The stereographic projection

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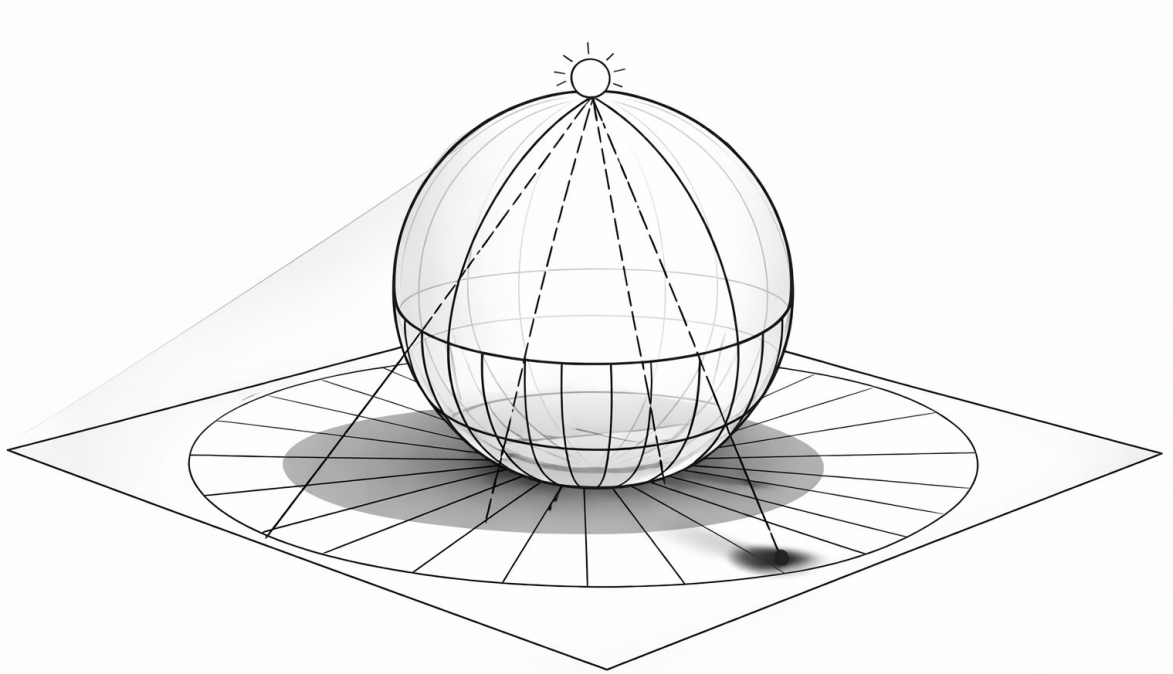
Basics

❖ **Light and shadow analogy:**

- A sphere is illuminated by a point light source
- The light source is placed at the north pole of the sphere
- Each point on the sphere is connected to the light by a straight ray
- The ray intersects a flat plane and creates a shadow point

↳ All shadow points form the stereographic projection



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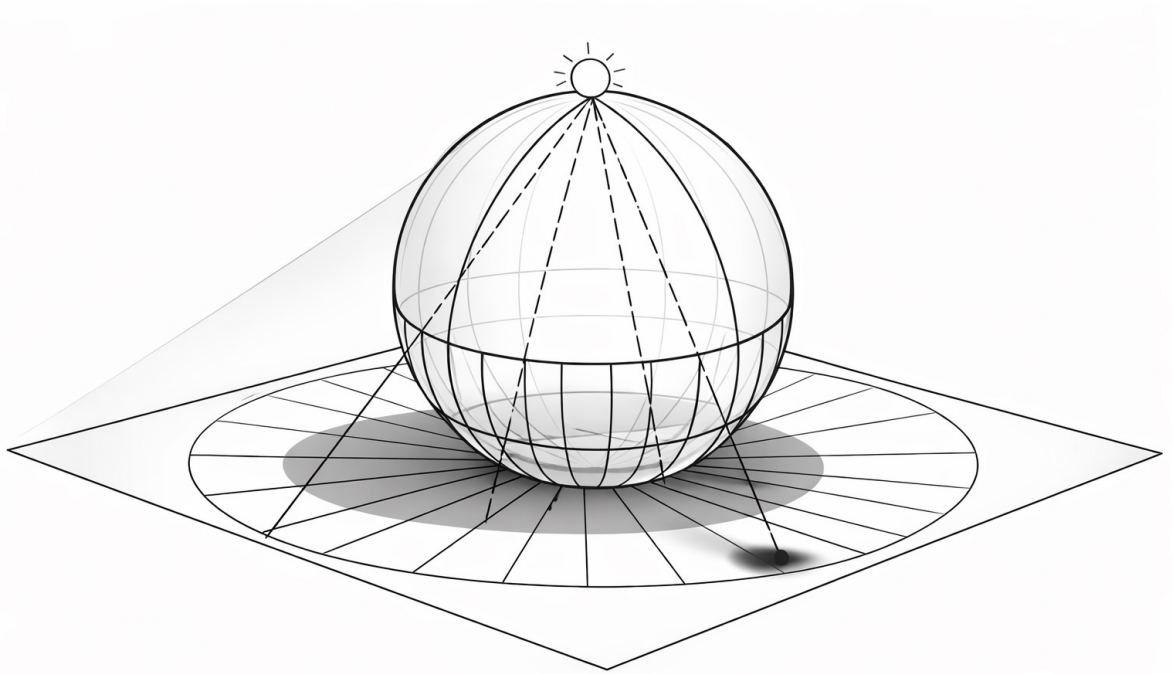
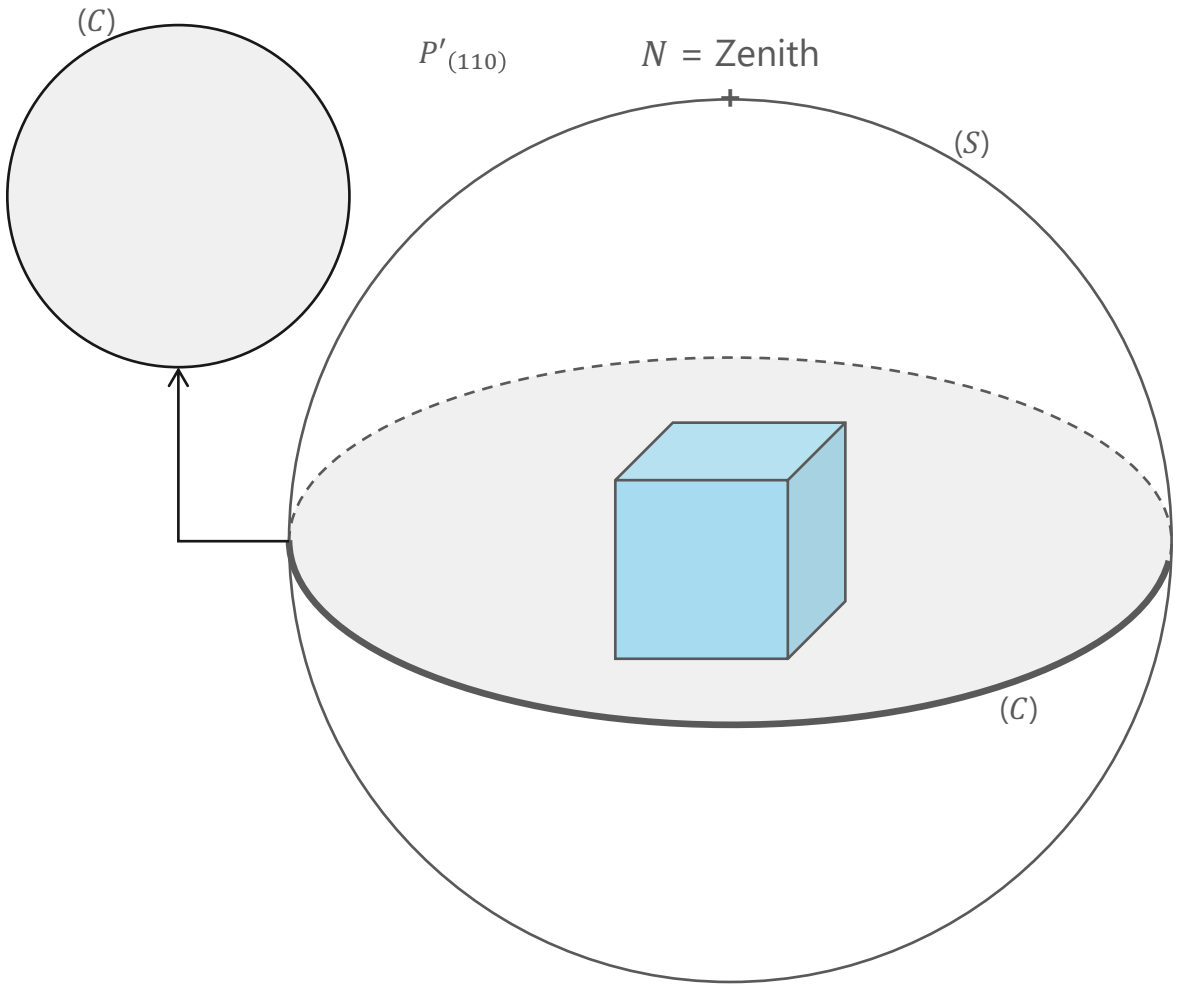


❖ Key geometric property:

- Angles between directions are preserved
- Lengths and areas are not preserved

Stereographic projection of a plane via its normal



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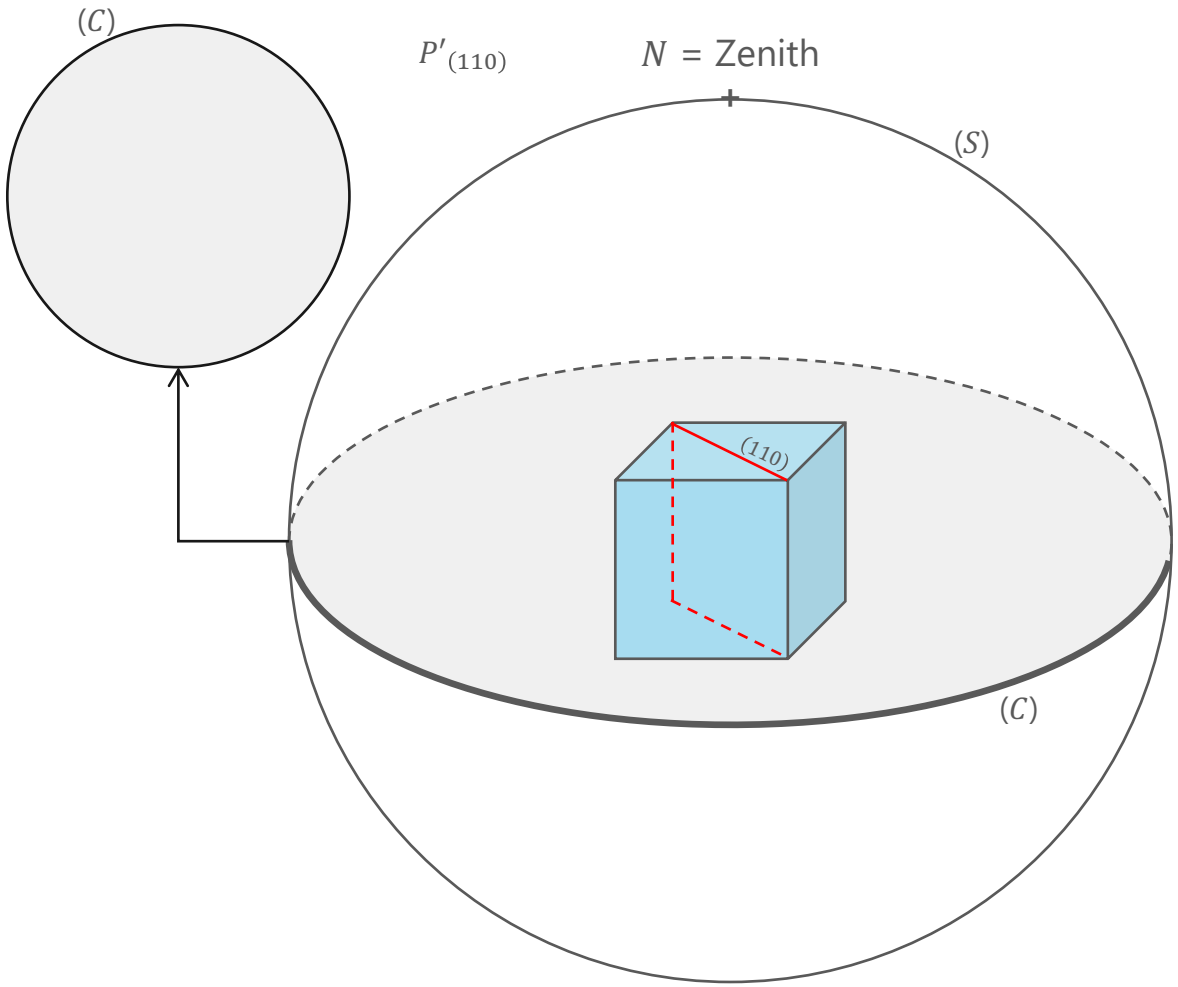

- Consider the crystal placed at the center of a reference sphere (S).
- Identify the crystallographic plane of interest, e.g., the (110) plane.

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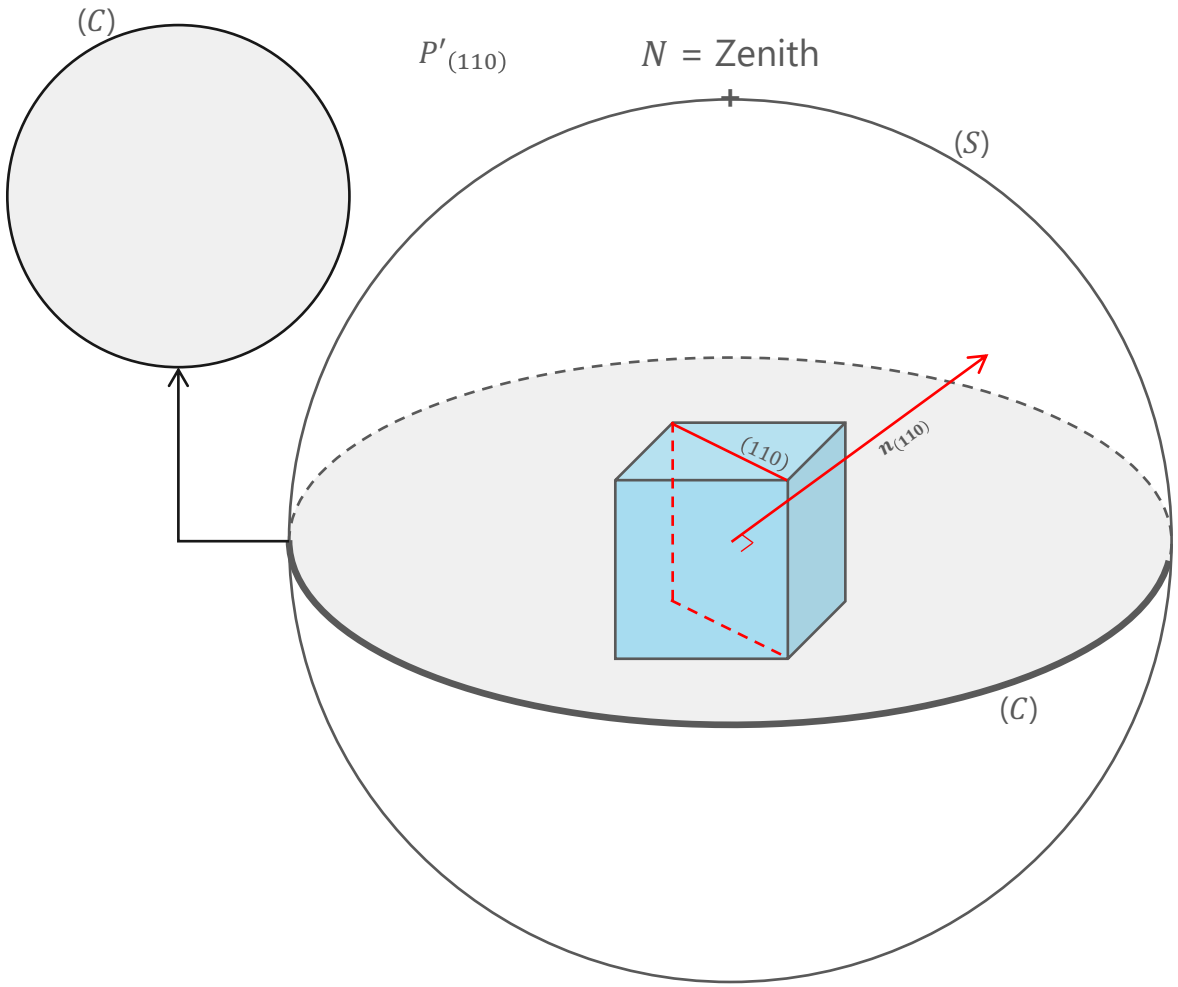

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- Draw the normal vector ($\boldsymbol{n}_{(110)}$) to this plane, starting from the center of the sphere.

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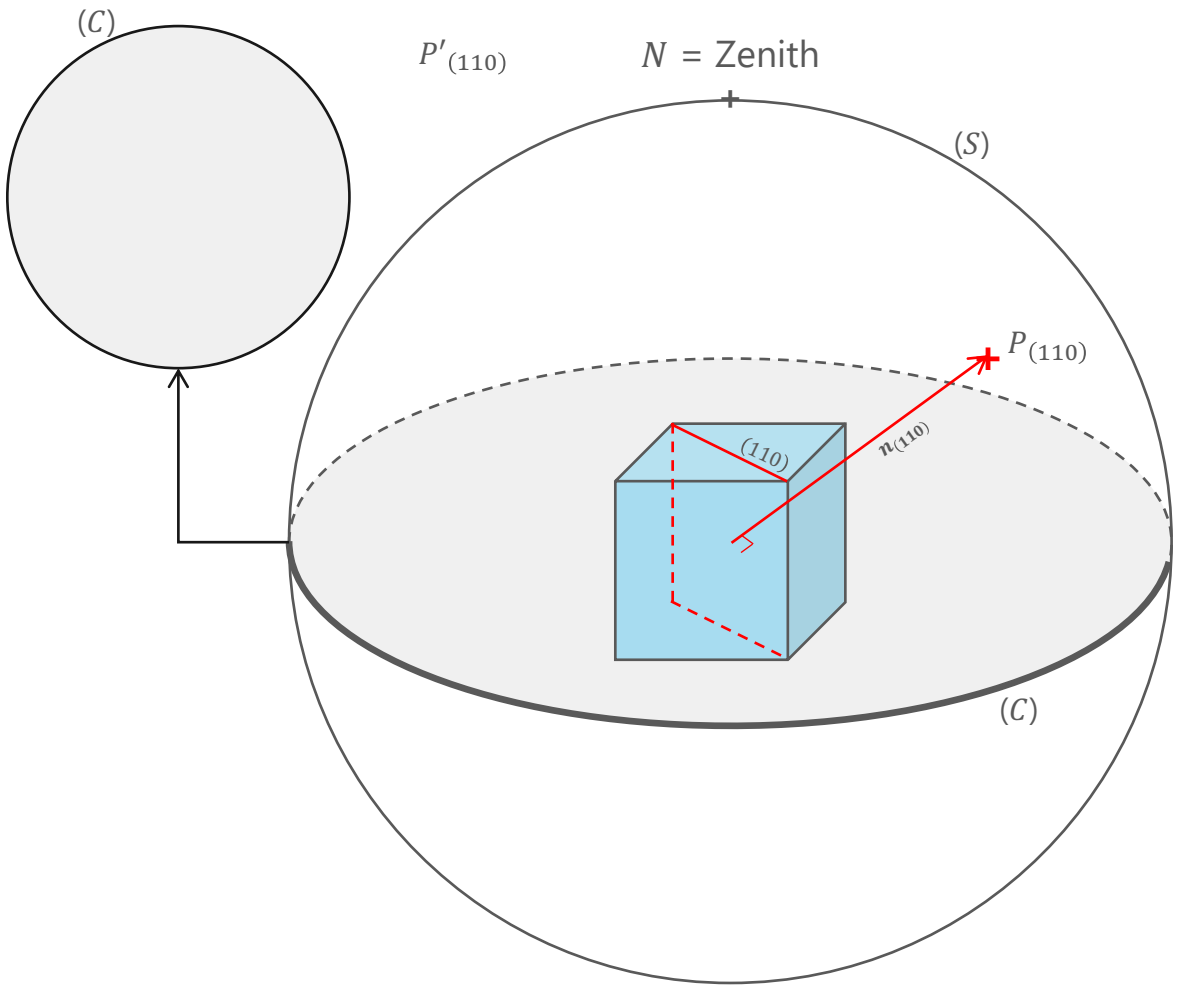

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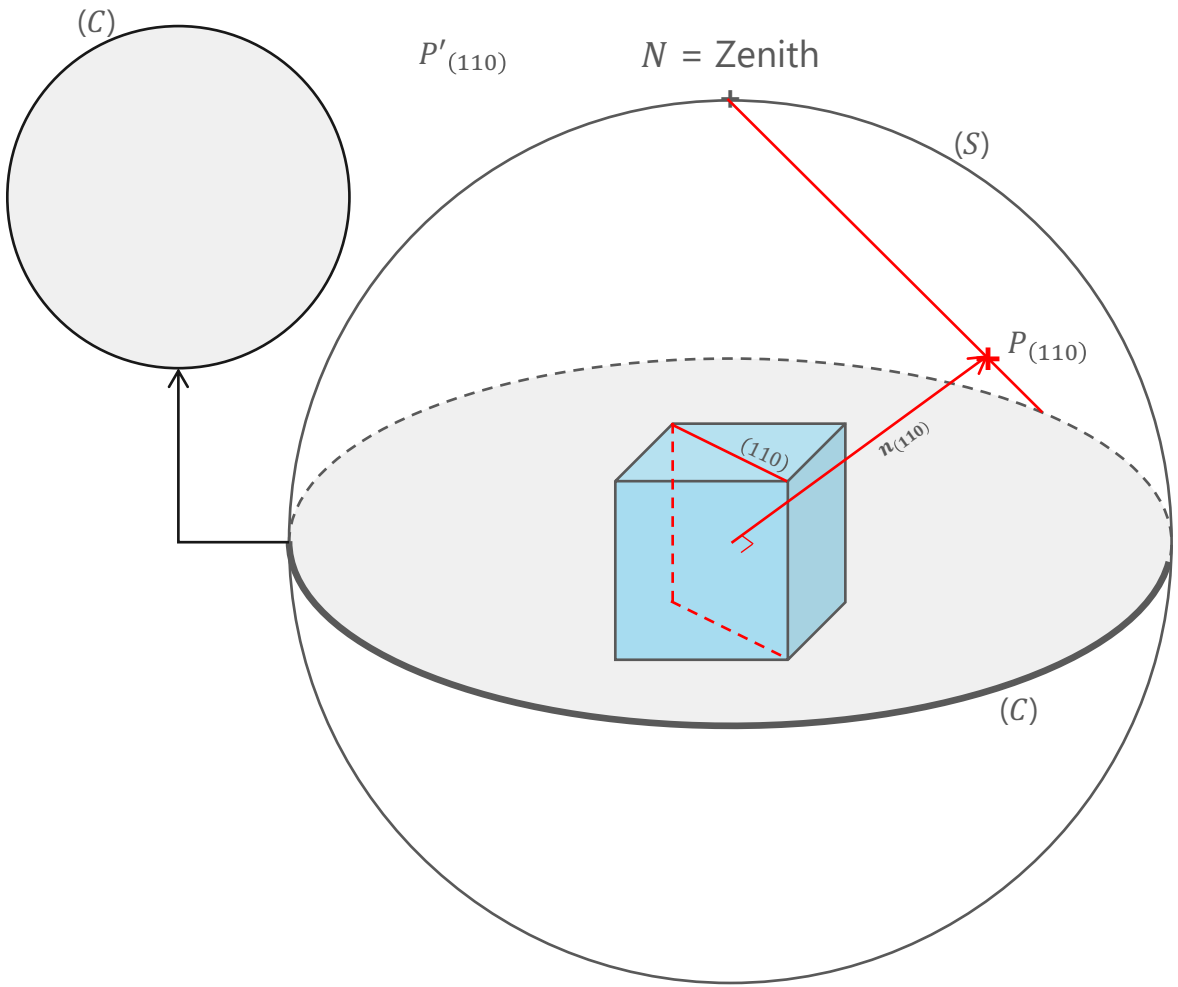

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- Draw the normal vector ($\boldsymbol{n}_{(110)}$) to this plane, starting from the center of the sphere.
- This normal intersects the sphere at a point $P_{(110)}$ on its surface.
- Project point $P_{(110)}$ onto the equatorial plane (C) along a straight line from the North pole (N) of the sphere.

↳The North pole is used as the projection point.
↳A direction in the upper hemisphere projects outside the primitive circle;
↳It is therefore replaced by its antipode ($\boldsymbol{n} \rightarrow -\boldsymbol{n}$), projected from the North pole onto the equatorial plane, and displayed as an open circle

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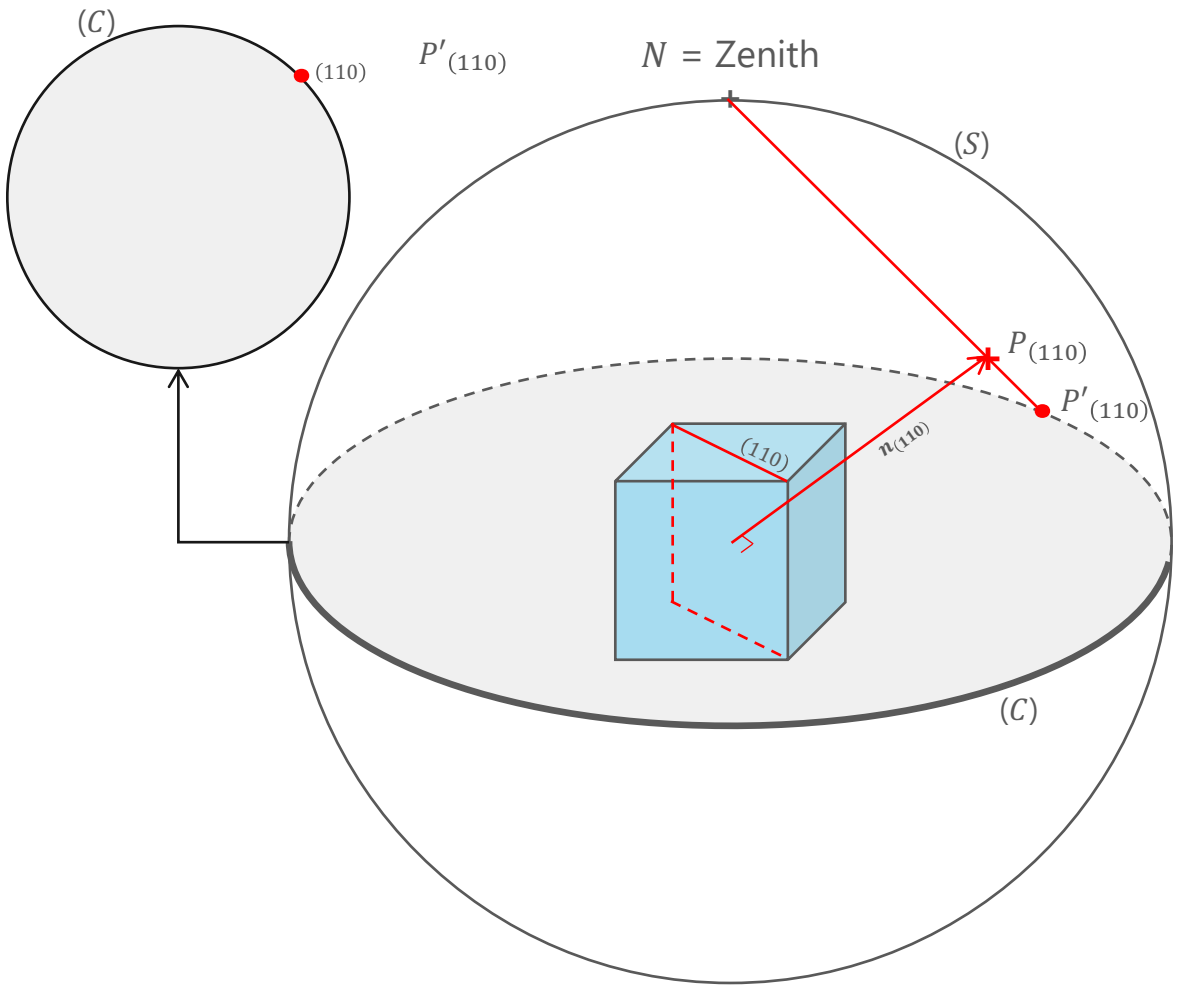

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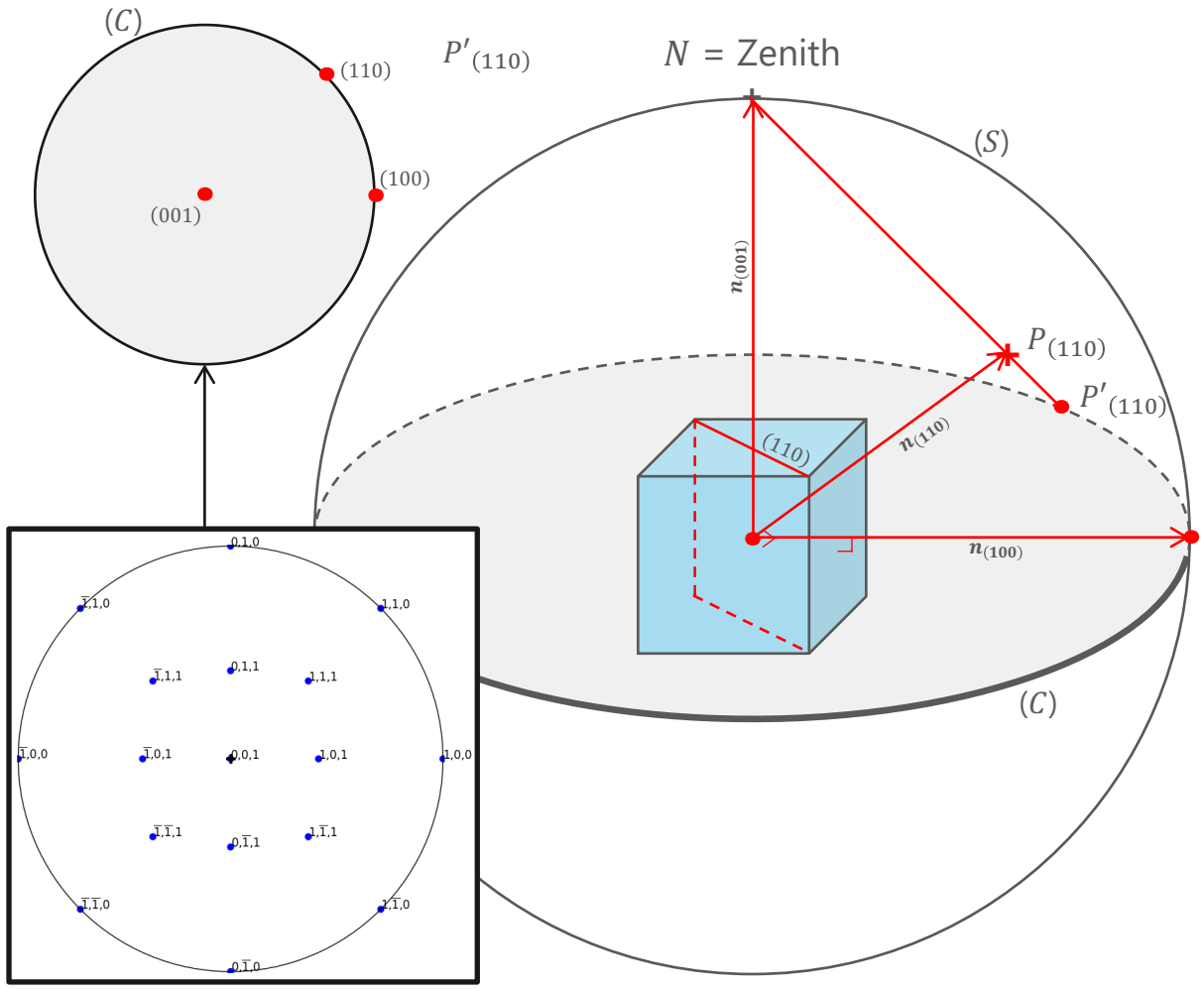

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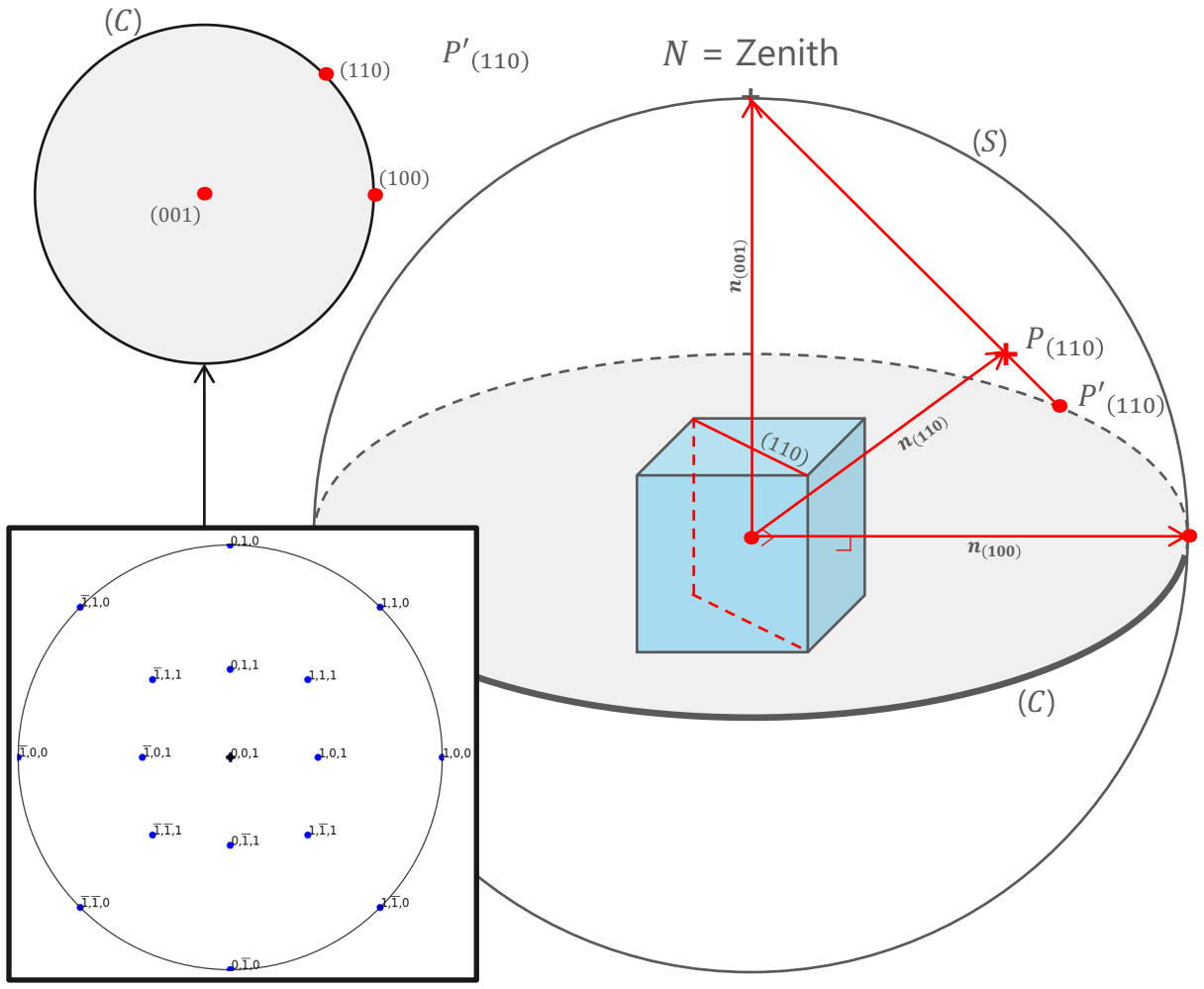

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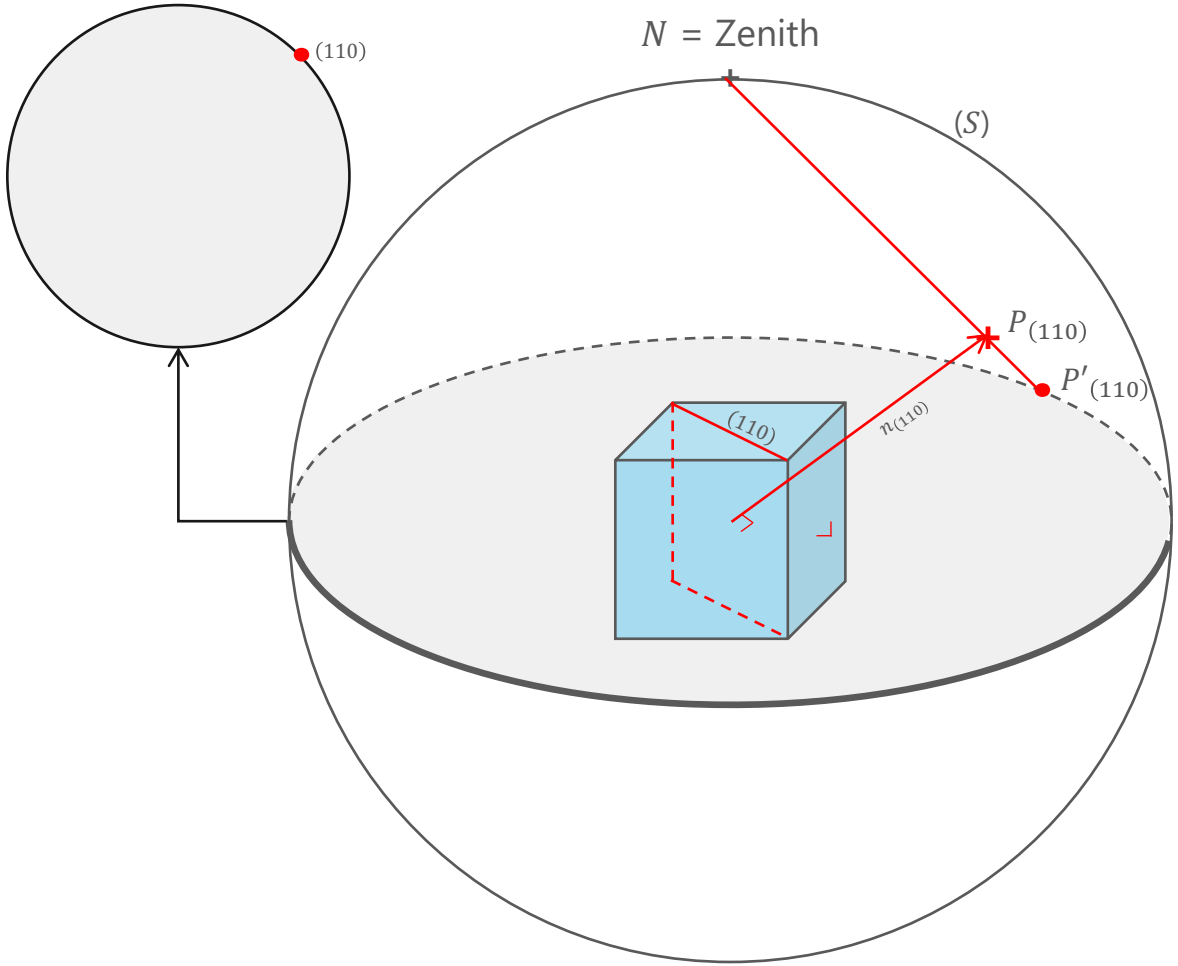
↳ Each crystallographic direction is represented by a single point on the stereographic projection

Stereographic projection of a plane via its trace



- Place the crystal at the center of the reference sphere (S).
- Identify the (110) crystallographic plane within the crystal.

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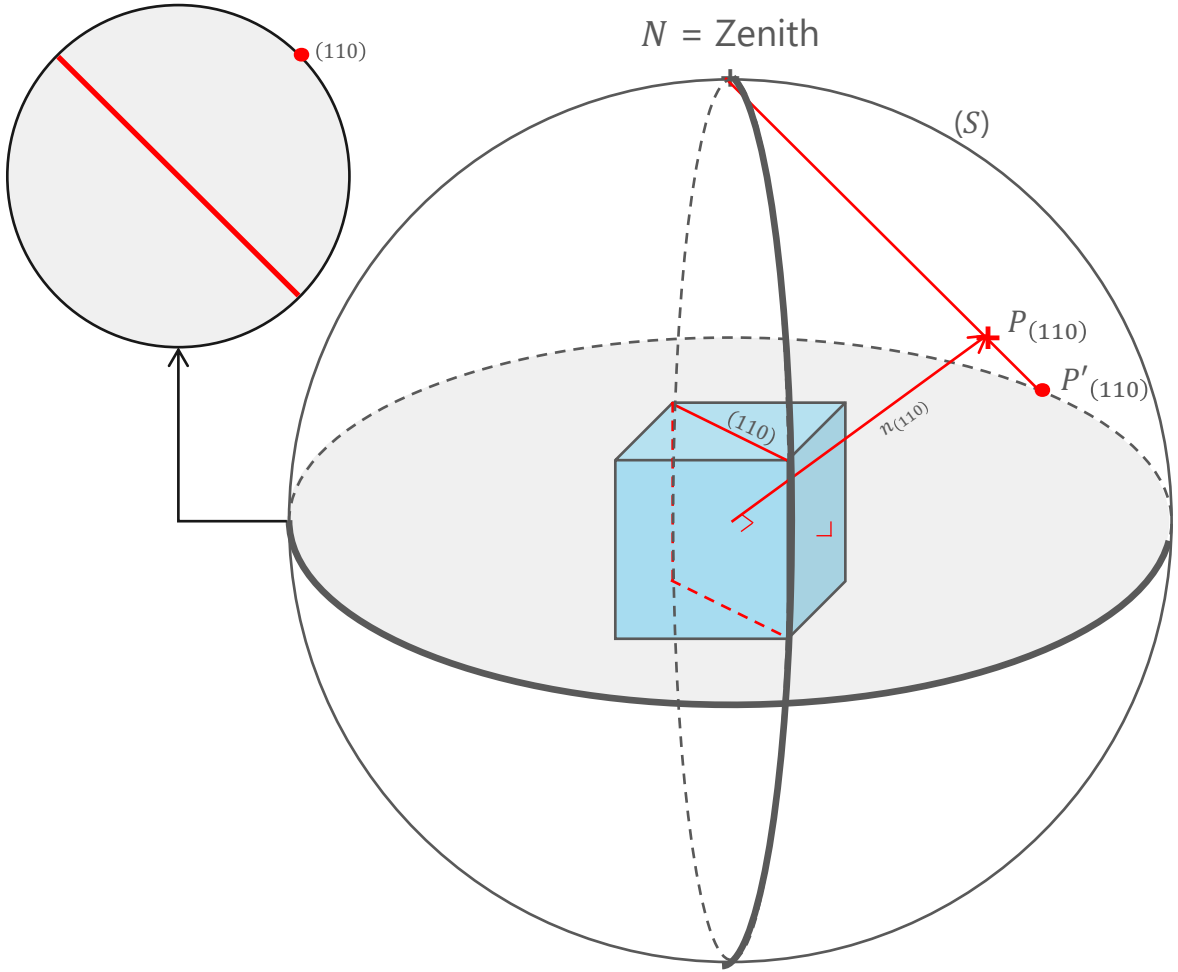

- Place the crystal at the center of the reference sphere (S).
- Identify the (110) crystallographic plane within the crystal.
- Imagine the intersection between this plane and the sphere: this defines a great circle on the sphere's surface.
- This great circle represents all directions lying within the (110) plane.

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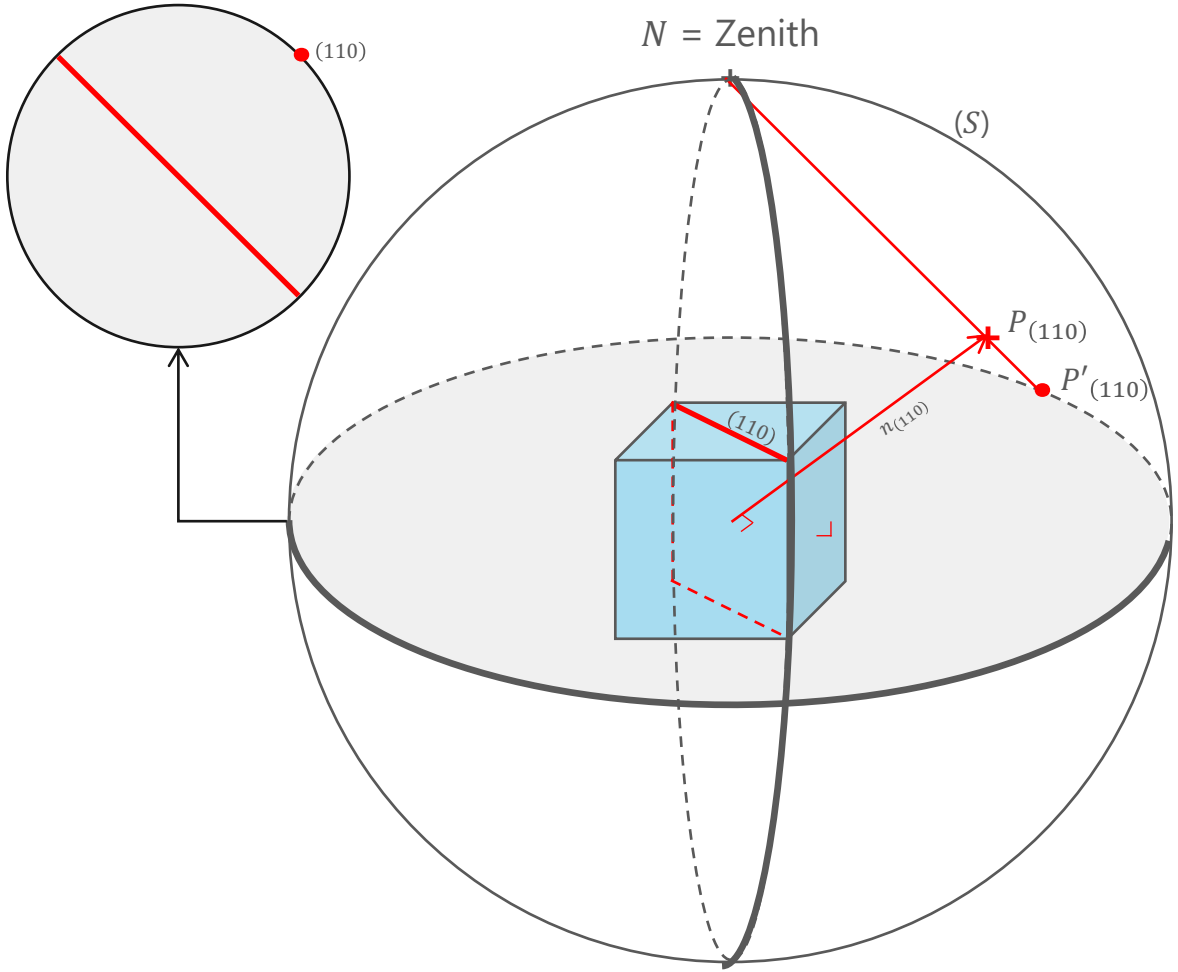

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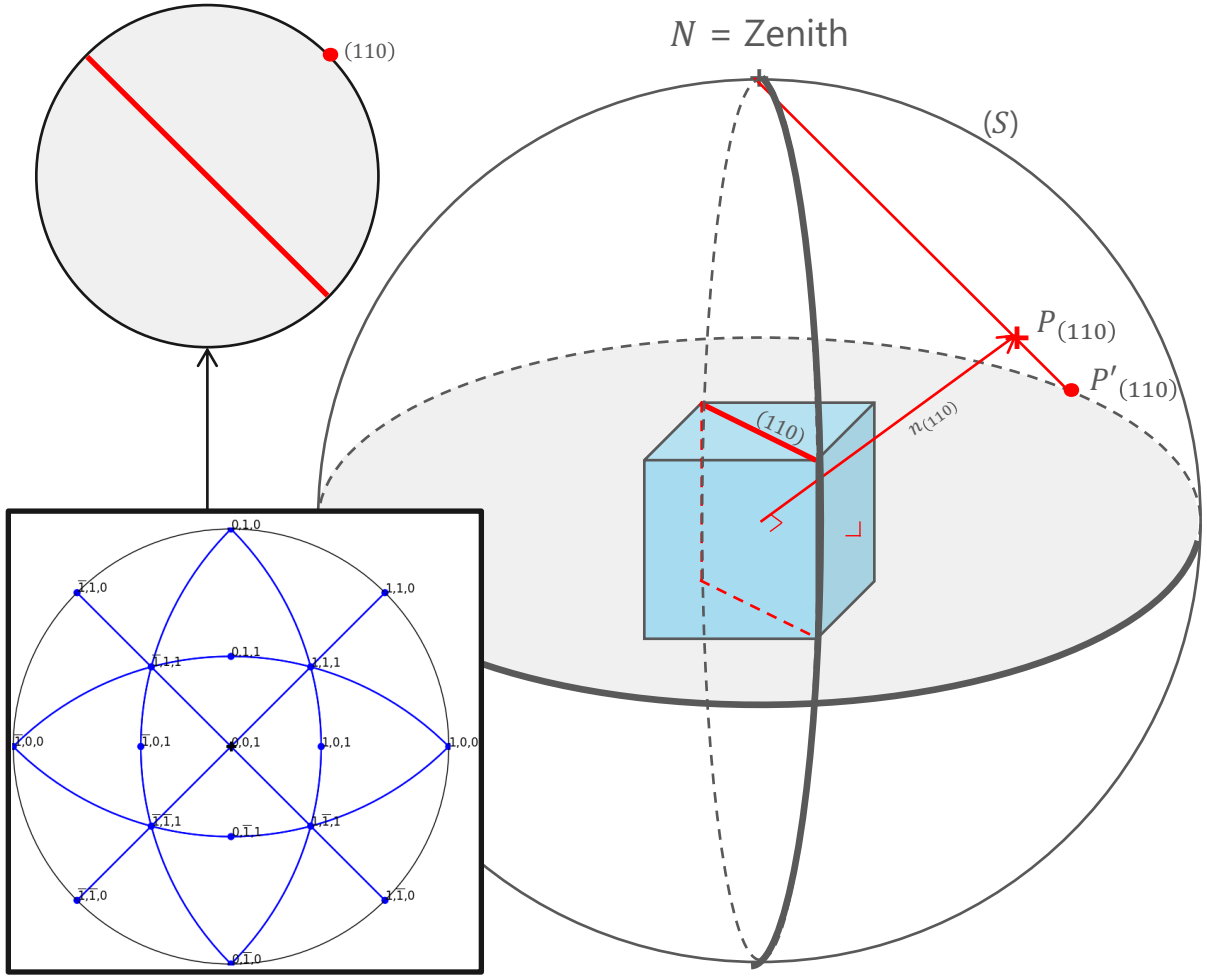

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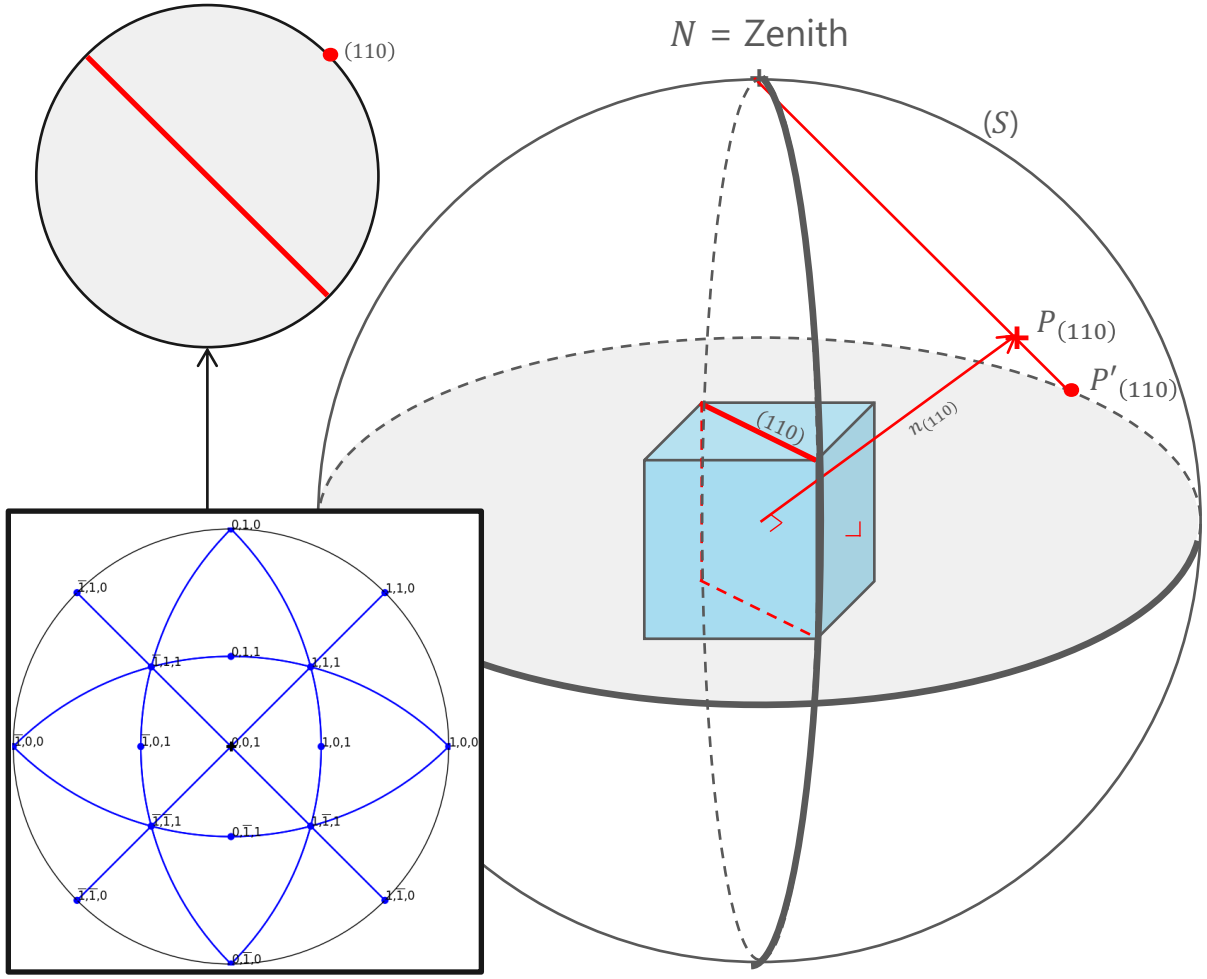

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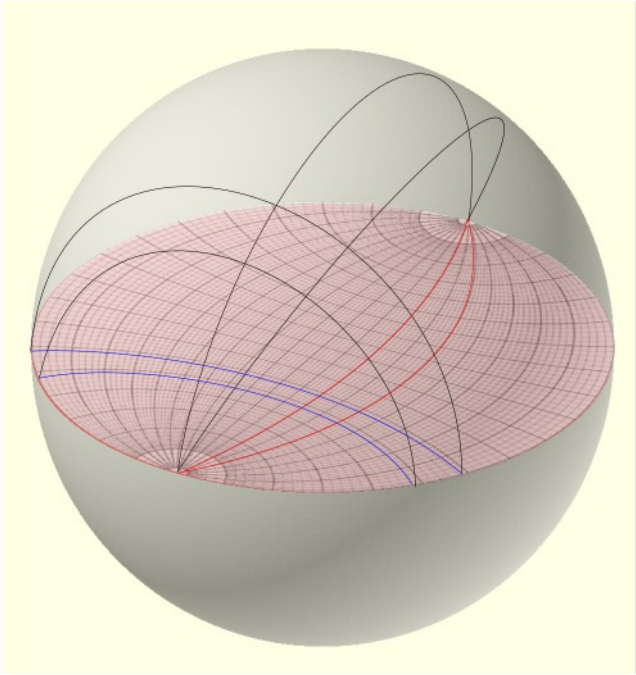
↳ The great circle trace shows all directions lying in the plane on the 2D projection.

The Wulff net



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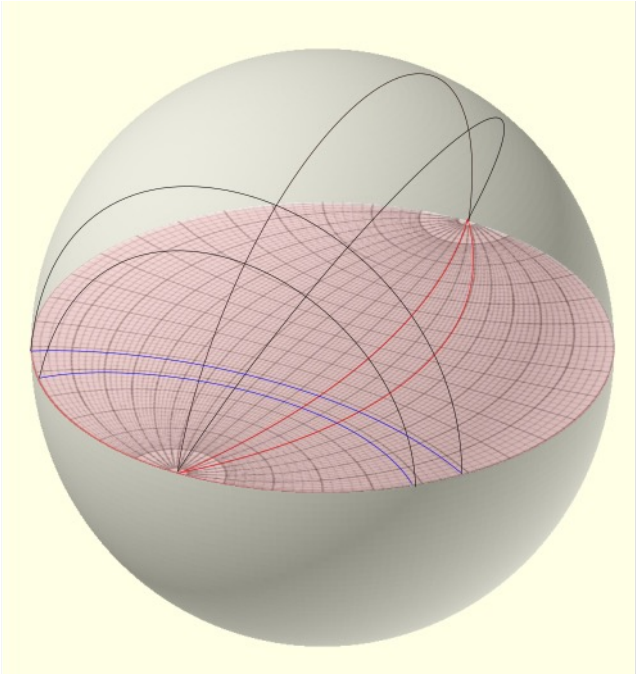

- The Wulff net is built using a stereographic projection of a reference sphere.
- It consists of a grid made up of two families of planes:
  - **Meridians:** represent planes with a constant azimuthal (rotation around the vertical axis) angle; they appear as circular arcs connecting the North and South poles.
  - **Parallels:** represent planes with a constant inclination angle from the vertical axis; they appear as concentric arcs or circles.

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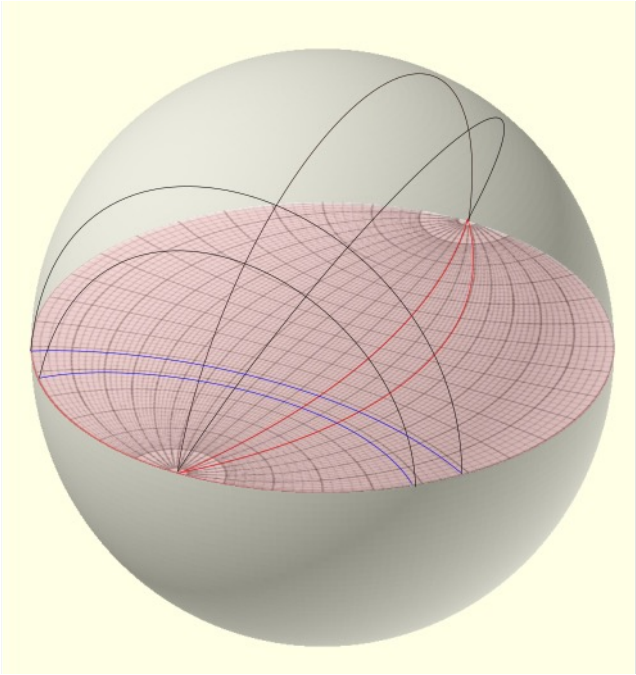

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- These meridians and parallels are equally spaced on the sphere before projection.

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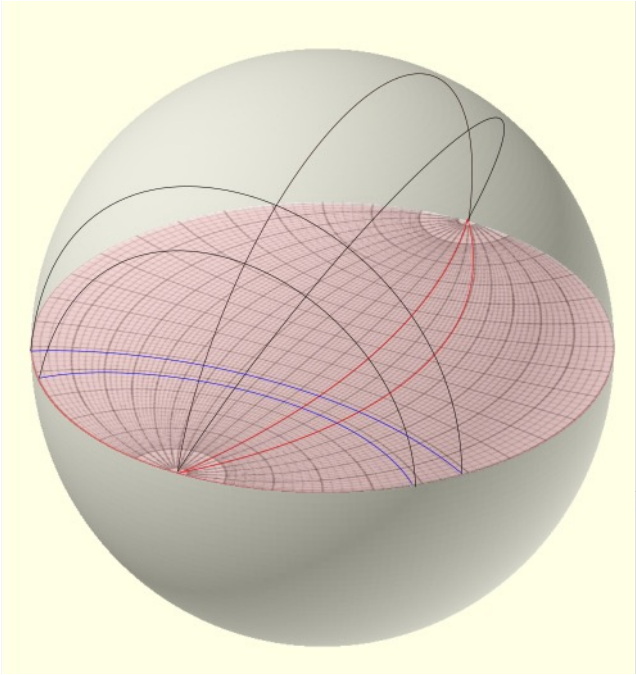

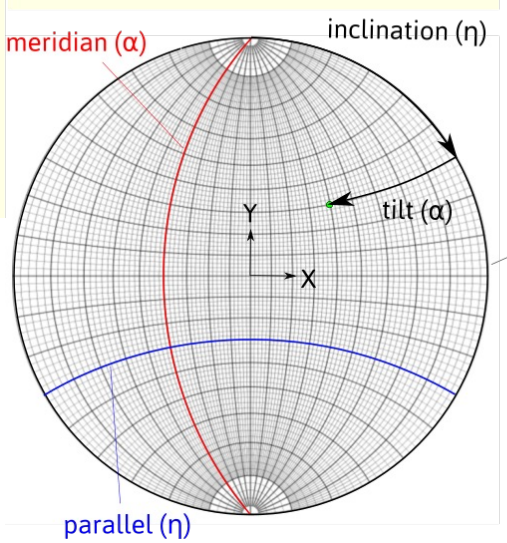

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- These meridians and parallels are equally spaced on the sphere before projection.
- The stereographic projection transforms them into the curved grid pattern of the Wulff net.

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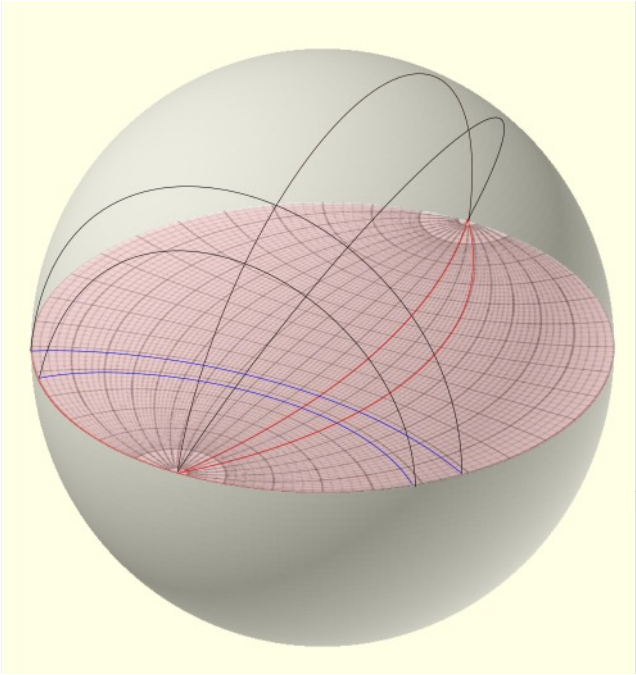

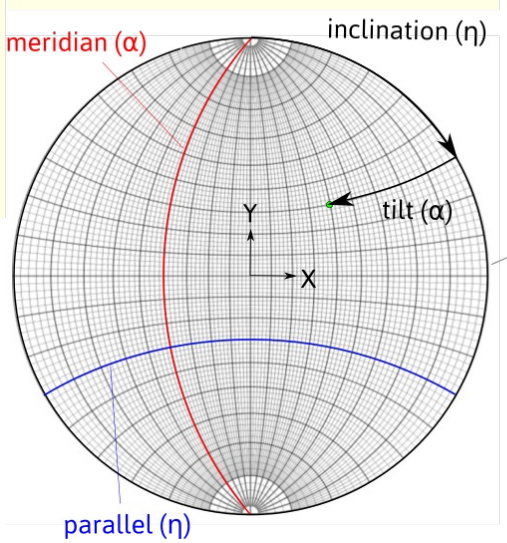

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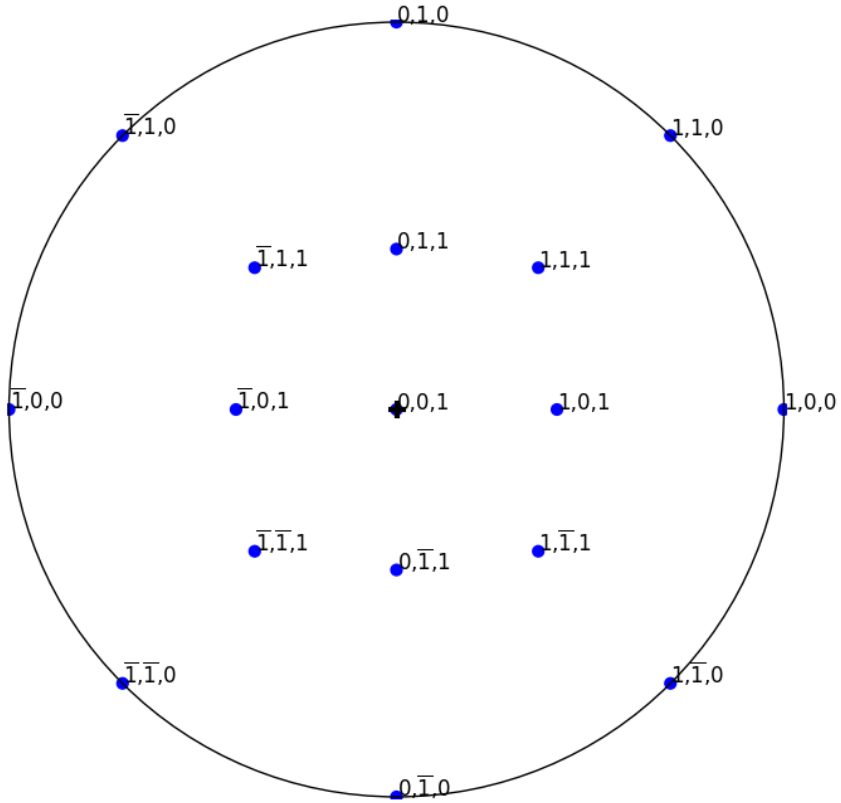
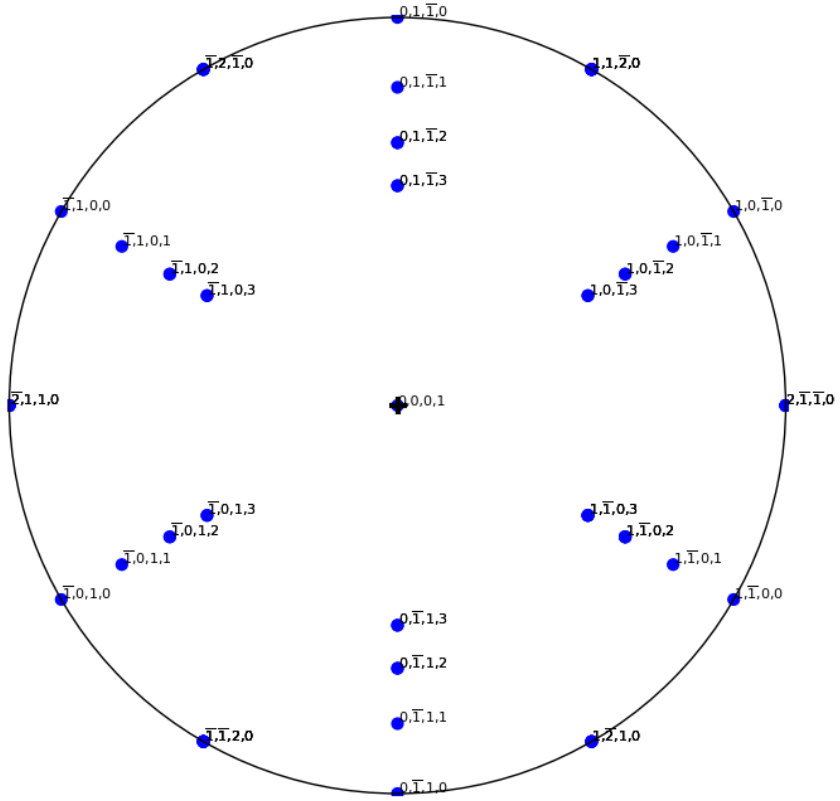
↳ The Wulff net serves as a reference tool to measure angles and construct stereographic projections accurately.

Some examples

Cubic system



Hexagonal system



Some exercises: Al-(001)

❖ Angles between directions

- Measure the angle between $[101]$ and $[100]$.
- Measure the angle between $[112]$ and $[011]$.
- Determine the angle between $[111]$ and $[1\bar{1}\bar{1}]$.

❖ Angles between planes:

- Measure the angle between (111) and (112).

❖ Direction-plane angles:

- Measure the angle between the direction $[111]$ and the plane (112).

❖ Great circles and plane traces

- Determine the trace of the plane (111) on the projection.
- Determine the trace of the plane (110).

❖ Intersections:

- Determine the intersection between the planes (111) and $(1\bar{1}1)$.
- Determine the intersection between (111) and (110).

❖ Belonging and families:

- Verify whether the direction $[1\bar{1}0]$ belongs to the plane (111).
- Identify all $\langle 110 \rangle$ directions contained in the plane (111).

❖ Crystal rotations and misorientation:

- Determine the rotation axis and rotation angle required to bring $[001]$ onto $[111]$.
- Determine the misorientation angle between crystal orientations $[001]$ and $[112]$, and identify the rotation axis.
- Determine the smallest rotation transforming $[111]$ into $[1\bar{1}1]$.

❖ Zone axes:

- Determine the zone axis common to the planes (111) and $(1\bar{1}1)$.
- Determine the zone axis common to the planes (111), $(1\bar{1}1)$ and $(11\bar{1})$.
- Identify all planes belonging to the zone axis $[110]$.

Thanks for your listening!

If you need further information:

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