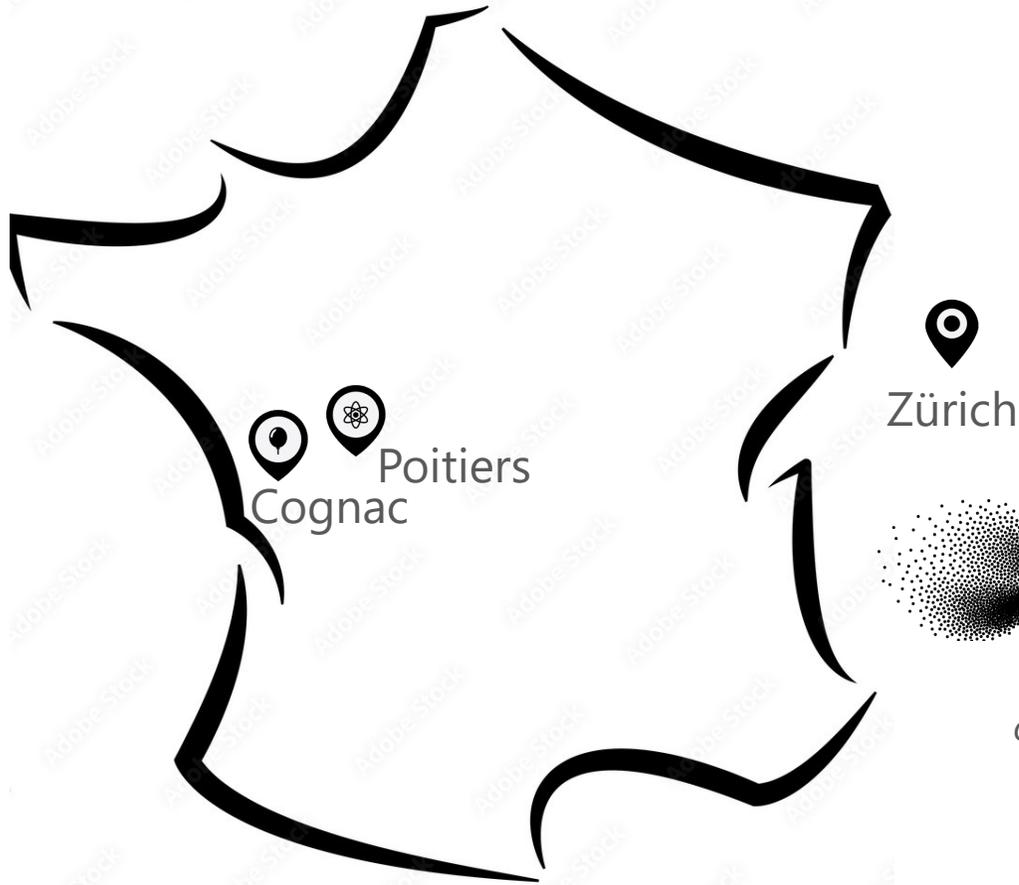
Who am I?

- From ~Cognac 🇫🇷
- Higher education in Poitiers 🇫🇷
- PhD in 2013 : *“Deformation mechanisms of MAX phases; a multiscale and aexperimental approach.”*

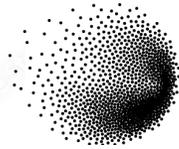


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Zürich



PSI EPFL

“Influence of strain-path changes on metal plasticity.”



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- Adjunct Professor at Georgia Institute of Technology (Atlanta 🇺🇸) since 2023



My laboratory

Laboratory for the Study of Microstructures and Mechanics of Materials (LEM3)

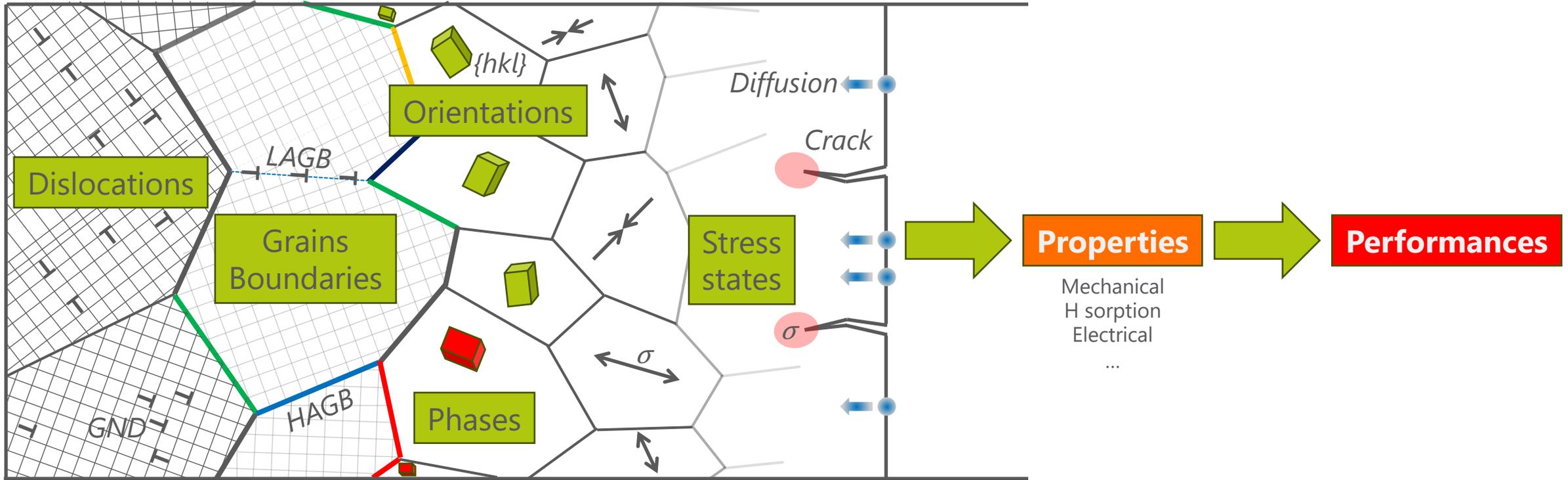


- Supervising institutions: Université de Lorraine, CNRS, Arts et Métiers
- 250 members
- 3 departments:
 - Mechanics of Materials, Structures, and Living Matter
 - **Microstructure Engineering, Anisotropy, and Behavior**
 - Thermomechanics of Processes and Tool-Material Interactions



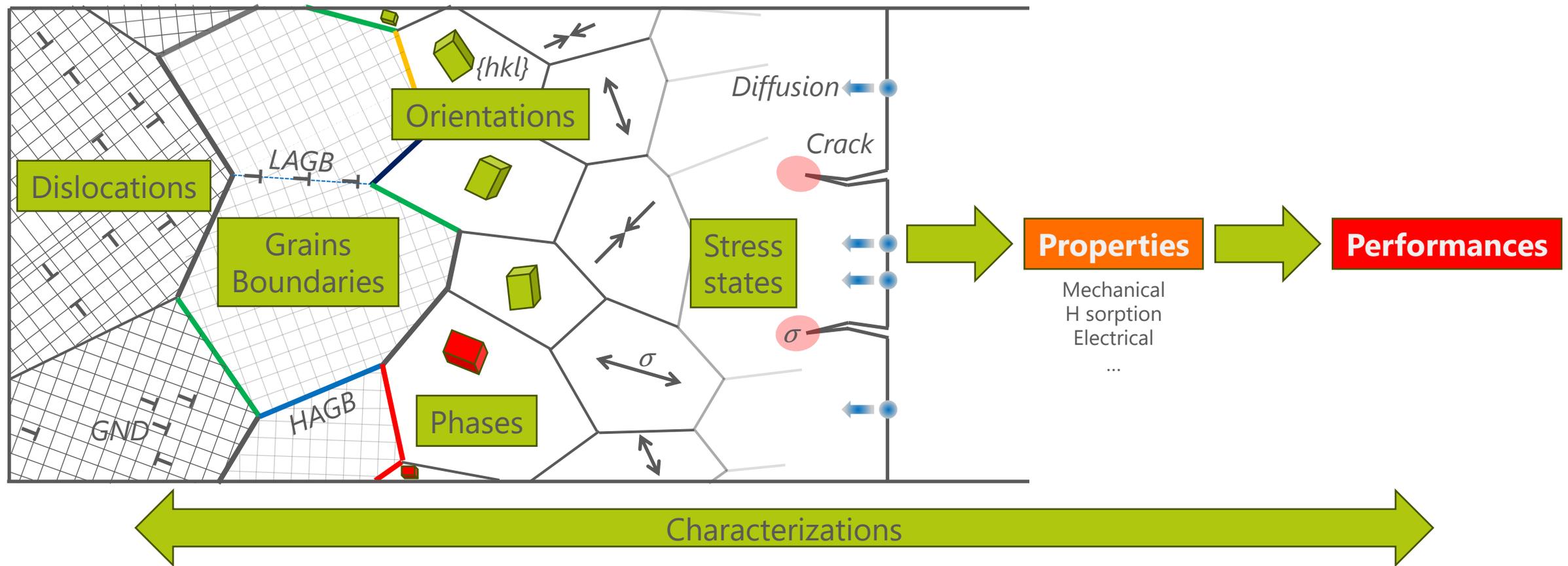
My research overview

Understanding and optimizing the physical properties of materials through the study and characterization of their defects.



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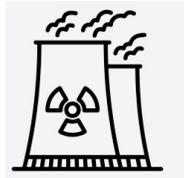
My research activities

Understanding and optimizing the physical properties of materials through the study and characterization of their defects.

Deformation properties

- Optimization of engines
- Service performance after irradiation
- Impact resistance

Metals, Ceramics



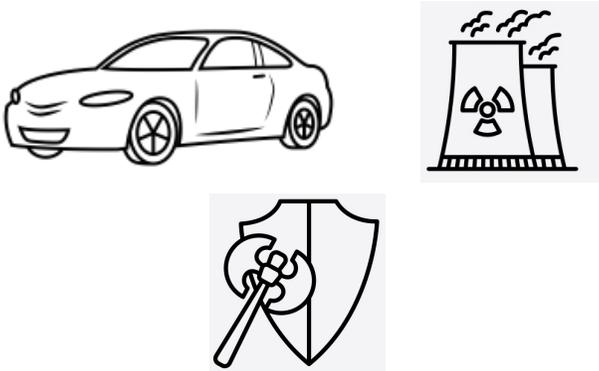
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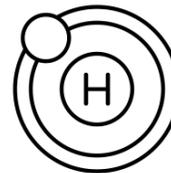
Metals, Ceramics



Hydrogen storage properties

- Optimization of storage/release capacities
- Optimization of storage/release rates

Metals



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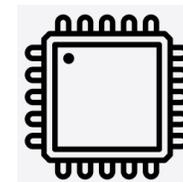
Metals

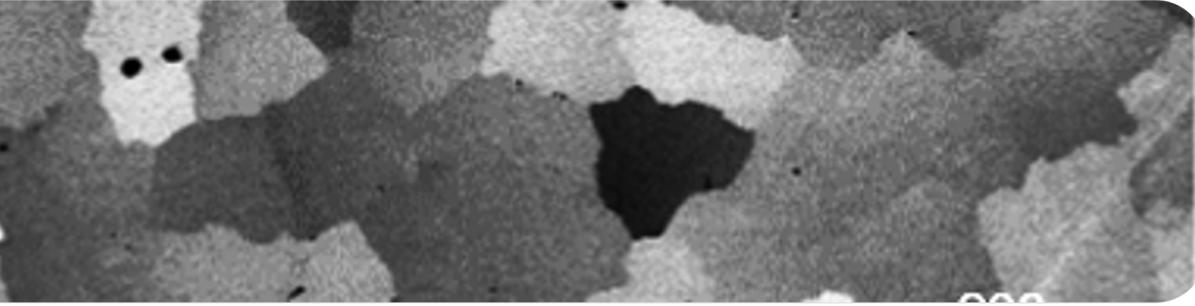


Electrical properties

- Defect detection
- Relationship between defects and electrical conductivity

Semiconductors





Mathematical tools



Why units matter?

Mars climate orbiter (1998)

❖ Specifications:

- Use metric system for software
- Use metrics system for engines

Why units matter?



* No, Mycroft... 30 inches are not the same as 30 cm...

Mars climate orbiter (1998)

❖ Specifications:

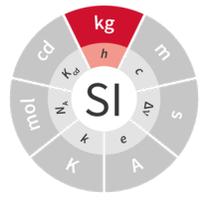
- Use metric system for software
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❖ Results:

- Metric system for software
- Imperial system for engines (British company)
- \$ 300'000'000 for nothing...

↪ An expensive mistake!

The international system of units (ISU)



Name: kilogram
 Symbol: kg
 Quantity: mass



Name: meter
 Symbol: m
 Quantity: length



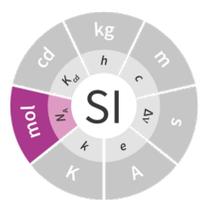
Name: second
 Symbol: s
 Quantity: time



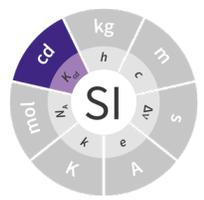
Name: ampere
 Symbol: A
 Quantity: intensity of electric current



Name: kelvin
 Symbol: K
 Quantity: temperature



Name: mole
 Symbol: mol
 Quantity: amount of substance



Name: candela
 Symbol: cd
 Quantity: luminous intensity

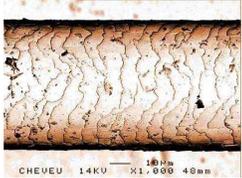
Conversion to ISU

Name	Conversion to ISU
Tera- (T-)	10^{12}
Giga- (G-)	10^9
Mega- (M-)	10^6
Kilo- (k-)	10^3
Milli- (m-)	10^{-3}
Micro- (μ -)	10^{-6}
Nano- (n-)	10^{-9}
Pico- (p-)	10^{-12}

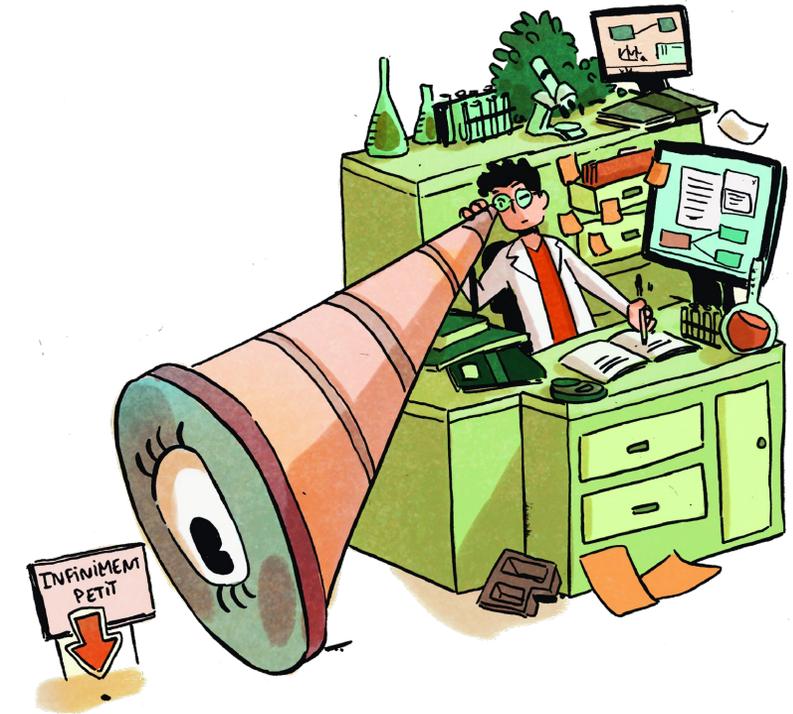


Some orders of magnitude

❖ Micrometer ($1 \mu\text{m} = 0.000001 \text{ m} = 10^{-6} \text{ m}$)

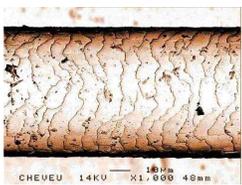


- Hair diameter = 50-100 μm
- Size of a bacteria = 0.1-10 μm



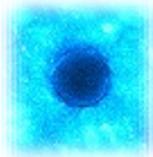
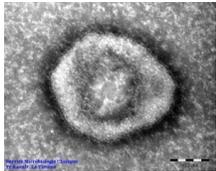
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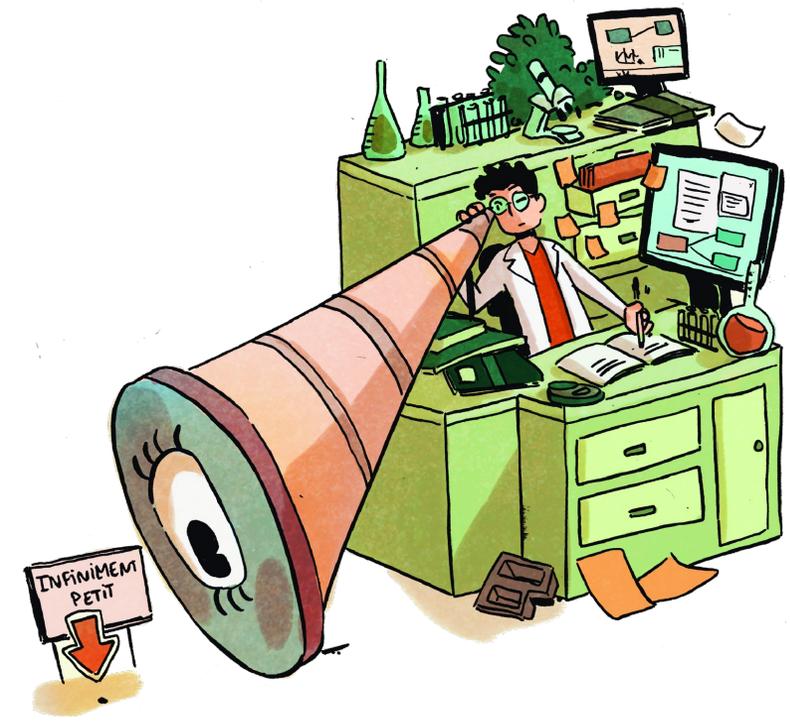


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❖ Nanometer (1 nm = 0.000000001 m = 10⁻⁹ m)

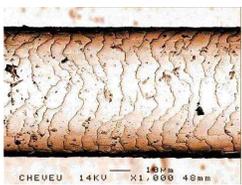


- Size of a virus = 20-450 nm



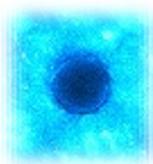
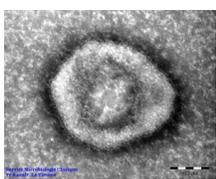
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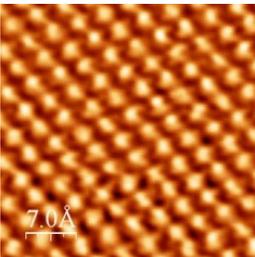
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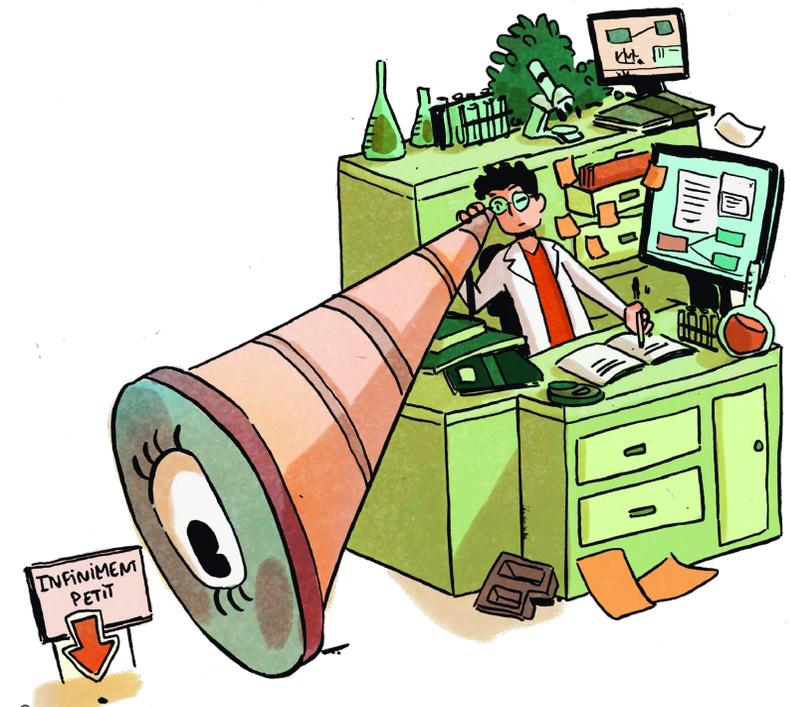


- Size of a virus = 20-450 nm

❖ Atomic scale (1 Å = 0.0000000001 m = 10⁻¹⁰ m)



- Distance between atoms = 1 nm = 10 Å
- Atom radius = 0.1 nm = 1 Å



This is neither copper nor gold!!!

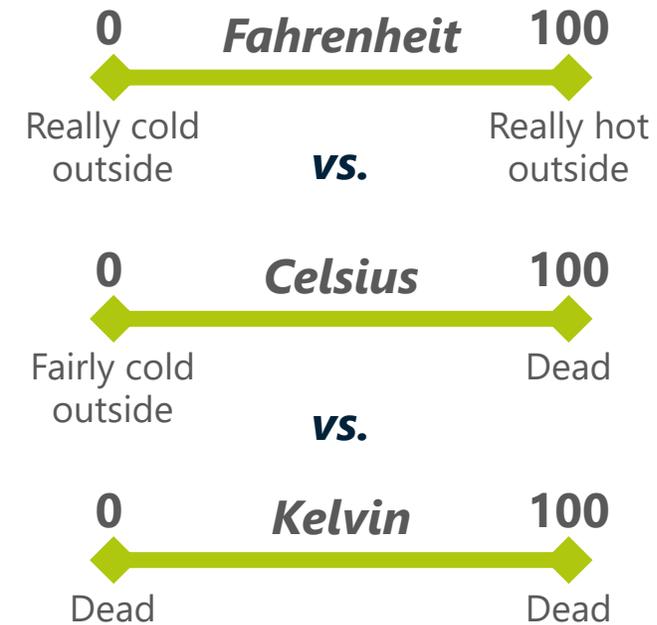
Weird units but useful sometimes

Name	Conversion to ISU
Angström (Å)	10^{-10} m
Electronvolt (eV)	$1.60218 \cdot 10^{-19}$ J
Bar (bar)	10^5 Pa
Celsius (°C)	$T(K) + 273.15$ K
Degree (°)	$\frac{\text{rad} \times 180}{\pi}$
Liter (L)	10^{-3} m ³
Calorie (cal)	4.18 J

↪ For calculations, convert to SI units!

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Liter (L)	10^{-3} m ³
Calorie (cal)	4.18 J
Fahrenheit (°F)	<i>Forget it...</i>
Inch (In)	<i>Forget it...</i>
Gallon (gal)	<i>Forget it...</i>



↪ For calculations, convert to SI units!

Exercise 1: Homogeneity of an equation

What is the unit of the quantity A ?

$$A = \sqrt{2} \frac{S_1}{S_2} \left(\sqrt{\frac{M}{\rho S_1 g}} - \sqrt{\frac{\frac{M}{\rho S_1} - h}{g}} \right)$$

(S_1 and S_2 : surfaces; M : mass; ρ : density; g : gravity acceleration; h : height)

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Correction:

$$[A] \mapsto \frac{[m^2]}{[m^2]} \left\{ \left(\frac{[kg]}{[kg \cdot m^{-3}][m^2][m \cdot s^{-2}]} \right)^{\frac{1}{2}} - \left(\frac{\frac{[kg]}{[kg \cdot m^{-3}][m^2]} - [m]}{[m \cdot s^{-2}]} \right)^{\frac{1}{2}} \right\}$$

$$[A] \mapsto (s^2)^{\frac{1}{2}} - \left(\frac{[m]}{[m \cdot s^{-2}]} \right)^{\frac{1}{2}}$$

$$[A] \mapsto \left(([s^2])^{\frac{1}{2}} - ([s^2])^{\frac{1}{2}} \right) \mapsto [s]$$

Exercise 2: Some orders of magnitude

Assign the correct orders of magnitude and units to the quantities below.

Atmospheric pressure	●	●	$2 \cdot 10^6$	●	●	J
Surface temperature of the sun	●	●	$1 \cdot 10^5$	●	●	Pa
Number of atoms in 6 g of aluminum	●	●	5778	●	●	m^3
Volume of a glass of water	●	●	$1.3 \cdot 10^{23}$	●	●	W
Number of moles in 6 g of aluminum	●	●	$3.7 \cdot 10^5$	●	●	K
Power of a sport car	●	●	0.2	●	●	mol
Energy content of a hamburger	●	●	$2.6 \cdot 10^{-4}$	●	●	⊘

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Trigonometry

Volume V of a pizza with radius z and thickness a :

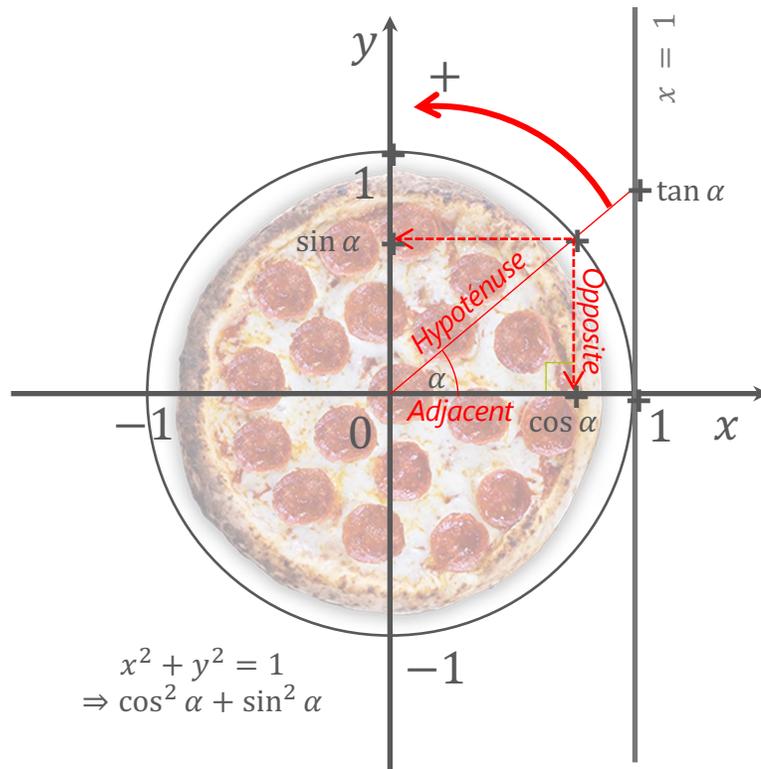
$$V = \pi z^2 a = \text{pi } z z a$$



Trigonometry

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❖ **Cosine:**

$$\cos \alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

❖ **Sine:**

$$\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$$

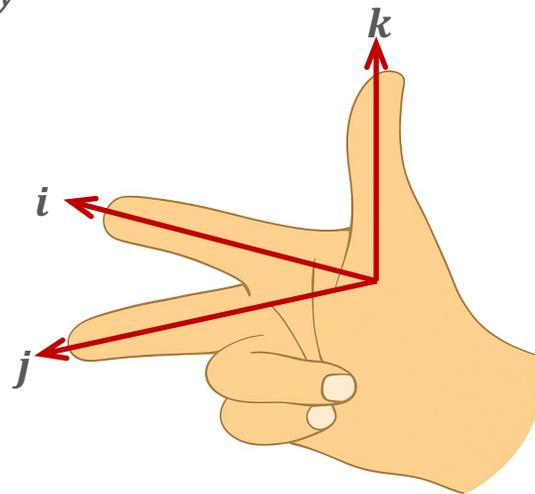
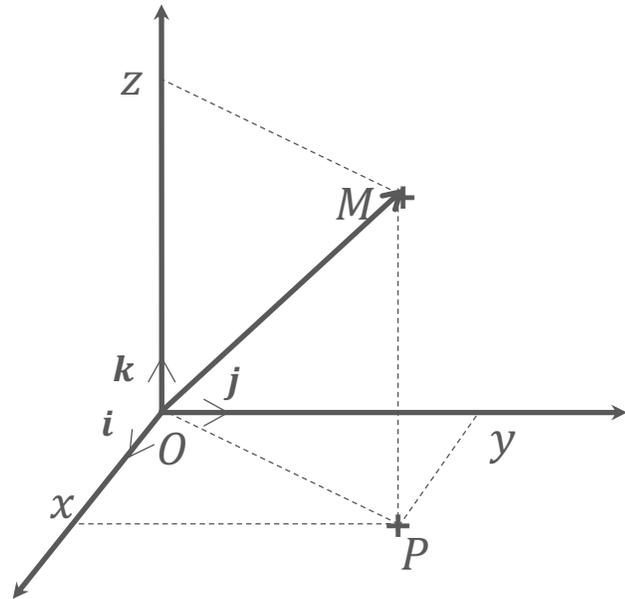
❖ **Tangent:**

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\sin \alpha}{\cos \alpha}$$

❖ **Angle conversion:**

$$\alpha [\text{rad}] = \frac{\alpha [\text{deg}] \times \pi}{180}; \quad \alpha [\text{deg}] = \frac{\alpha [\text{rad}] \times 180}{\pi}$$

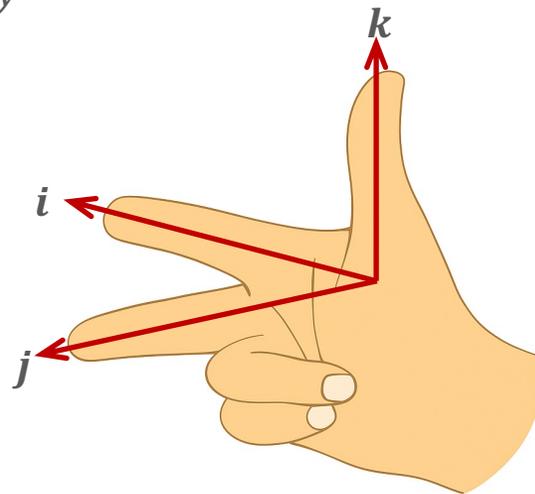
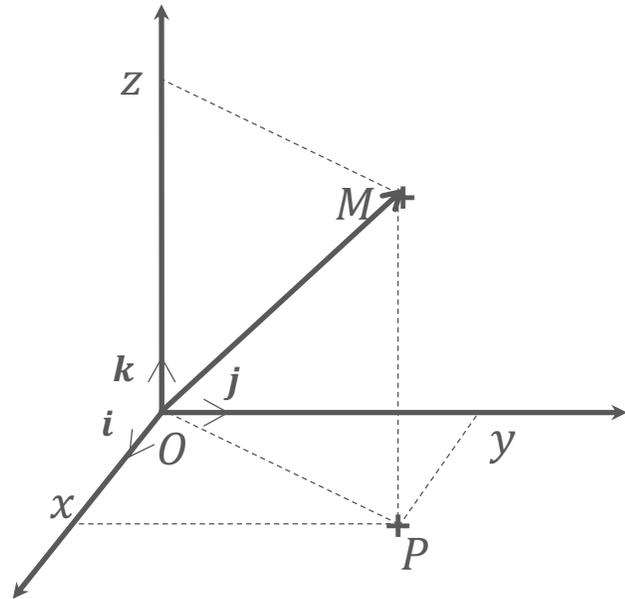
Review of Cartesian coordinates



❖ Coordinate system:

- Defined by three mutually orthogonal unit vectors $\mathcal{R} = (O, \mathbf{i}, \mathbf{j}, \mathbf{k})$:
 - \mathbf{i} : unit vector along x
 - \mathbf{j} : unit vector along y
 - \mathbf{k} : unit vector along z
- Such as $\mathbf{k} = \mathbf{i} \times \mathbf{j}$ and $\|\mathbf{i}\| = \|\mathbf{j}\| = \|\mathbf{k}\| = 1$
 - ⇒ This is a direct orthonormal basis.

Review of Cartesian coordinates



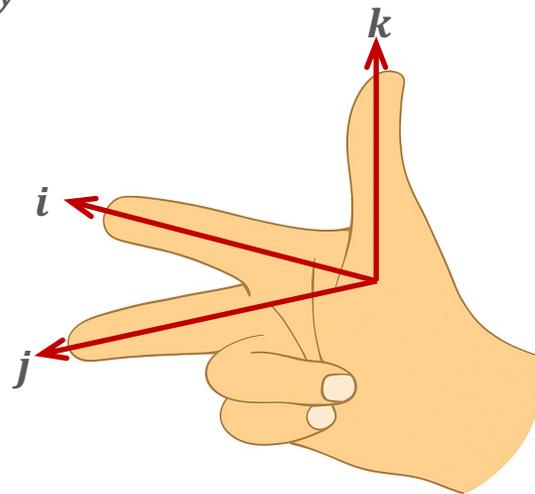
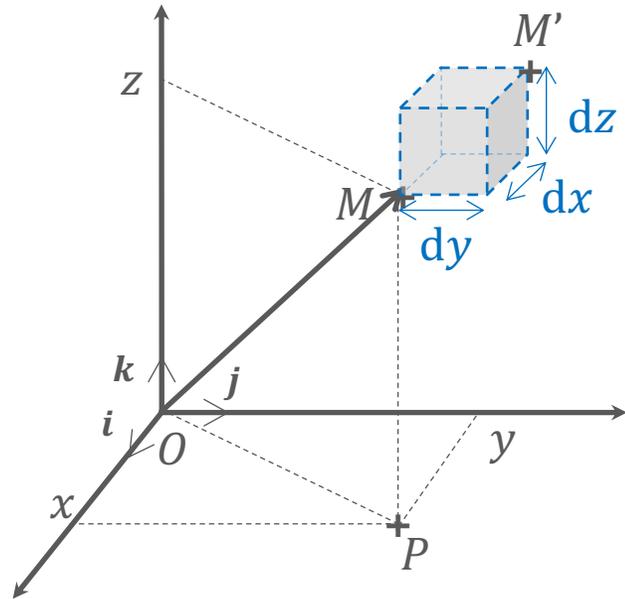
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❖ Position vector:

$$\mathbf{OM} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Review of Cartesian coordinates



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$$\mathbf{OM} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

❖ Elementary displacement $M \rightarrow M'$:

$$d\ell = \mathbf{MM}' = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

❖ Elementary surface:

$$dS = dx dy = dy dz = dx dz$$

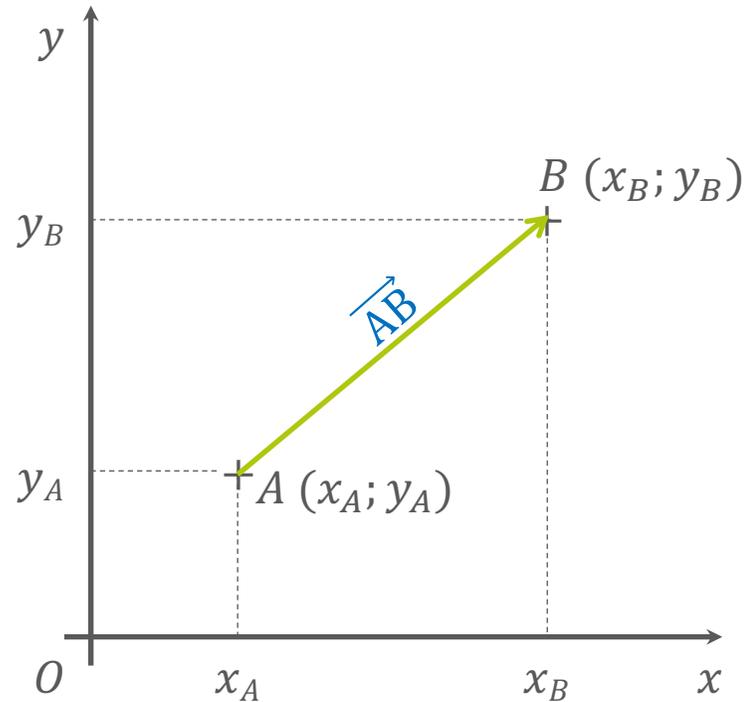
❖ Elementary volume:

$$dV = dx dy dz$$

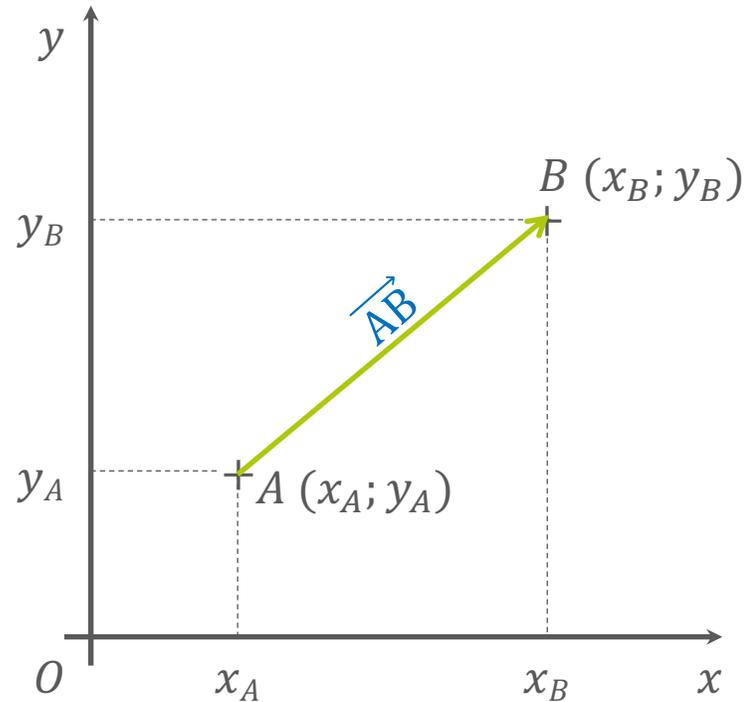
Vectors

❖ A vector is defined by:

- Its direction (vertical, horizontal...)
- Its orientation (from bottom to top...)
- Its magnitude (12 m).



Vectors



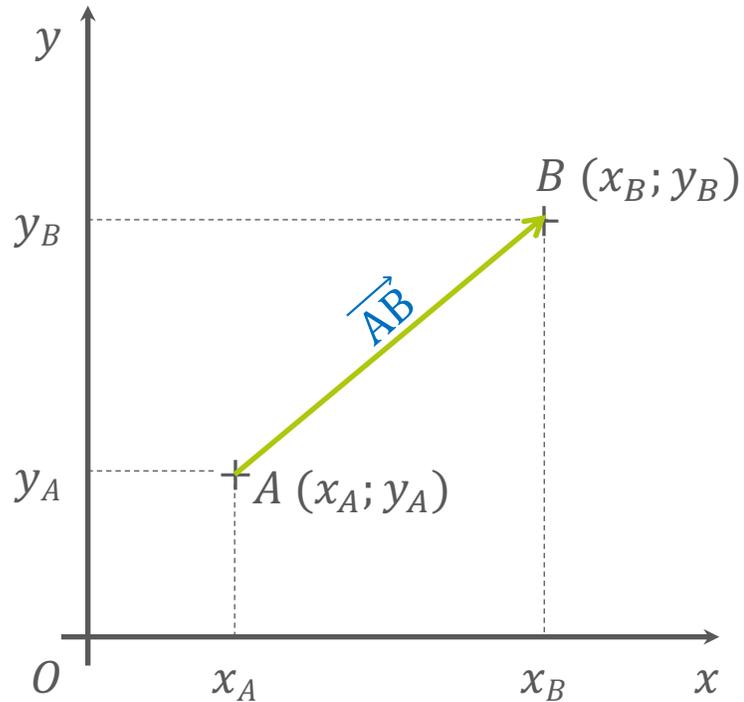
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❖ Coordinates of the vector AB (\vec{AB}):

$$\vec{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix} \begin{array}{l} \longrightarrow \text{along } i \\ \longrightarrow \text{along } j \\ \longrightarrow \text{along } k \end{array}$$

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❖ Norm of the vector \overrightarrow{AB} :

$$\|\overrightarrow{AB}\| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

Vector calculations in an orthonormal basis

❖ For two vectors $\in \mathbb{R}^3$:

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

❖ Dot product:

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \sum_{i=1}^3 a_i b_i$$

↪ (vector 1; vector 2) → 1 scalar value

Vector calculations in an orthonormal basis

❖ For two vectors $\in \mathbb{R}^3$:

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

❖ Dot product:

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \sum_{i=1}^3 a_i b_i$$

↪ (vector 1; vector 2) → 1 scalar value

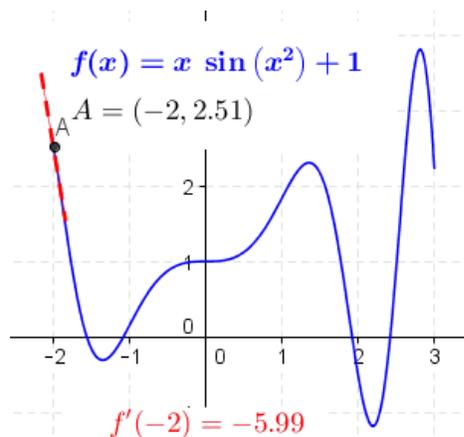
❖ Cross product:

$$\mathbf{c} = \mathbf{u} \times \mathbf{v} = \mathbf{u} \wedge \mathbf{v} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}; c_i = \epsilon_{ijk} a_j b_k$$

$$\text{where } \epsilon_{ijk} = \begin{cases} +1, & \text{if } (i, j, k) \text{ is an even permutation} \\ -1, & \text{if } (i, j, k) \text{ is an odd permutation} \\ 0, & \text{if any two indices are equal} \end{cases} \quad (\text{Levi-Civita})$$

↪ (vector 1; vector 2) → 1 vector

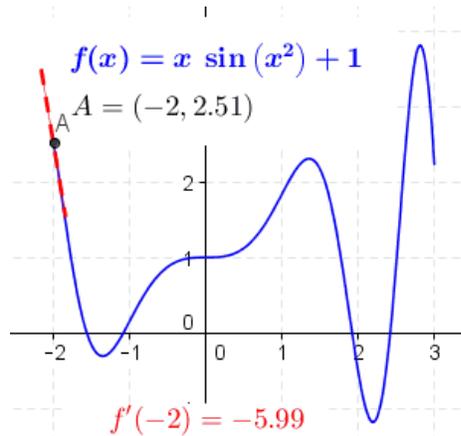
Derivative and integral



❖ Derivative at point A

- ⇒ Slope of the tangent at point A.
- Positive when the curve is increasing.
 - Negative when the curve is decreasing.
 - Zero when the curve reaches a local extremum.

Derivative and integral

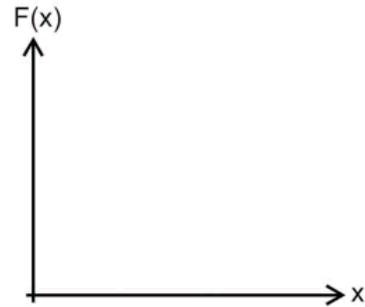


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❖ Integral

- ⇒ Area under the curve.
- Positive when the curve lies above the x -axis.
 - Negative when the curve lies below the x -axis.
 - Zero when the positive and negative areas cancel each other out.



Mechanical actions

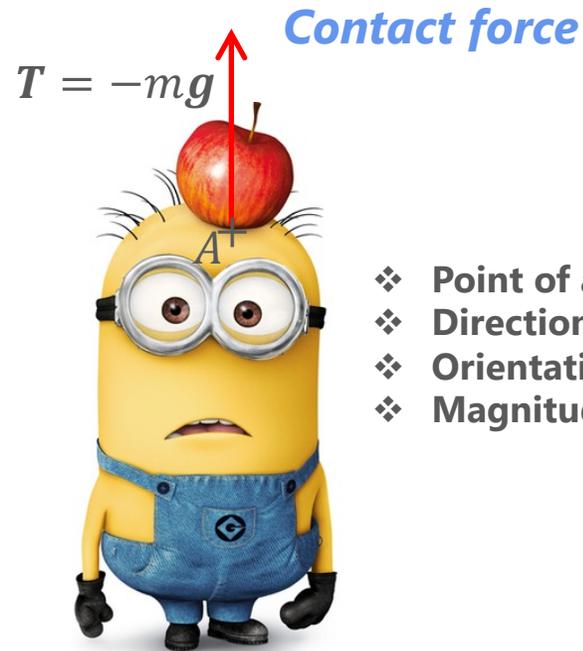
❖ Force:

- Modeling an interaction.
- Resulting from the action of one object on another.
- Represented by a vector (a direction, an orientation, and a magnitude or intensity) with a point of application.

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- ❖ **Point of application:** A
- ❖ **Direction:** vertical
- ❖ **Orientation:** upward
- ❖ **Magnitude:** $m \times g$

Mechanical actions

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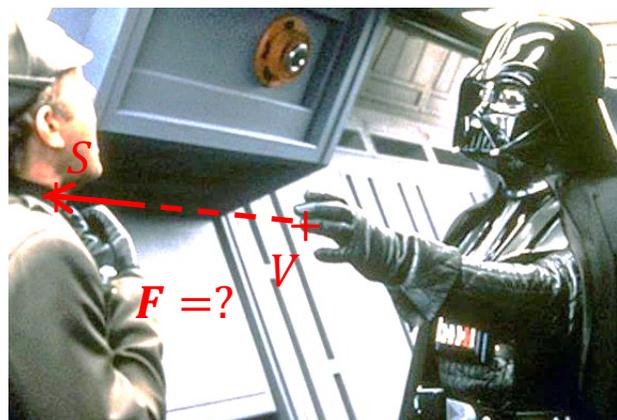
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$T = -mg$ *Contact force*



- ❖ **Point of application:** A
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Distance force



- ❖ **Point of application:** S
- ❖ **Direction:** along (VS)
- ❖ **Orientation:** From V to S
- ❖ **Magnitude:** ?

Mechanical actions

❖ Moment:

- Physical quantity describing the ability of a force F applied at point P to make an object rotate around a point O .
- Represented by a pseudo-vector expressed as:

$$\mathbf{M}_{F/O} = \mathbf{OP} \times \mathbf{F}$$

Mechanical actions

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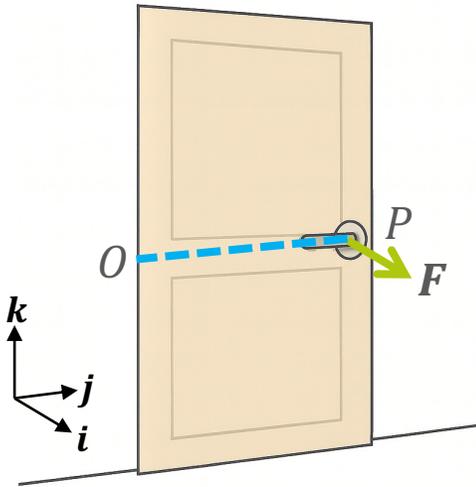
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How to open a door efficiently?

❖ Let's identify:

- Force
 - $\mathbf{F} = F\mathbf{i}$
 - Application point: P
- Lever arm
 - $\mathbf{OP} = l\mathbf{j}$



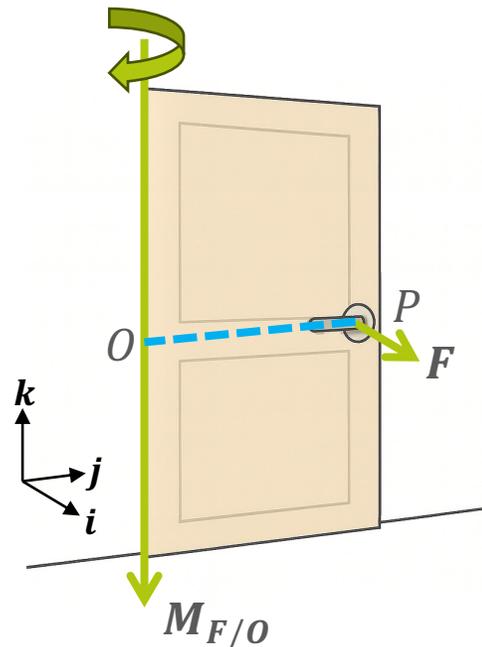
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How to open a door efficiently?



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- Lever arm
 - $OP = lj$

❖ Moment calculation:

$$\mathbf{M}_{F/O} = \mathbf{OP} \times \mathbf{F} = lj \times Fi = -lFk$$

⇒ clockwise rotation when viewed from above.

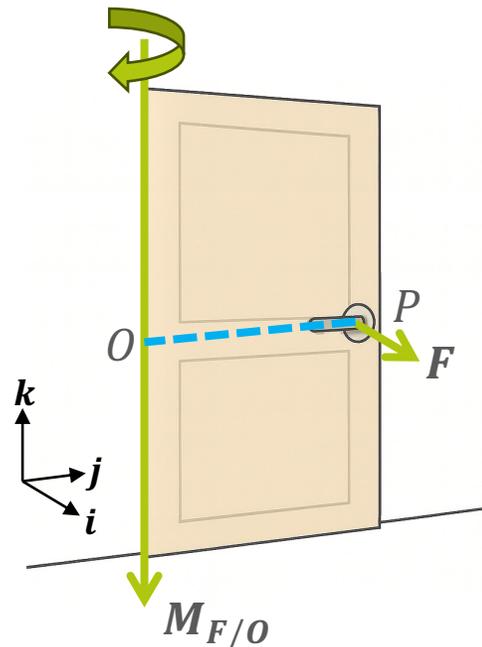
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⇒ clockwise rotation when viewed from above.

⇒ Therefore, to open a door efficiently, you must maximize the lever arm. Apply the force as far from the hinges as possible.

The static tensor (wrench)

❖ The mechanical actions of a system, expressed as a resultant force (\mathbf{R}_A) and a resultant moment (\mathbf{M}_A) at point A in a given reference frame, can be represented compactly using a table known as the static torsor $\{\mathcal{T}_A\}$.

$$\{\mathcal{T}_A\} = \left\{ \begin{array}{l} \mathbf{R}_A = R_{A,x}\mathbf{x} + R_{A,y}\mathbf{y} + R_{A,z}\mathbf{z} \\ \mathbf{M}_A = M_{A,x}\mathbf{x} + M_{A,y}\mathbf{y} + M_{A,z}\mathbf{z} \end{array} \right\}$$

$$\{\mathcal{T}_A\} = \left\{ \begin{array}{ll} R_{A,x} & M_{A,x} \\ R_{A,y} & M_{A,y} \\ R_{A,z} & M_{A,z} \end{array} \right\}$$

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❖ To move a torsor from point A to point B , the transport equation must be used:

$$\{\mathcal{T}_A\} \rightarrow \{\mathcal{T}_B\} = \begin{Bmatrix} \mathbf{R}_B = \mathbf{R}_A = \mathbf{R} \\ \mathbf{M}_B = \mathbf{M}_A + \mathbf{BA} \times \mathbf{R} \end{Bmatrix}$$

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & M_{A,x} \\ R_{A,y} & M_{A,y} \\ R_{A,z} & M_{A,z} \end{Bmatrix}$$



Thanks for your listening!

If you need further information:

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