Mechanics of deformable bodies
COE – 3001
Traction and compression
Homework #2

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Modeling assumptions

Unless otherwise stated, the following assumptions of strength of materials are adopted throughout this assignment:

- material is continuous, homogeneous, and isotropic,
- deformations are small,
- linear elastic behavior,
- loads and supports are perfectly idealized.

Exercise I Questions on the course

Do you feel confident about your knowledge of the course? Please answer each question to assess your understanding.

1. Engineering strain uses the instantaneous length.

☐ True ☐ False

2. True stress accounts for changes in cross section during deformation.

☐ True ☐ False

3. True strain is based on the logarithm of the stretch ratio.

☐ True ☐ False

4. For small strains, engineering strain and true strain are approximately equal.

☐ True ☐ False

5. For small strains, engineering stress and true stress are approximately equal.

☐ True ☐ False

6. Engineering stress and true stress are identical in the elastic regime for any deformation level.

☐ True ☐ False

7. Under uniaxial tension, the axial strain is positive.

☐ True ☐ False

8. Under uniaxial compression, the axial strain is negative.

☐ True ☐ False

9. Under uniaxial tension, the transverse strain is usually negative for common materials.

☐ True ☐ False

10. Under uniaxial compression, the transverse strain is usually negative for common materials.

☐ True ☐ False

11. Transverse strain is related to Poisson's ratio.

☐ True ☐ False

12. In linear elasticity, Poisson's ratio can be computed as the ratio of transverse strain to axial strain with a minus sign.

☐ True ☐ False

13. For an isotropic linear elastic material, E, and ν are independent and can be chosen arbitrarily.

☐ True ☐ False

14. For an isotropic linear elastic material, the admissible range of Poisson's ratio is $-1 < \nu < 0.5$.

☐ True ☐ False

15. In a uniaxial tensile or compressive test, the stress is uniform over the cross section only if Saint-Venant's principle applies.

☐ True ☐ False

16. In uniaxial loading, shear stresses on planes perpendicular to the loading axis are zero.

☐ True ☐ False

☐ True ☐ False

17. Hooke's law in uniaxial tension can be written $\sigma = E\varepsilon$ in the linear elastic regime.

☐ True ☐ False

18. Young's modulus is the slope of the engineering stress versus engineering strain curve in the elastic regime.

☐ True ☐ False

19. A material can be stiff and weak at the same time.

☐ True ☐ False

20. Yield stress is a measure of stiffness.

☐ True ☐ False

21. Plastic deformation is fully reversible after unloading.

☐ True ☐ False

22. Brittle materials are generally stronger in compression than in tension.

☐ True ☐ False

23. In a compression test, friction at the platens can affect the measured response.

☐ True ☐ False

24. In a tensile test, necking typically causes the true stress to increase while the engineering stress may decrease after maximum load.

☐ True ☐ False

25. If the cross sectional area decreases during tension, the true stress is always lower than the engineering stress.

☐ True ☐ False

26. True stress is defined as the applied force divided by the instantaneous cross sectional area.

☐ True ☐ False

27. Uniaxial tensile testing is the most common mechanical test.

☐ True ☐ False

28. Engineering stress at a given load is always greater than true stress at the same load.

☐ True ☐ False

Exercise II Young's modulus

A cylindrical test specimen with a diameter of 12 mm and an initial length of 30 mm is subjected to uniaxial tensile loading in the elastic regime. Under an applied force of 15 kN, the specimen elongates by 0.089 mm.

1. Write the expressions of the engineering stress and engineering strain.

2. Determine the Young's modulus E of the material.

3. Comment on the order of magnitude of the obtained value.

Exercise III Lateral strain under compression

A steel bar with the following properties:

- Young's modulus: $E = 205\,\text{GPa}$,
- Poisson's ratio: $\nu = 0.30$,
- Diameter: 5 mm,
- Length: 2 m,

is subjected to a compressive force of 5000 N.

1. Determine the lateral (transverse) strain of the bar.

2. Deduce the corresponding change in diameter.

3. Explain physically why a transverse deformation appears.

Exercise IV Strain transformation under uniaxial loading

Before loading, a straight line is drawn on the surface of a test specimen (see Figure 1). The specimen is then subjected to a uniaxial normal stress of

$$\sigma = 130\,\text{MPa}.$$

Given:

- Young's modulus: $E = 70\,\text{GPa}$,
- Poisson's ratio: $\nu = 0.33$,

Assume plane stress conditions.

1. Determine the initial slope of the line from Figure 1.

2. Determine the axial and transverse strains.

3. Express the strain tensor.

4. Determine the new slope of the line after loading.

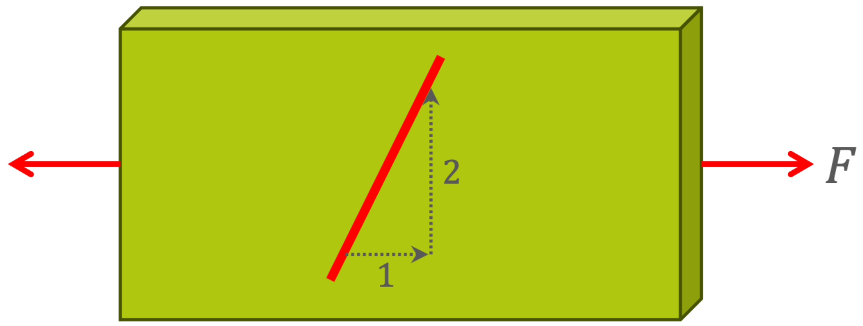

Figure 1: Line drawn on the specimen surface before loading.

Exercise V Bearing stress in a masonry wall and foundation

Consider a masonry wall resting on a foundation, as shown in Figure 2. Assume a uniform stress distribution and neglect eccentricity effects.

The material densities are:

- Masonry: $\rho_{\text{masonry}} = 2000\,\text{kg}\,\text{m}^{-3}$,
- Foundation: $\rho_{\text{foundation}} = 2500\,\text{kg}\,\text{m}^{-3}$.

The allowable bearing stresses are:

- Masonry: $\sigma_{\text{masonry}} = 100\,\text{N}\,\text{cm}^{-2}$,
- Foundation: $\sigma_{\text{foundation}} = 1000\,\text{N}\,\text{cm}^{-2}$,
- Soil: $\sigma_{\text{soil}} = 50\,\text{N}\,\text{cm}^{-2}$.

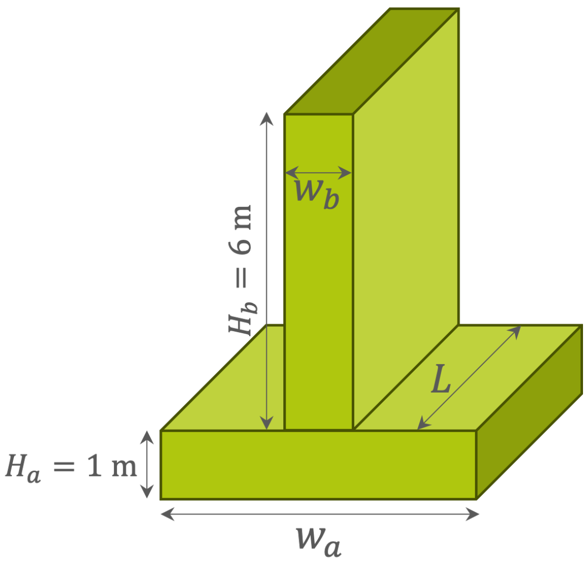

Figure 2: Masonry wall resting on a foundation (bearing stress problem).

The axial load applied at the top of the wall is 300 kN per meter length.

Determine the required wall width w_a and the foundation width w_b such that all allowable stress limits are satisfied.

Exercise VI Rigid beam supported by elastic bars

Consider a rigid horizontal beam AB of length L, connected at points A and B to two vertical elastic cylindrical bars AA' and BB', which are fixed at their lower ends (see Figure 3). The self weights of the beam and the bars are neglected.

The beam is subjected to a vertical concentrated load P applied at point C, such that:

$$AC = \frac{AB}{3} = \frac{L}{3}.$$

Given data:

- $P = 3000\,\mathrm{N}$,
- Length of bar AA': $L_A = 500\,\mathrm{mm}$,
- Length of bar BB': $L_B = 700\,\mathrm{mm}$,
- Young's modulus: $E = 2.0 \times 10^5\,\mathrm{N\,mm^{-2}}$,
- Initial cross sectional areas: $S_A = S_B = 40\,\mathrm{mm}^2$,
- Yield stress: $\sigma_y = 300\,\mathrm{N\,mm^{-2}}$,
- Safety factor: $s = 6$.

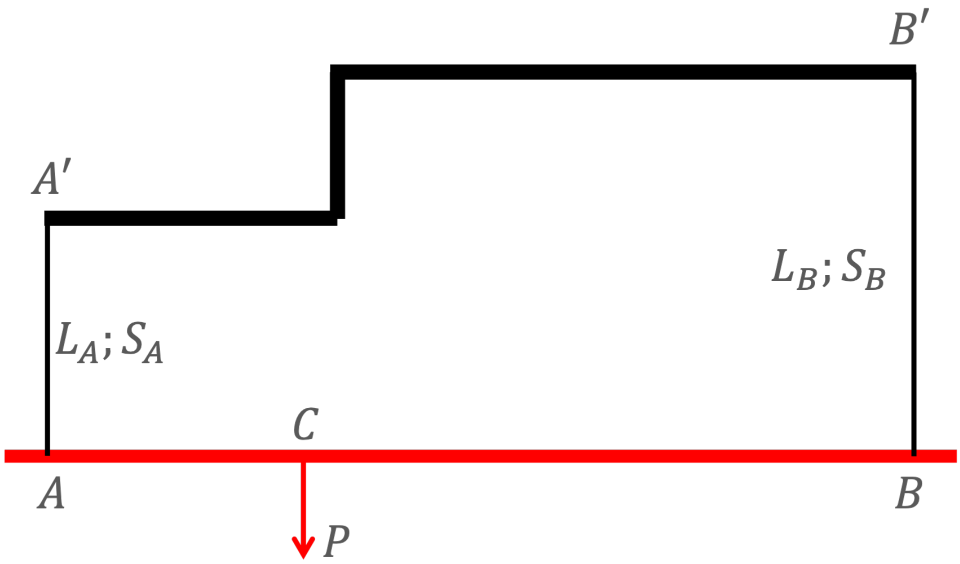

Figure 3: Rigid beam supported by two vertical elastic bars.

1. Compute the elongations ΔL_A and ΔL_B of bars AA' and BB'. Sketch the deformed shape of the beam AB after loading.
2. Determine the cross sectional area S_B required to keep the beam AB in a horizontal position.

3. Compute the allowable (working) stress σ_p.
4. Determine the cross sectional areas S_A and S_B such that both bars remain within the elastic range.

Exercise VII Determination of Young's modulus from a compression curve

A material is subjected to a uniaxial compression test. The measured stress–strain curve presents a linear elastic regime, followed by a deviation from linearity.

In the elastic regime, the measured slope E_{mes} of the stress–strain curve is affected by the compliance of the testing machine. The following relation holds:

$$\frac{1}{E_{\text{mes}}} = \frac{1}{E} + \frac{1}{M},$$

where:

- E is the Young's modulus of the material,
- M is the apparent stiffness of the testing system, given by

$$M = \frac{R\,h_0}{S_0},$$

- R is the stiffness of the machine,
- h_0 is the initial height of the specimen,
- S_0 is the initial cross-sectional area of the specimen.

The specimen has the following geometry:

- initial height: $h_0 = 5.02\,\text{mm}$,
- cross-sectional area: $S_0 = 3.87\,\text{mm}^2$.

The stiffness of the testing machine is:

$$R = 71\,\text{MPa}\,\text{m}^{-1}.$$

From the measured stress–strain curve, the apparent Young's modulus is:

$$E_{\text{mes}} = 70\,\text{GPa}.$$

1. Explain why the measured modulus E_{mes} is different from the true Young's modulus E of the material.
2. Compute the stiffness M of the testing system.
3. Deduce the true Young's modulus E of the material.
4. Comment on the order of magnitude of the obtained value.
5. Explain why it is important to correct experimental stress–strain curves when determining elastic properties.

Exercise VIII Deformation of a thin GaN film on sapphire

A thin gallium nitride (GaN) film is grown by epitaxy on a thick sapphire substrate. During cooling after growth, a thermal expansion mismatch generates stresses in the film and induces bending of the wafer.

- Substrate (sapphire): $t_s = 430\,\mu\text{m}$, $E_s = 345\,\text{GPa}$, $\nu_s = 0.25$, $\alpha_s = 7.5 \times 10^{-6}\,\text{K}^{-1}$.
- Film (GaN): $t_f = 2.0\,\mu\text{m}$, $E_f = 300\,\text{GPa}$, $\nu_f = 0.20$, $\alpha_f = 5.6 \times 10^{-6}\,\text{K}^{-1}$.

1. Assuming linear isotropic elasticity, equi-biaxial in-plane stress ($\sigma_x = \sigma_y = \sigma_f$) and plane stress ($\sigma_z = 0$), show that:

$$\varepsilon = \frac{1 - \nu_f}{E_f}\sigma_f + \alpha_f \Delta T.$$

2. Compute the relative thermal strain between the film and the substrate.

3. Determine the biaxial stress in the film.