Mechanics of deformable bodies
COE – 3001
Torsion
Homework #4

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Modeling assumptions

Unless otherwise stated, the following assumptions of strength of materials are adopted throughout this assignment:

- material is continuous, homogeneous, and isotropic,
- deformations are small,
- linear elastic behavior,
- stresses and strains are uniformly distributed,
- loads and supports are perfectly idealized.

Exercise I Questions on the course

Do you feel confident about your knowledge of shear loading? Please answer each question to assess your understanding.

1. In a circular shaft subjected to a torque T, the shear stress varies linearly with the radial coordinate ρ.

☐ True ☐ False

2. The shear stress τ is zero at $\rho = 0$.

☐ True ☐ False

3. The maximum shear stress is given by $\tau_{\max} = \dfrac{TR}{I_G}$.

☐ True ☐ False

4. The torsion relation for a circular shaft can be written $\dfrac{T}{I_G} = \dfrac{\tau}{\rho} = \dfrac{G\phi}{L}$.

☐ True ☐ False

5. The polar moment of inertia I_G depends only on the material.

☐ True ☐ False

6. For a solid circular shaft of radius R, $I_G = \dfrac{\pi R^4}{2}$.

☐ True ☐ False

7. The angle of twist ϕ is proportional to the applied torque T.

☐ True ☐ False

8. The angle of twist is given by $\phi = \dfrac{TL}{GI_G}$.

☐ True ☐ False

9. Increasing the shaft length L decreases the angle of twist ϕ, all other parameters being constant.

☐ True ☐ False

10. The shear strain in torsion is given by $\gamma = \rho\dfrac{\phi}{L}$.

☐ True ☐ False

11. The maximum shear strain occurs at $\rho = R$.

☐ True ☐ False

12. In pure torsion of a circular shaft, cross sections remain plane and perpendicular to the axis after deformation.

☐ True ☐ False

13. In non-circular shafts, torsion generally causes warping of cross sections.

☐ True ☐ False

14. The torsional stiffness of a shaft is defined as $k_t = \dfrac{T}{\phi} = \dfrac{GI_G}{L}$.

☐ True ☐ False

15. If T is constant along the shaft, the shear stress distribution does not vary along L.

☐ True ☐ False

16. For two shaft segments in series subjected to the same torque T, the total angle of twist is the sum of the individual angles of twist.

☐ True ☐ False

17. In SI units, torque T is expressed in N.

☐ True ☐ False

18. The shear modulus G affects the angle of twist but not the shear stress distribution $\tau(\rho)$.

☐ True ☐ False

19. If the radius R of a solid circular shaft is doubled, the polar moment of inertia I_G is multiplied by 16.

☐ True ☐ False

20. The elastic torsion theory presented here is valid only within the linear elastic regime.

☐ True ☐ False

Exercise II Torsion of a circular cylinder

A circular cylinder of diameter $D = 50\,\text{mm}$ and length $L = 500\,\text{mm}$ is subjected to a torsional torque $C = 2.5\,\text{kN m}$ (see Figure 1). The material has a shear modulus $G = 78\,\text{GPa}$.

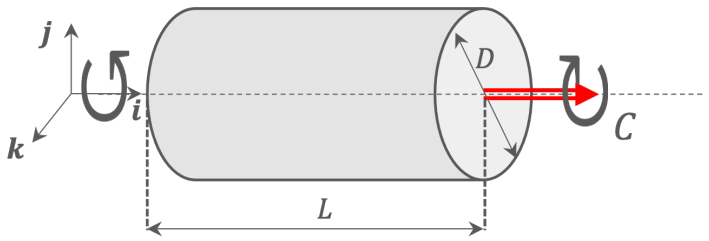

Figure 1: Circular cylinder subjected to a torsional torque C.

1. Compute the maximum shear stress in the cylinder.
2. Determine the maximum shear strain of the generatrices, $\gamma_{\max}$, expressed in radians and in degrees.
3. Determine the maximum angle of twist of the end cross sections, $\phi_{\max}$, expressed in degrees.

Exercise III Design of a transmission shaft in torsion

A shaft AB of length $L = 1.2\,\text{m}$, with a constant circular cross section, must transmit a power $P = 24\,\text{kW}$ from an electric motor to a coupling sleeve at a rotational speed $n = 1600\,\text{min}^{-1}$ (rpm).

Assume an elastic limit in shear $\tau_e = 200\,\text{MPa}$, a safety factor $s = 5$, and a shear modulus $G = 80\,\text{GPa}$.

1. Determine the required shaft diameter.
2. Determine the unit angle of twist θ between A and B.

Exercise IV Torsion of two connected shafts

The maximum shear stress in shafts AB and CD is $\tau_{\max} = 55\,\text{MPa}$ (see Figure 2). Compute:

1. the applied torque C_0,
2. the diameter d,
3. the rotation of cross section A (in °) relative to cross section D, assuming $G = 80\,\text{GPa}$.

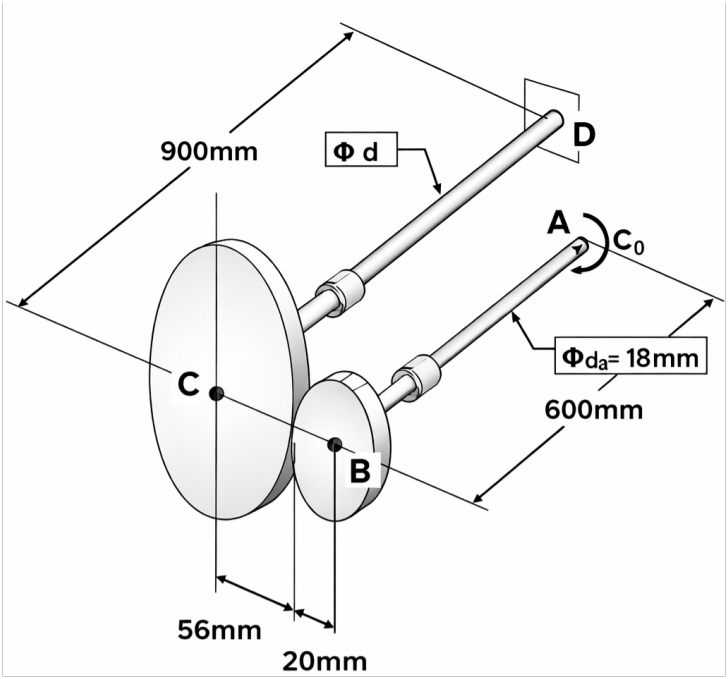

Figure 2: Two connected shafts AB and CD in torsion.

Exercise V Stepped steel shaft in pure torsion

Consider a stepped steel shaft with shear modulus $G = 80\,\text{GPa}$ and total length $L = 1.20\,\text{m}$. The shaft consists of three consecutive segments with diameters $D_1 = 40\,\text{mm}$, $D_2 = 30\,\text{mm}$ and $D_3 = 20\,\text{mm}$ (see Figure 3). The shaft is subjected to pure torsion by a torque M_t.

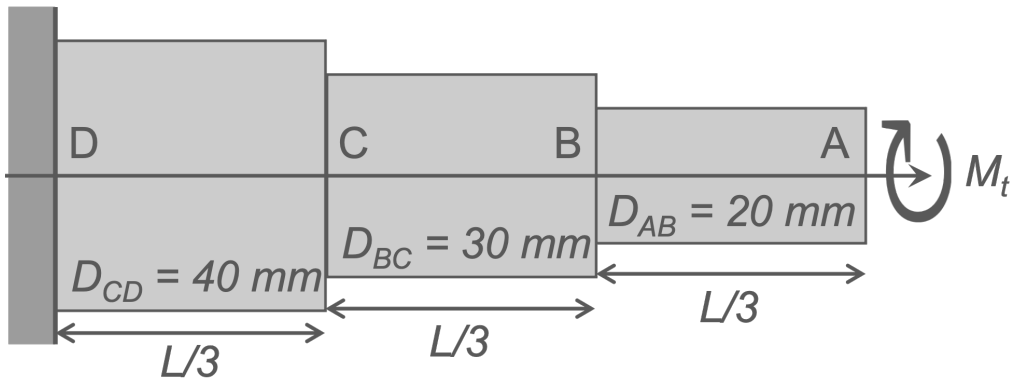

Figure 3: Stepped shaft in pure torsion.

1. Provide the expression of the shear stress in a cross section S of the shaft.
2. What torque magnitude M_t is required so that the end cross sections S_A and S_D rotate by 10° relative to each other?
3. What is the maximum shear stress?

Exercise VI Comparison between solid and hollow shafts in torsion

To transmit a torque $T = 400\,\text{N m}$, a cylindrical shaft is considered, either solid or hollow. Both shafts are made of the same steel with shear yield strength $\tau_e = 240\,\text{MPa}$ and shear modulus $G = 80\,\text{GPa}$.

In both cases, the same safety factor $s = 3$ is adopted.

The solid shaft has diameter D_1. The hollow shaft has outer diameter D_2 and inner diameter d_2 such that $d_2 = kD_2 = 0.6D_2$.

1. Determine the required diameter D_1 of the solid shaft and the angular deformation between two cross sections separated by 300 mm.

2. Determine the required diameters D_2 and d_2 of the hollow shaft and the angular deformation between two cross sections separated by 300 mm. Compare with part 1.

3. Determine the mass ratio λ of the two shafts. Conclude on the mass saving.